

# COMPARISON OF TRANSITION RATES WITH THE LG (0, 1)\* SPIRAL-PHASE MODE FIELD DISTRIBUTION IN THE FRAME OF THREE IONIZATION THEORIES

TATJANA B. MILADINOVIĆ<sup>1</sup>, NEBOJŠA S. DANILOVIĆ<sup>2</sup>

<sup>1</sup>Institute for Information Technologies Kragujevac, University of Kragujevac, Kragujevac, Serbia

<sup>2</sup>Faculty of Science, University of Kragujevac, Kragujevac, Serbia

## ABSTRACT

We discussed the tunneling ionization of an Argon atom placed in a strong low-frequency field of Ti: Sapphire laser. The transition rate of the electron obtained with radial polarization LG (0, 1)\* spiral-phase mode field distribution included are compared to the basic transition rate. All analyses are given in the frame of the three different ionization theories – Keldysh, PPT, and ADK. We demonstrated that the tunneling transition rate is sensitive to laser polarization and a set field distribution. As well as changes in the azimuthal angle as a parameter on which the given distribution depends.

**Keywords:** Tunneling ionization, Transition rate, Radially polarized LG (0, 1)\* spiral-phase mode.

## INTRODUCTION

The discovery of the laser in 1960 and the development of laser technology enabled research in several of areas. The study of the behavior of atomic and molecular systems that are exposed to the strong electromagnetic field of a high-power laser with a long or short pulse is one of those areas. First scientific papers dealing with these topics were written in the early 1970's (Keldysh, 1965; Voronov et al., 1966). During years the research was continued and the subject was expanded from the tunnel to multiphoton ionization (Xiong et al., 1991; Mainfray et al., 1991), barrier suppression (BSI) (Krainov et al., 1995) and above threshold ionization (ATI) (Eberly et al., 1988). These problems are still of interest to researchers, as evidenced by the latest work (Calvert et al., 2016; Lai et al., 2017; Shvetsov-Shilovski et al., 2019).

In order to explain the physical picture of laser interaction with an atom or molecule, it was necessary to give a theoretical basis. Over the years, several methods have created that can explain this physical phenomenon. Some of them are: numerically solving of the full-dimensional time-dependent Schrödinger equation (TDSE) (Parker et al., 1998; Parker et al., 2006), reduced-dimensional TDSE, strong field approximation (SFA) (Lappas et al., 1998; Liu et al., 1999), semiclassical method (Schafer et al., 1993; Corkum, 1993) and classical ensemble method (Panfili et al., 2001; Ho et al., 2006).

A semiclassical model, in which the atom is treated as a quantum object while the electromagnetic field is classical, will be used in this paper. The electron leaves the atom by tunneling while its trajectory in the field is classical. Landau and Lifshitz gave the basis and fundamental equations for the

semiclassical model for the ionization of the hydrogen atom in the ground state, exposed to the influence of the electromagnetic field (Landau & Lifshitz, 1991). Among many later papers, Keldysh's work should be highlighted. Keldysh modified the given formula by including the influence of the electromagnetic field on the free electron (Keldysh, 1965). Also, he showed that multiphoton and tunnel ionization are two processes that are very similar in nature. As the boundary between these two processes, the Keldysh defined a parameter known as the Keldysh parameter  $\gamma = (\omega\sqrt{2I_p})/F$ , where  $\omega$  is the laser frequency,  $F$  the laser electric field strength and  $I_p$ , the ionization potential of the atom. When the parameter is  $\gamma \gg 1$ , multiphoton ionization occurs, while in the case of  $\gamma \ll 1$ , tunnel ionization is the dominant process. Perelomov, Popov and Terent'ev (PPT) derived an equation that gives the tunneling transition rate of an arbitrary atom when changes in the external field can be neglected (Perelomov et al., 1966). Twenty years later, Ammosov, Delone and Krainov (ADK) extended the PPT theory to complex atoms and atomic ions (Ammosov et al., 1986).

Our research is focused on the influence of the shape of the laser pulse on the tunneling transition rate of the weakest bound electron of an Argon atom exposed to a strong laser field. The influence of Gaussian and Lorentzian beam shapes and different polarizations on ionization processes has been presented in numerous papers (Delone et al., 2000; Eichmann et al., 2009; Ooi et al., 2012; Ciappina et al., 2020). Since it has not been shown so far how Laguerre-Gaussian (LG) (0, 1)\* beam with radial polarization affects this value, our intention was to investigate it. The radially-polarized pulse can be focused to a very small size (Dorn et al., 2003), which is ideal for electron acceleration and has an influence on its ionization. Also, we wanted to compare the behavior of the

\*Corresponding author: [tanja.miladinovic@gmail.com](mailto:tanja.miladinovic@gmail.com)

transition rates calculated using Keldysh, PPT, and ADK theory for linear and LG (0,1)\* spiral phase mode. Comparing these transition rates for elliptical and circular polarization are shown in the paper (Guo et al., 2019). Through the paper, atomic units were used ( $m_e = |e| = \hbar = 1$ ).

## THEORETICAL PART

The tunneling ionization process of a quantum system (atom) is considered. The Theory of strong-field ionization, Keldysh Theory, gave the foundations for an understanding of this process. In the case of a ground state of a hydrogen atom, for large the field strength  $F$  and low the photon energy  $\omega$ , the ionization rate (i.e., the probability of ionization per unit time)  $w_{Keldysh}$  can be written (Keldysh, 1965):

$$w_{Keldysh} = \frac{\sqrt{3\pi Z F_{lin}}}{2^{7/4}} \times \exp\left[\frac{2Z^3}{3F_{lin}} \times \left(1 - \frac{1}{10} \frac{\gamma^2 Z^2}{2I_p}\right)\right] \quad (1)$$

$Z$  is the charge of the ionized system,  $\gamma$  is the Keldysh parameter, while  $I_p$  is the ionization energy. The  $lin$  index in quantity  $F$  denotes that the observed laser field is linear polarized.

Perelomov, Popov and Terent'ev obtained an equation for transition rate, not only for ionization from the ground state but also from excited states. (Perelomov et al., 1966; Delone et al., 1998):

$$w_{PPT} = \sqrt{\frac{3n^3 F_{lin}}{\pi}} \frac{(2\ell+1)(\ell+|m|)! 2^{4n-2|m|-2} n^{-6n+3|m|}}{(n+1)!(n-\ell-1)! (|m|)! (\ell-|m|)!} \times \frac{\exp\left[-\frac{2}{3n^3 F_{lin}}\right]}{F_{lin}^{2n-|m|-1}} \quad (2)$$

where is  $n$  principal,  $\ell$  orbital and  $m$  magnetic quantum number. Eq. (2) is applicable when the condition  $F_{lin} < Z^3/16n^4$  is fulfilled (Bauer, 2006; Delone et al., 1999). Laser field strength is, smaller compared to the atomic field strengths for highly excited states of the hydrogen atom ( $1/16n^4$ ) even with the most intense lasers available today (Delone et al., 1999).

Amosov, Delone and Krainov included certain modifications in Eq. (2), so the formula could be used to calculate the transition rate in the case of complex atoms. For rough calculation of the factorials, Stirling's formula was used. Main quantum number  $n$  has been replaced by an effective

quantum number  $n^* \equiv Z/\sqrt{2I_p}$  (Ammosov et al., 1986; Delone et al., 1998):

$$w_{ADK} = \sqrt{\frac{2n^{*3} F_{lin}}{\pi Z^3}} \frac{F_{lin}}{8\pi Z} \left(\frac{4\gamma(2I_p)^{3/2}}{\omega Z}\right)^{2n^*} \times \exp\left[-\frac{2}{3} \frac{2I_p \gamma}{\omega}\right] \quad (3)$$

Eqs. 1, 2, and 3 give the transition rates in a linearly polarized laser field. In this case, the relation between the intensity and the strength of the laser field is  $F_{lin} : \sqrt{I}$  (in atomic units). The laser beam can have different profiles, for example, Gaussian and Lorentzian. Each of them has a specific field distribution  $F_G = F \exp[-2(\rho/R)^2]$ , (Tokarev et al., 2003)  $F_L = F / (1 + (\rho/R)^2)$  (David et al., 2005) where  $R$  is the diameter of Gaussian beam and has the smallest value for  $z = 0$ , while  $\rho = R\sqrt{1 + (\lambda z/\pi R^2)^2}$  is axial coordinate normal to the light ray (Zhang, 2010),  $\lambda$  is laser wavelength. The influence of these distributions on ionization processes has already been studied (Boutu et al., 2011; Ishkhanyan et al., 2015). We wanted to see how the quantities that describe these processes will behave when the field distribution is given by LG (0, 1)\* spiral-phase mode (Machavariani et al., 2007):

$$F_{sp}(r, \phi) = F_0 \sqrt{\rho_{sp}} e^{-\rho_{sp}/2} e^{i\phi} \quad (4)$$

where  $r$  and  $\phi$  are the cylindrical coordinates. Spiral is presented by polar equation  $r(\phi) = ae^{k\phi}$  ( $a, k$  are parameters),  $\rho_{sp} = 2r^2/R^2$  and sign  $\pm$  depend on the chosen helicity. It is important to emphasize that LG (0, 1)\* mode can appear with linear, circular or elliptical polarization (Machavariani et al., 2007; Shealya, et al., 2005).

For linear polarization, the electric field distribution of a spiral-phase LG (0, 1)\* has the form:

$$\vec{F}_{lin,sp}(r, \phi) = F_{lin} \sqrt{\rho} e^{-\rho/2} e^{-i\phi} (\vec{e}_x + \vec{e}_y) \quad (5)$$

$\vec{e}_x$  and  $\vec{e}_y$  presented the unite vectors along the  $x$  and  $y$  axis. In scalar form, Eq. (5) is given with the equation:

$$F_{lin,sp}(r, \phi) = F_{lin} \sqrt{\rho} e^{-\rho/2} \cos \phi \vec{e}_x \cdot \vec{e}_x \quad (6)$$

taking into account that is  $\vec{e}_x \cdot \vec{e}_x = 1$ .

In order to estimate the influence of LG (0, 1)\* spiral-phase mode field's distribution with radial polarization on the transition rates given by Keldysh, PPT and ADK theories, we had to include Eq. (6) into Eqs. (1), (2) and (3).

## NUMERICAL RESULTS

We discussed the tunnel ionization of the Argon atom exposed to the light of the Ti: Sapphire laser wavelength of  $\lambda = 800nm$ , while the photon energy in atomic units is  $\omega = 0.5696$  a.u.. The binding energy of the first electron in the valence shell, of the Ar atom, in atomic units, is  $I_p = 0.5791$  a.u. and the charge of this system will be  $Z = 1$ . The intensity of the laser field within the ionization process is observed in range  $I = 10^{14} - 10^{17} W/cm^2$ . The intensity of the laser field is given by  $I$  in  $W/cm^2$ , while the strength of this field with the linear polarization  $F_{lin}$  in  $V/cm$ , which is one of the ways to apply the semi-classical theory since other quantities are represented in the atomic system of units. Formally, the relationship between field strength and intensity can be represented by an expression  $F_{lin} = 27.5\sqrt{I}$ . In the case of LG (0, 1)\* spiral-phase mode with radial polarization field strength is given by the Eq. (6).

Ti: Sapphire lasers can produce beams with different diameters, which can be the order of millimeters (15–30)mm or micrometer (3–60) $\mu m$  (Dorn et al., 2003; Ahmmed et al. 2014). Radially polarized laser light produces a spot of smaller diameter. In our paper, we will assume that  $R = 3\mu m$  ( $5.7 \times 10^4$  in atomic units).  $r(\phi) = ae^{k\phi}$  is the cylindrical coordinate and is a function of parameters  $a$  and  $k$ , also on azimuthal angle  $\Phi$ .  $\Phi$  lies in a specific interval span of  $360^\circ$ , such as  $[-180^\circ, +180^\circ]$  or  $[0^\circ, +360^\circ]$ .  $a = 0.57$  ( $1.08 \times 10^4$ ) when  $r$  is in  $\mu m$ ,  $k = \tan \theta$ . Angle  $\theta$  can take a value in the range of  $[-90^\circ, 90^\circ]$  and defines spiral geometry (Ouyang, et al., 2015).

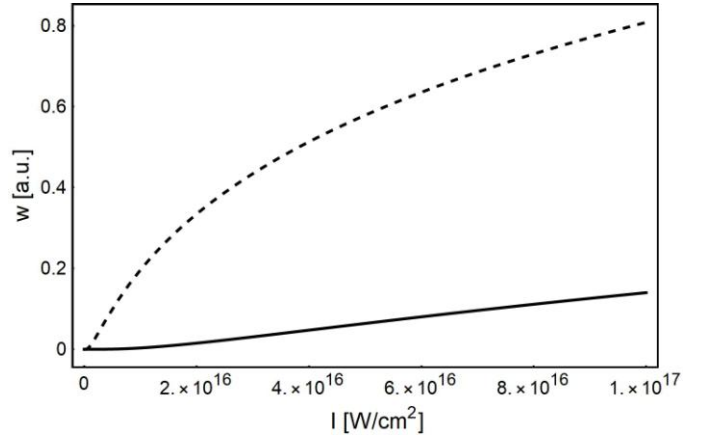
First, we focused on the comparison of the basic transition rate and the transition rate calculated with field distribution of LG (0, 1)\* spiral-phase mode is included. The behavior of these rates as a function of laser intensity is observed.

In Figs. 1, 2 and 3 are shown obtained curves calculated using equations for Keldysh, PPT and ADK transition rates. In a given range of laser intensities, we can notice that the inclusion of the LG (0, 1)\* spiral-phase field distribution leads to a decrease in the transition rate in all three observed cases. The analysis shown in Figures 1, 2, and 3 was done for azimuthal angle fixed to the value  $\phi = 30^\circ$  and laser intensity in range  $I = 10^{14} - 10^{17} W/cm^2$ . The values of the transition rates shown on the y-axis are given in arbitrary units.

At lower field intensities  $I < 10^{15} W/cm^2$ , the difference between the transition rates shown in Fig. 1 is imperceptible,

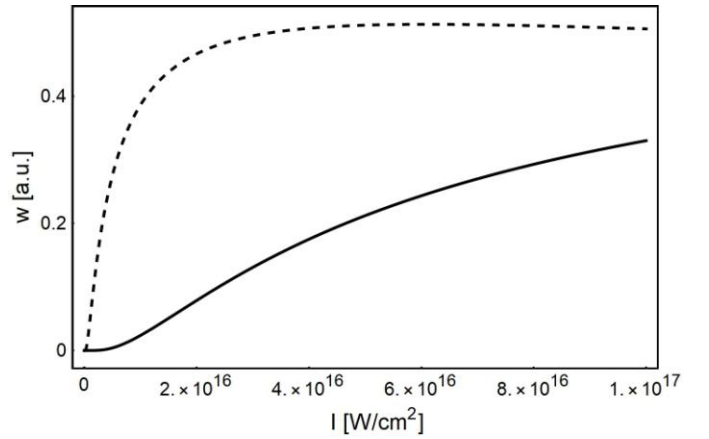
the largest one is calculated for  $I = 1.5 \times 10^{16} W/cm^2$ , when

$$w_{Keldysh} = 0.272a.u. \text{ and } w_{Keldysh}^{sp} = 0.008a.u..$$



**Figure 1.** Keldysh tunneling transition rate as a function of laser intensity.  $w_{Keldysh}$  (dashed line) and  $w_{Keldysh}^{sp}$  (solid line) corresponds to the transition rate with linear polarization and with LG (0, 1)\* spiral-phase mode, respectively.

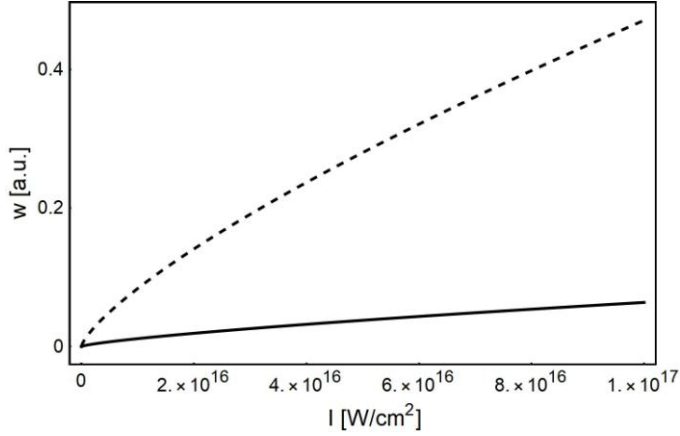
We calculated that for  $I = 10^{14} W/cm^2$ , the difference between  $w_{PPT}$  and  $w_{PPT}^{sp}$  is negligible. With an increase in laser intensity, the difference becomes more noticeable. For  $I = 1.75 \times 10^{16} W/cm^2$ ,  $w_{PPT}^{sp}$  reaches only 15% of the value of  $w_{PPT}$  (Fig. 2.).



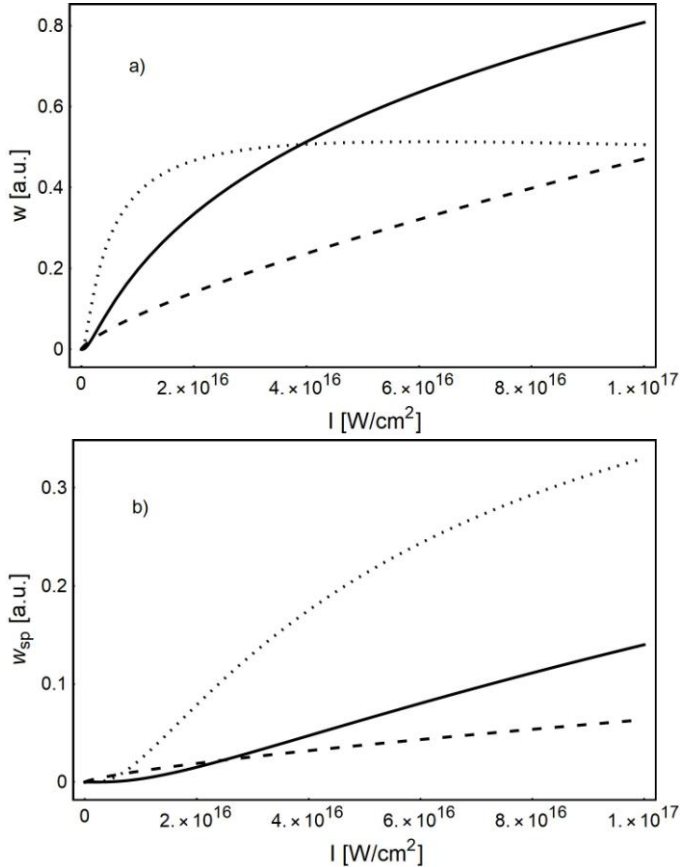
**Figure 2.** PPT tunneling transition rate as a function of laser intensity.  $w_{PPT}$  (dashed line) and  $w_{PPT}^{sp}$  (solid line) corresponds to the transition rate with linear polarization and with LG (0, 1)\* spiral-phase mode, respectively.

In Fig. 3 can be noticed that the basic tunnel ADK transition rate has a higher value and that the inclusion of a specific field distribution decreases the transition rate significantly. The obtained result is very interesting; although is not immediately visible, the difference between the values that reach these two transition rates is almost constant.  $w_{ADK}^{sp}$ ,

in the entire laser intensities interval, reaches a value that is in the interval of 86.4976%–86.4981% the lower than  $w_{ADK}$ .

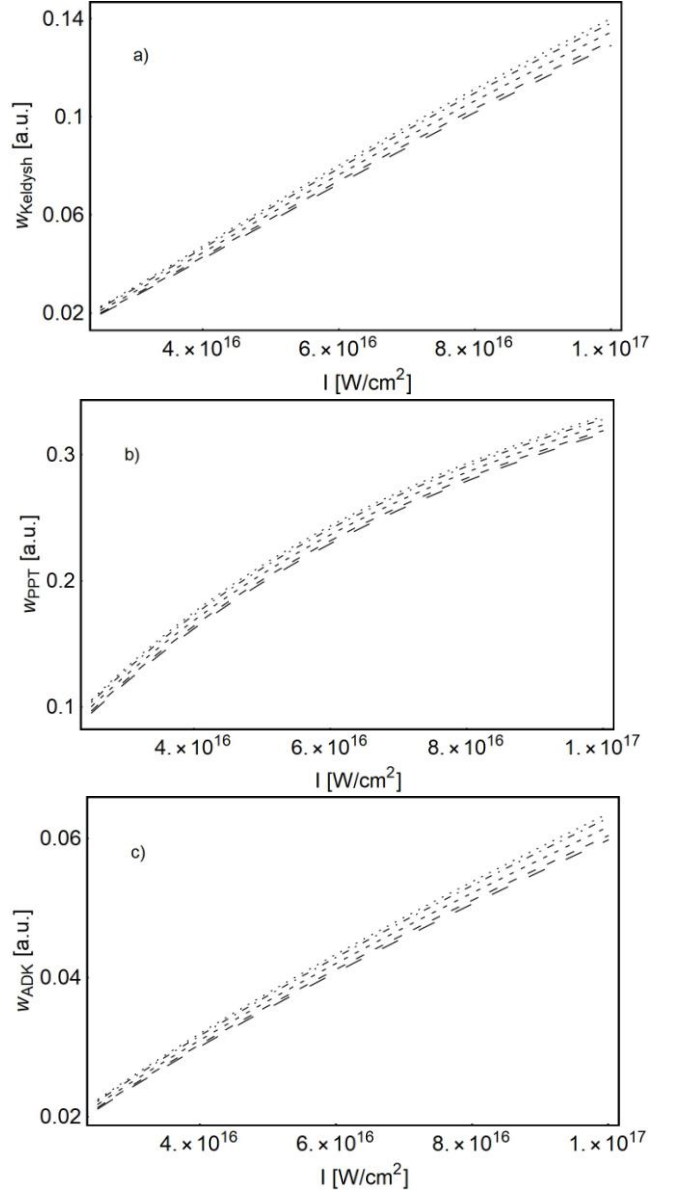


**Figure 3.** ADK tunneling transition rate as a function of laser intensity.  $w_{ADK}$  (dashed line) and  $w_{ADK}^{sp}$  (solid line) corresponds to the transition rate with linear polarization and with LG (0, 1)\* spiral-phase mode, respectively.



**Figure 4.** Transition rates as a function of laser intensity; Keldysh theory (solid line), PPT theory (dotted line), ADK theory (dashed line); Pane a) and pane b) corresponds to the transition rate,  $\omega$  with linear polarization and  $\omega_{sp}$  with LG (0, 1)\* spiral-phase mode, respectively.

We also wanted to show the transition rates given using all three theories on a single graph (Fig. 4). From this figure, it can be seen that, for the same values of the laser intensity, the transition rates given by different theories have various values. We can also notice that the inclusion of LG (0, 1)\* spiral-phase field distribution affects transition rates differently. With the inclusion of the specific field distribution  $W_{Keldysh}$  and  $w_{PPT}$  change, both shapes and values,  $w_{ADK}$  retains it's shape, but the values of the transition rates are much smaller.



**Figure 5.** Panes a)  $w_{Keldysh}^{sp}$ , b)  $w_{PPT}^{sp}$  and c)  $w_{ADK}^{sp}$  tunneling transition rates as a function of laser intensity for five different azimuthal angles  $\phi = -30^\circ$  (dashed large),  $\phi = -20^\circ$  (dashed medium),  $\phi = 0^\circ$  (dashed small),  $\phi = 20^\circ$  (dot-dashed),  $\phi = 30^\circ$  (dotted).

At the very end, in order to complete the discussion, we have shown how the transition rates, after LG (0, 1)\* spiral-phase field distribution was included, depend on the azimuthal angle (Fig. 5). The azimuthal angle took values  $\phi = \{-30^\circ, -20^\circ, 0^\circ, 20^\circ, 30^\circ\}$ , while the intensity of the laser field was in the range  $I = 10^{14} - 10^{17} \text{ W/cm}^2$ . In Fig. 5, it is not possible to see clearly, but for the lower intensities, the changes of the azimuthal angle affect all transition rates. Also, we can observe that in the case of all of three transition rates is the lowest value for  $\phi = -30^\circ$ , while the highest value is reached for the  $\phi = 30^\circ$ .

## CONCLUSION

The process when an electron leaves an atom by tunneling was analyzed by observing the behavior of the transition rate as a function of laser intensity. We performed our calculation for two physical situations; for basic transition rate with the LG (0, 1)\* spiral-phase mode field distribution included. The results show that the Keldysh, PPT and ADK transition rate after inclusion of specific field distributions has a lower value. Also, we must emphasize that this field distribution affects to the different extent within various theories. The greatest decrease is observed for the Keldysh transition rate, and that can be clearly seen in the figure which shows together all the transition rates. It is shown that the value of the transition rate depends on the azimuthal angle. These dependencies are very similar for all three theories. The lowest and highest values transition rates reached for the same angle values.

The analysis clearly establishes the influence of the different laser pulse shapes on all three observed transition rates. Although our result shows a general approach to the tunneling ionization process, it is important for the interpretation and understanding of relevant experimental and theoretical results.

## ACKNOWLEDGMENTS

The authors acknowledge funding provided by the University of Kragujevac - Institute for Information Technologies (the contract 451-03-9/2021-14/200378), University of Kragujevac - Faculty of Science (the contract 451-03-9/2021-14/200122) through the grants by the Ministry of Education, Science and Technological Development of the Republic of Serbia.

## REFERENCES

Ahmed, K. M. T., Grambow, C. & Kietzig, A. M. 2014. Fabrication of Micro/Nano Structures on Metals by

- Femtosecond Laser Micromachining. *Micromachines*, 5(4), pp. 1219-1253. doi.org/10.3390/mi5041219
- Ammosov, V. M., Delone, N. B. & Krainov, V. P. 1986. Tunnel ionization of complex atoms and of atomic ions in an alternating electromagnetic field. *Sov. Phys. JETP*, 64(6), pp. 1191-1194.
- Bauer, D., 2006. *Theory of Laser-Matter Interaction*, Max-Planck Institute, Heidelberg.
- Boutu, W., Auguste, T., Boyko, O., Sola, I., Balcou, Ph., Binazon, L., Gobert, O., Merdji, H., Valentin, C., Constant, E., Mével, E. & Carré, B. 2011. High-order-harmonic generation in gas with a flat-top laser beam, *Phys. Rev. A*, 84(6), pp. 063406-6. DOI:10.1103/PhysRevA.84.063406
- Calvert, J., Xu, H., Palmer, A., Glover, R., Laban, D., Tong, X., Kheifets, A., Bartschat, K., Litvinyuk, I., Kielpinski, D. & Sang, R. 2016. The interaction of excited atoms and few-cycle laser pulses. *Sci. Rep.*, 6(1), 34101-1-34101-9. doi.org/10.1038/srep34101
- Ciappina, M. F., Peganov, E. E. & Popruzhenko, S. V. 2020. Focal-shape effects on the efficiency of the tunnel-ionization probe for extreme laser intensities. *Matter Radiat. Extremes* 5(4), pp. 044401-10. doi.org/10.1063/5.0005380
- Corkum, P. B. 1993, Plasma perspective on strong field multiphoton ionization. *Phys. Rev. Lett.*, 71(13), pp.1994-1997. doi.org/10.1103/PhysRevLett.71.1994
- Delone, N. B. & Krainov, V. P. 2000. *Multiphoton processes in atoms*: Springer.
- Delone, N. B. & Krainov, V. P. 1999. AC Stark shift of atomic energy levels, *Phys. Usp.*, 42(7), pp. 669-687. https://doi.org/10.1070/PU1999v042n07ABEH000557
- Delone, N. B. & Krainov, V. P. 1998. Tunneling and barrier-suppression ionization of atoms and ions in a laser radiation field. *Physics Uspekhi*, 41(5), pp. 469-485. doi.org/10.1103/PhysRevLett.83.520
- Dorn, R., Quabis, S. & Leuchs, G. 2003. Sharper Focus for a Radially Polarized Light Beam. *Phys. Rev. Lett.*, 91(23), pp. 233901. doi.org/10.1103/PhysRevLett.91.233901
- Eberly, J. H. & Javanainen, J. 1988. Above-threshold ionization. *Eur. J. Phys.*, 9(4), pp. 265-275. doi.org/10.1088/0143-0807/9/4/004
- Eichmann, U., Nubbemeyer, T., Rottke, H. & Sandner W. 2009. Acceleration of neutral atoms in strong short-pulse laser fields. *Nature*, 461(7268), pp. 1261-1264. doi.org/10.1038/nature08481
- Guo, L., Hu, S., Liu, M., Shu, Z., Liu, X., Li, J., Yang, W., Lu, R., Han, S. & Chen, J. 2019. Accuracy of the semiclassical picture of photoionization in intense laser fields (Source: arXiv:1905.00213)
- Ho, Phay J. & Eberly, J. H. 2006. In-Plane Theory of Nonsequential Triple Ionization. *Phys. Rev. Lett.*, 97(8), pp. 083001-4. doi: 10.1103/PhysRevLett.97.083001
- Ishkhanyan, A M. & Krainov, V P. 2015. Non-exponential tunneling ionization of atoms by an intense laser field. *Laser Phys. Lett.*, 12(4), 046002 (6pp). DOI: 10.1088/1612-2011/12/4/046002

- Keldysh, L. V. 1965. Ionization in the Field of a Strong Electromagnetic Wave. *Sov. Phys. JETP*, 20, pp. 1307-1314.
- Krainov, V. P. & Shokri, B. 1995. Energy and angular distributions of electrons resulting from barrier-suppression ionization of atoms by strong low-frequency radiation. *JETP*, 80(4), pp. 657-661.
- Lai, Y. H., Xu, J., Szafruga, U. B., Talbert, B. K., Gong, X., Zhang, K., Fuest, H., Kling, M. F., Blaga, C. I., Agostini, P. & DiMauro, L. F. 2017. Experimental investigation of strong-field-ionization theories for laser fields from visible to midinfrared frequencies, *Phys. Rev. A*, 96(6), pp. 063417-10. doi.org/10.1103/PhysRevA.96.063417
- Landau, L. D. & Lifshitz, E. M. 1991. *Course of Theoretical Physics. Vol. 3: Quantum Mechanics: Non Relativistic Theory*: Pergamon, Oxford.
- Lappas, D. G. & Van Leeuwen, R. 1998. Electron correlation effects in the double ionization of He. *J. Phys. B*, 31(6), pp. L249-L256. doi:10.1088/0953-4075/31/6/001
- Liu, W.-C., Eberly, J. H., Haan, S. L. & Grobe, R. 1999. Correlation Effects in Two-Electron Model Atoms in Intense Laser Fields. *Phys. Rev. Lett.*, 83(3), pp. 520-523.
- Machavariani, G., Davidson, N., Lumer, Y. et al., 2007 European Conference on Lasers and Electro-Optics and the International Quantum Electronics Conference, Munich, p. 1.
- Machavariani, G., Lumer, Y., Moshe, I. & Jackel, S. 2007. Effect of the spiral phase element on the radial-polarization (0, 1)\* LG beam. *Opt. Commun.*, 271, pp. 190-196. doi:10.1016/j.optcom.2006.10.013
- Mainfray, G. & Manus, G. 1991. Multiphoton ionization of atoms. *Rep. Prog. Phys.*, 54(10), pp. 1333-1372. doi.org/10.1088/0034-4885/54/10/002
- Ooi, C. H. Raymond, Ho, WaiLoon, & Bandrauk, A. D. 2012. Photoionization spectra by intense linear, circular, and elliptic polarized lasers. *Phys. Rev. A*, 86(2), pp. 023410-6. DOI:10.1103/PhysRevA.86.023410
- Ouyang, J., Perrie, W., Allegre, O. J., Heil, T., Jin, Y., Fearon, E., Eckford, D., Edwardson, S. P. & Dearden G. 2015. Tailored optical vector fields for ultrashort-pulse laser induced complex surface plasmon structuring. *Opt. Express*, 23(10), pp. 12562-12572. DOI:10.1364/OE.23.012562
- Panfili, R., Eberly, J. H. & Haan, S. L. 2001. Comparing classical and quantum dynamics of strong-field double ionization. *Opt. Express*, 8(7), pp. 431-435. doi.org/10.1364/OE.8.000431
- Parker, J. S., Doherty, B. J. S., Taylor, K. T., Schultz, K. D., Blaga, C. I. & DiMauro, L. F. 2006. High-Energy Cutoff in the Spectrum of Strong-Field Nonsequential Double Ionization. *Phys. Rev. Lett.*, 96(13), pp. 133001-4. DOI:10.1103/PhysRevLett.96.133001
- Parker, J. S., Smyth, E. S. & Taylor, K. T. 1998. Intense-field multiphoton ionization of helium. *J. Phys. B*, 31(14), pp. L571-L578. doi.org/10.1088/0953-4075/31/14/001
- Perelomov, A. M., Popov, V. S. & Terent'ev, M. V. 1966. Ionization of Atoms in an Alternating Electric Field. *Sov. Phys. JETP*, 23(5), pp. 924-934.
- Schafer, K. J., Yang, B., DiMauro, L. F. & Kulander, K. C. 1993. Above threshold ionization beyond the high harmonic cutoff. *Phys. Rev. Lett.*, 70(11), pp. 1599-1602. doi.org/10.1103/PhysRevLett.70.1599
- Shealya, D. L. & Hoffnagle, J. A. 2005. Beam shaping profiles and propagation, *Proceedings SPIE Conf. Laser Beam Shaping VI*, (San Diego, California, USA), pp. 5876-13. <http://people.cas.uab.edu/~dls/publications/spie5876/spie5876-13-electronic.pdf>
- Shvetsov-Shilovski, N. I., Lein, M. & Tökési, K. 2019. Semiclassical two-step model for ionization of the hydrogen molecule by strong laser field. *Eur. Phys. J. D*, 73(2), pp. 1-8. doi.org/10.1140/epjd/e2018-90527-6
- Tokarev, V. N., Lopez, J., Lazare, S. & Weisbuch, F. 2003. High-aspect-ratio microdrilling of polymers with UV laser ablation: experiment with analytical model, *Applied Physics A*, 76(3), pp. 385-396. doi.org/10.1007/s00339-002-1511-8
- Voronov, G. S. & Delone, N. B. 1966. Many-photon ionization of the xenon atom by ruby laser radiation, *Sov. Phys. JETP*, 23, pp. 54-58.
- Xiong, W. & Chin, S. L. 1991. Tunnel ionization of potassium and xenon atoms in a high-intensity CO<sub>2</sub> laser radiation field. *Sov. Phys. JETP*, 72, pp. 268-271.
- Zhang, L. 2010. Intensity spatial profile analysis of a Gaussian laser beam at its waist using an optical fiber system, *Chinese Physics Letters*, 27(5), pp. 054207-3. DOI:10.1088/0256-307X/27/5/054207