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ON THE SECOND ORDER STATISTICS OF THE RATIO OF TWO FISHER-SNEDECOR RANDOM VARIABLES AND ITS APPLICATION TO INTERFERENCE LIMITED COMMUNICATIONS

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ABSTRACT

The paper investigates the higher order statistics of the ratio of two Fisher-Snedecor F (FS-F) random variables (RVs) and its application to wireless communications in the presence of co-channel interference. Namely, the work provides novel expressions for probability density function (PDF), cumulative density function (CDF), level crossing rate (LCR) and average fade duration (AFD) of the ratio of two FS-F RVs. Numerical examples of the analytically derived statistical measures in terms of FS-F multi-path and shadowing parameters are shown and examined. Moreover, the impact of the number of interferes on the considered measures are further presented and examined.

Keywords: 5G, Fisher-Snedecor F (FS-F) distribution, Second order statistics.

INTRODUCTION

A Fisher-Snedecor F (FS-F) distribution has been recently proposed as a tractable and experimentally tested composite fading model that can be efficiently applied for 5G and beyond 5G wireless communication systems (WCS) (Yao et al., 2021; Zhang et al., 2019; Makarfi et al., 2020).

Namely, the FS-F distribution has been introduced in (Yoo et al., 2017). In (Badarneh et al., 2018a), authors examined the sum of FS-F RVs and its application to maximal-ratio-combining (MRC) while selection combining (SC) and switch-and-stay combining (SSC) techniques over FS-F fading channels are considered in (Al-Hmood & Al-Raweshidy, 2020; Cheng et al., 2021), respectively. Dual-hop relay-assisted WCS over FS-F channels is examined in (Zhang et al., 2020). Physical layer security over FS-F distribution is further investigated in (Kong & Kaddoum, 2018). The first order statistics (FO-S) of cascaded N-FS-F distribution is considered in (Badarneh et al., 2018b) while the FO-S of the ratio of FS-F distribution is further considered in (Badarneh et al., 2020; Du et al., 2019). The first order performance analysis of communication systems in interference limited environments where the transmission signal as well as interference signal are modeled with FS-F distribution are given in (Alshawaqfeh et al., 2022). The second order statistics (SO-S) in terms of level crossing rate (LCR) and average fade duration (AFD) of FS-F distribution are provided in (Yoo et al., 2019) while the LCR and AFD of the product of two FS-F RVs are provided in (Stefanovic et al., 2021a). SO-S of N-FS-F fading model and their application to multihop communications are provided in (Stefanovic et al., 2021b, 2022b). Moreover, SO-S over cascaded N-Gamma-gamma (N-GG) fading channels are investigated in (Stefanović et al., 2021). The SO-S of SC combining systems with the co-channel interference over various fading channels are investigated in (Hadzi-Velkov, 2006, 2007a,b; Stefanović et al., 2012). Furthermore, SO-S of Unmanned Aerial vehicle (UAV)-to-ground, mobile-to-mobile (M2M), vehicle-to-infrastructure (V2I) and macro-diversity communications in interference limited scenarios have been investigated, respectively in (Stefanovic et al., 2022a; Đošić et al., 2022; Milosevic et al., 2018b,a; Stefanovic et al., 2018; Suljović et al., 2020).

Since the co-channel interference (CCI) is one of the major factors that can negatively impact the system performances, this paper examines LCR and AFD of ratio of two FS-F RVs and its application to the interference limited communication scenarios. To the best of the author's knowledge there is no paper that investigates SO-S of the ratio of the FS-F F RVs.

SYSTEM MODEL

The ratio of two independent Fisher-Snedecor F (FS-F) RVs, $Z_{\mathcal{F},1}$ and $Z_{\mathcal{F},2}$ can be mathematically given as:

$$Z_{\mathcal{F}} = Z_{\mathcal{F},1}/Z_{\mathcal{F},2} = (X_{n,1}Y_{In,1})/(X_{n,2}Y_{In,2}) = (X_{n,1}/Y_{n,1})/(X_{n,2}/Y_{n,2}) \quad (1)$$

where $X_{n,1}$ and $Y_{In,1}$ are Nakagami-m and inverse normalised Nakagami-m RVs given in (Yoo et al., 2017). Thus, the PDFs of $X_{n,1}$ and $Y_{n,1} = 1/Y_{In,1}$ are, respectively:

$$p_{X_{n,1}}(x_{n,1}) = \frac{2(\frac{m_{m,1}}{\Omega_1})^{m_{m,1}} x_{n,1}^{2m_{m,1}-1}}{\Gamma(m_{m,1})} e^{-m_{m,1}x_{n,1}^2} \quad (2)$$

$$p_{Y_{n,1}}(y_{n,1}) = \frac{2(m_{s,1}-1)^{m_{s,1}} y_{n,1}^{2m_{s,1}-1}}{\Gamma(m_{s,1})} e^{-(m_{s,1}-1)y_{n,1}^2} \quad (3)$$

where the multi-path and shadowing parameters are $m_{m,1}$ and $m_{s,1}$, respectively, whereas Ω_1 is the shaping parameter.

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Similarly, the PDFs of the interference signals of Nakagami-m distribution $X_{n,2}$ (Hadzi-Velkov, 2007a) and normalised Nakagami-m $Y_{n,2}$, respectively can be given as:

$$p_{X_{n,2}}(x_{n,2}) = \frac{2(\frac{m_{m,2}N_m}{\Omega_2})^{m_{m,2}} x_{n,2}^{2m_{m,2}N_m-1}}{\Gamma(m_{m,2}N_m)} e^{-m_{m,2}x_{n,m}^2} \quad (4)$$

$$p_{Y_{n,2}}(y_{n,2}) = \frac{2((m_{s,2}-1)N_s)^{m_{s,2}} x_{n,2}^{(2m_{s,2}-1)N_s}}{\Gamma(m_{m,s}N_s)} e^{-(m_{s,2}-1)y_{n,2}^2} \quad (5)$$

where the CCI multi-path and shadowing parameters are $m_{m,2}$ and $m_{s,2}$, respectively, Ω_2 is the CCI shaping parameter, N_m and N_s characterize the number of interfers.

The PDF of the $Z_{\mathcal{F},1} = X_{n,1}/Y_{n,i}$, $i = 1, 2$ can be expressed as:

$$p_{Z_{\mathcal{F},i}}(z_{\mathcal{F},i}) = \int_0^\infty \left|\frac{dx_{n,i}}{dz_{\mathcal{F},i}}\right| p_{X_{n,i}}(z_{\mathcal{F},i}y_{n,i}) p_{Y_{n,i}}(y_{n,i}) dy_{n,i} \quad (6)$$

where $\left|\frac{dx_{n,i}}{dz_{\mathcal{F},i}}\right| = y_{n,i}$. Based on (1-4), and using (Gradshteyn & Ryzhik, 2014), the PDF of $Z_{\mathcal{F},1}(z_{\mathcal{F},1})$ can be written as:

$$p_{Z_{\mathcal{F}},1}(z_{\mathcal{F},1}) = \frac{2\,(m_{m,1}/\Omega_1)^{m_{m,1}}\,(m_{s,1}-1)^{m_{s,1}}\Omega_1^{m_{m,1}+m_{s,1}}}{B(m_{m,1}, m_{s,1})} \times \frac{z_{\mathcal{F},1}^{2m_{m,1}-1}}{\left(m_{m,1}z_{\mathcal{F},1}^2 + \Omega_1(m_{s,1}-1)\right)^{m_{m,1}+m_{s,1}}} \quad (7)$$

while $p_{Z_{\mathcal{F}},2}(z_{\mathcal{F},2})$ is:

$$p_{Z_{\mathcal{F}},2}(z_{\mathcal{F},2}) = \frac{2\,(m_{m,2}/\Omega_2)^{m_{m,2}N_m}\,(m_{s,2}-1)^{m_{s,2}N_s}}{B(m_{m,2}N_m, m_{s,2}N_s)} \times \frac{z_{\mathcal{F},2}^{2m_{m,2}N_m-1}\Omega_2^{m_{m,2}N_m+m_{s,2}N_s}}{\left(m_{m,2}z_{\mathcal{F},2}^2 + \Omega_2(m_{s,2}-1)\right)^{m_{m,2}N_m+m_{s,1}N_s}} \quad (8)$$

Similarly, the PDF of a ratio of FS-F RVs $Z_{\mathcal{F}} = Z_{\mathcal{F},1}/Z_{\mathcal{F},2}$ can be obtained from:

$$p_{Z_{\mathcal{F}}}(z_{\mathcal{F}}) = \int_0^\infty \left|\frac{dz_{\mathcal{F},1}}{dz_{\mathcal{F}}}\right| p_{Z_{\mathcal{F},1}}(z_{\mathcal{F}}z_{\mathcal{F},2}) p_{Z_{\mathcal{F},2}}(z_{\mathcal{F},2}) dz_{\mathcal{F},2} \quad (9)$$

where $\left|\frac{dz_{\mathcal{F},1}}{dz_{\mathcal{F}}}\right| = z_{\mathcal{F},2}$. Finally, PDF of $p_{Z_{\mathcal{F}}}(z_{\mathcal{F}})$ can be written as:

$$p_{Z_{\mathcal{F}}}(z_{\mathcal{F}}) = \frac{4m_{m,1}{}^{m_{m,1}}(m_{s,1}-1)^{m_{s,1}}}{B(m_{m,1}, m_{s,1})B(m_{m,2}N_m, m_{s,2}N_s)} \times m_{m,2}{}^{m_{m,2}N_m}(m_{s,2}-1)^{m_{s,2}N_s}\Omega_1{}^{m_{s,1}}\Omega_2{}^{m_{s,2}N_s} z_{\mathcal{F}}^{2m_m-1} \times \int_0^\infty \frac{z_{\mathcal{F},2}^{2m_{m,1}+2m_{m,2}N_m-1}}{(m_{m,1}(z_{\mathcal{F}}z_{\mathcal{F},2})^2 + (m_{s,1}-1)\Omega_1)^{m_{m,1}+m_{s,1}}} \times \frac{1}{(m_{m,2}z_{\mathcal{F},2}^2 + (m_{s,2}-1)\Omega_2)^{m_{m,2}N_m+m_{s,2}N_s}} dz_{\mathcal{F},2} \quad (10)$$

After using (Gradshteyn & Ryzhik, 2014), a closed-form PDF expression $p_{Z_{\mathcal{F}}}(z_{\mathcal{F}})$ of a ratio of two FS-F in terms of Gaussian hyper-geometric function ${}_2F_1(\cdot, \cdot; \cdot; \cdot)$ (Gradshteyn & Ryzhik, 2014) is derived and presented as (11) at the top of the next page.

The CDF of a ratio of FS-F RVs can be obtained from $F_{Z_{\mathcal{F}}}(z_{\mathcal{F}}) = \int_0^{z_{\mathcal{F}}} p_{Z_{\mathcal{F}}}(x)dx$. Thus, $F_{Z_{\mathcal{F}}}(z_{\mathcal{F}})$ can be given as:

$$F_{Z_{\mathcal{F}}}(z_{\mathcal{F}}) = \frac{4m_{m,1}{}^{m_{m,1}}(m_{s,1}-1)^{m_{s,1}}}{B(m_{m,1}, m_{s,1})B(m_{m,2}N_m, m_{s,2}N_s)} \times m_{m,2}{}^{m_{m,2}N_m}(m_{s,2}-1)^{m_{s,2}N_s}\Omega_1{}^{m_{s,1}}\Omega_2{}^{m_{s,2}N_s} \times \int_0^\infty dz_{\mathcal{F},2} \frac{z_{\mathcal{F},2}^{2m_{m,1}+2m_{m,2}N_m-1}}{(m_{m,2}z_{\mathcal{F},2}^2 + (m_{s,2}-1)\Omega_2)^{m_{m,2}N_m+m_{s,2}N_s}} \times \int_0^{z_{\mathcal{F}}} \frac{x^{2m_{m,1}-1}}{(m_{m,1}x^2 z_{\mathcal{F},2}^2 + (m_{s,1}-1)\Omega_1)^{m_{m,1}+m_{s,1}}} dx \quad (12)$$

The level crossing rate ($N_{z_{\mathcal{F}}}$) for the predetermined envelope threshold z_{th}, $N_{z_{\mathcal{F}}}(z_{th})$ can be given as:

$$N_{z_{\mathcal{F}}}(z_{th}) = \int_0^\infty \dot{z}_{\mathcal{F}} p_{z_{\mathcal{F}}\dot{z}_{\mathcal{F}}}(z_{th}, \dot{z}_{\mathcal{F}}) d\dot{z}_{\mathcal{F}} \quad (13)$$

where, $p_{z_{\mathcal{F}}\dot{z}_{\mathcal{F}}}(z_{\mathcal{F}}, \dot{z}_{\mathcal{F}})$ is the joint distribution of the signal-to-interference ratio (SIR) envelope, $z_{\mathcal{F}}$ and its first derivative $\dot{z}_{\mathcal{F}}$.

Since we can express the received SIR as $Z_{\mathcal{F}} = Z_{\mathcal{F},1}/Z_{\mathcal{F},2}$, where $Z_{\mathcal{F},1}$ and $Z_{\mathcal{F},2}$ can be further expressed as $Z_{\mathcal{F},1} = X_{n,1}Y_{In,1} = X_{n,1}/Y_{n,1}$ and $Z_{\mathcal{F},2} = X_{n,2}Y_{In,2} = X_{n,2}/Y_{n,2}$, respectively, the $p_{z_{\mathcal{F}}\dot{z}_{\mathcal{F}}}(z_{th}, \dot{z}_{\mathcal{F}})$ can be given as an integral-form expression of a joint PDF of independent and identically distributed (i.i.d) RVs, $Z_{\mathcal{F}}, \dot{Z}_{\mathcal{F}}, Y_{n,1}, X_{n,2}$ and $Y_{n,2}$, can be given as:

$$p_{z_{\mathcal{F}}\dot{z}_{\mathcal{F}}}(z_{\mathcal{F}}, \dot{z}_{\mathcal{F}}) = \int_0^\infty dy_{n,1} \int_0^\infty dx_{n,2} \times \int_0^\infty p_{z_{\mathcal{F}}\dot{z}_{\mathcal{F}}y_{n,1}x_{n,2}y_{n,2}}(z_{\mathcal{F}}\dot{z}_{\mathcal{F}}y_{n,1}x_{n,2}y_{n,2}) dy_{n,2} \quad (14)$$

where $p_{z_{\mathcal{F}}\dot{z}_{\mathcal{F}}y_{n,1}x_{n,2}y_{n,2}}(z_{\mathcal{F}}\dot{z}_{\mathcal{F}}y_{n,1}x_{n,2}y_{n,2})$ can be further simplified and expressed through independent conditional and individual PDFs as:

$$\begin{aligned} p_{z_{\mathcal{F}}\dot{z}_{\mathcal{F}}y_{n,1}x_{n,2}y_{n,2}}(z_{\mathcal{F}}\dot{z}_{\mathcal{F}}y_{n,1}x_{n,2}y_{n,2}) &= p_{\dot{z}_{\mathcal{F}}|z_{\mathcal{F}}y_{n,1}x_{n,2}y_{n,2}}\left(\dot{z}_{\mathcal{F}}|z_{\mathcal{F}}y_{n,1}x_{n,2}y_{n,2}\right) \\ &\times p_{z_{\mathcal{F}}y_{n,1}x_{n,2}y_{n,2}}\left(z_{\mathcal{F}}y_{n,1}x_{n,2}y_{n,2}\right) \\ &= p_{\dot{z}_{\mathcal{F}}|z_{\mathcal{F}}y_{n,1}x_{n,2}y_{n,2}}\left(\dot{z}_{\mathcal{F}}|z_{\mathcal{F}}y_{n,1}x_{n,2}y_{n,2}\right) \\ &\times p_{z_{\mathcal{F}}|y_{n,1}x_{n,2}y_{n,2}}\left(z_{\mathcal{F}}|y_{n,1}x_{n,2}y_{n,2}\right) \\ &\times p_{y_{n,1}}\left(y_{n,1}\right) p_{x_{n,2}}\left(x_{n,2}\right) p_{y_{n,2}}\left(y_{n,2}\right) \end{aligned} \quad (15)$$

The conditional distribution $p_{z_{\mathcal{F}}|y_{n,1}x_{n,2}y_{n,2}}\left(z_{\mathcal{F}}|y_{n,1}x_{n,2}y_{n,2}\right)$ is then transformed into:

$$p_{z_{\mathcal{F}}|y_{n,1}x_{n,2}y_{n,2}}\left(z_{\mathcal{F}}|y_{n,1}x_{n,2}y_{n,2}\right) = \left|\frac{dx_{n,1}}{dz_{\mathcal{F}}}\right| p_{x_{n,1}}\left(\frac{z_{\mathcal{F}}y_{n,1}x_{n,2}}{y_{n,2}}\right) \quad (16)$$

From (13-16), the $N_{z_{\mathcal{F}}}$ of SIR envelope threshold in FS-F propagation environments is expressed as:

$$N_{z_{\mathcal{F}}}(z_{th}) = \int_0^\infty dy_{n,1} \int_0^\infty dx_{n,2} \int_0^\infty dy_{n,2} \times \left|\frac{dx_{n,1}}{dz_{\mathcal{F}}}\right| p_{x_{n,1}}\left(\frac{z_{\mathcal{F}}y_{n,1}x_{n,2}}{y_{n,2}}\right) \times p_{y_{n,1}}\left(y_{n,1}\right) p_{x_{n,2}}\left(x_{n,2}\right) p_{y_{n,2}}\left(y_{n,2}\right) \times \int_0^\infty \dot{z}_{\mathcal{F}} p_{\dot{z}_{\mathcal{F}}|z_{\mathcal{F}}y_{n,1}x_{n,2}y_{n,2}}\left(\dot{z}_{\mathcal{F}}|z_{\mathcal{F}}y_{n,1}x_{n,2}y_{n,2}\right) d\dot{z}_{\mathcal{F}} \quad (17)$$

$$
\begin{aligned}
p_{z_\mathcal{F}}(z_\mathcal{F}) = & \frac{2(m_{m,1}/\Omega_1)^{m_{m,1}}(m_{m,2}/\Omega_2)^{m_{m,2}}}{B(m_{m,1},m_{s,1})B(m_{m,2}N_m,m_{s,2}N_s)(m_{s,1}-1)^{m_{m,1}}(m_{s,2}-1)^{m_{m,2}N_m}} z_\mathcal{F}^{2m_{m,1}-1} B(m_{m,1}+m_{m,2}N_m, m_{s,1}+m_{s,2}N_s) \\
& \times \left(\frac{m_{m,2}}{(m_{s,2}-1)\Omega_2}\right)^{-m_{m,1}-m_{m,2}N_m} \left({}_2F_1\left(m_{m,1}+m_{s,1}, m_{m,1}+m_{m,2}N_m; m_{m,1}+m_{s,1}+m_{m,2}N_m+m_{s,2}N_s; 1-\frac{m_{m,1}(m_{s,2}-1)\Omega_2 z_\mathcal{F}}{(m_{m,2})(m_{s,1}-1)\Omega_1}\right)\right)
\end{aligned}
\tag{11}
$$

where,

$$
\int_0^\infty \dot{z}_\mathcal{F}\, p_{\dot{z}_\mathcal{F}|z_\mathcal{F} y_{n,1} x_{n,2} y_{n,2}}\left(\dot{z}_\mathcal{F}|z_\mathcal{F} y_{n,1} x_{n,2} y_{n,2}\right) d\dot{z}_\mathcal{F} = \frac{\sigma_{\dot{z}_\mathcal{F}}}{\sqrt{2\pi}}
\tag{18}
$$

The $\sigma^2_{\dot{z}_\mathcal{F}}$ is the variance of $\dot{z}_\mathcal{F}$. Furthermore, $\dot{z}_\mathcal{F}$ can be written as:

$$
\dot{z}_\mathcal{F} = \frac{y_{n,2}}{y_{n,1}x_{n,2}}\dot{x}_{n,1} - \frac{x_{n,1}y_{n,2}}{y_{n,1}^2 x_{n,2}}\dot{y}_{n,1} - \frac{x_{n,1}y_{n,2}}{y_{n,1}x_{n,2}^2}\dot{x}_{n,2} + \frac{y_{n,1}}{y_{n,1}x_{n,2}}\dot{y}_{n,2}
\tag{19}
$$

where $\dot{x}_{n,1}$, $\dot{y}_{n,1}$, $\dot{x}_{n,2}$ and $\dot{y}_{n,2}$ are the first derivatives of $x_{n,1}$, $y_{n,1}$, $x_{n,2}$ and $y_{n,2}$, respectively. We assume that $\dot{z}_\mathcal{F}$ is a zero-mean Gaussian RV whose variance after some transformations can be expressed as:

$$
\begin{aligned}
\sigma^2_{\dot{z}_\mathcal{F}} = & \frac{y_{n,2}^2}{y_{n,1}^2 x_{n,2}^2}\sigma_{\dot{x}^2_{n,1}}\left(1 + \frac{z_\mathcal{F}^2 x_{n,2}^2}{y_{n,2}^4}\frac{\sigma_{\dot{y}^2_{n,1}}}{\sigma_{\dot{x}^2_{n,1}}}\right. \\
& \left. + \frac{z_\mathcal{F}^2 y_{n,1}^2}{y_{n,2}^2}\frac{\sigma_{\dot{x}^2_{n,2}}}{\sigma_{\dot{x}^2_{n,1}}} + \frac{z_\mathcal{F}^2 y_{n,1}^2 x_{n,2}^2}{y_{n,2}^4}\frac{\sigma_{\dot{y}^2_{n,2}}}{\sigma_{\dot{x}^2_{n,1}}}\right)
\end{aligned}
\tag{20}
$$

where $\sigma_{\dot{x}^2_{n,1}}$, $\sigma_{\dot{y}^2_{n,1}}$, $\sigma_{\dot{x}^2_{n,2}}$ and $\sigma_{\dot{y}^2_{n,2}}$ are the variances of $\dot{x}_{n,1}$, $\dot{y}_{n,1}$, $\dot{x}_{n,2}$ and $\dot{y}_{n,2}$, respectively. Finally, $N_{z_\mathcal{F}}(z_{th})$ of a SIR threshold envelope for $\sigma_{\dot{x}^2_{n,1}} = \pi^2 f_m^2(\Omega_1/m_{m,1})$ can be written as:

$$
\begin{aligned}
N_{z_\mathcal{F}}(z_{th})/f_m = & \frac{16 m_{m,1}^{m_{m,1}-1/2}(m_{m,2}N_m)^{m_{m,2}}(m_{s,1}-1)^{m_{s,1}}}{\sqrt{2\pi}\Omega_m^{m_{m,1}-1/2}\Omega_s^{m_{s,1}}\Gamma(m_{m,1})\Gamma(m_{s,1})\Gamma(m_{m_2}N_m)} \\
& \times \frac{(m_{s,2}N_s-1)^{m_{s,2}}}{\Gamma(m_{s,2}N_s)} z_{th}^{2m_{m_1}-1} I_1
\end{aligned}
\tag{21}
$$

where I_1 is provided as:

$$
\begin{aligned}
I_1 = & \int_0^\infty dy_{n,1}\int_0^\infty dx_{n,2}\int_0^\infty dy_{n,2} \\
& \times \left(1 + \frac{z_\mathcal{F}^2 x_{n,2}^2}{y_{n,2}^4}\frac{\sigma_{\dot{y}^2_{n,1}}}{\sigma_{\dot{x}^2_{n,1}}} + \frac{z_\mathcal{F}^2 y_{n,1}^2}{y_{n,2}^2}\frac{\sigma_{\dot{x}^2_{n,2}}}{\sigma_{\dot{x}^2_{n,1}}} + \frac{z_\mathcal{F}^2 y_{n,1}^2 x_{n,2}^2}{y_{n,2}^4}\frac{\sigma_{\dot{y}^2_{n,2}}}{\sigma_{\dot{x}^2_{n,1}}}\right)^{\frac{1}{2}} \\
& \times x_{n,2}^{2m_{m_1}+2m_{s_1}-2}\, y_{n,1}^{2m_{m_2}N_m+2m_{m_1}-2}\, y_{n,2}^{2m_{s_2}N_s-2m_{m_1}} \\
& \times e^{-\frac{m_{m_1}}{\Omega_1}\frac{z_{th}^2 y_{n,1}^2 x_{n,2}^2}{y_{n,2}^2} - \frac{m_{m_2}}{\Omega_2}x_{n,2}^2 - (m_{s_1}-1)y_{n,1}^2 - (m_{s_2}-1)y_{n,2}^2}
\end{aligned}
\tag{22}
$$

Average fade duration (AFD), $A_{z_\mathcal{F}}(z_{th})$ is calculated as:

$$
A_{Z_\mathcal{F}}(z_{th}) = \frac{F_{Z_\mathcal{F}}(z_{th})}{N_{Z_\mathcal{F}}(z_{th})}
\tag{23}
$$

where $F_{Z_\mathcal{F}}$ is provided in (12) and $N_{Z_\mathcal{F}}$ in (21).

NUMERICAL RESULTS

The first and second order statistics of the ratio of two FS-F RVs in terms of the PDF, LCR and AFD are presented on Figs 1-3. The $p_{z_\mathcal{F}}(z_\mathcal{F})$ for $\Omega_1 = \Omega_2 = 1$ and for various $m_{m,1}$, $m_{s,1}$, $m_{m,2}$, $m_{s,2}$ and $N = N_m = N_s$ is shown in Fig. 1. Normalised LCR, $N_{z_\mathcal{F}}(z_{th})/f_m$ is presented in Fig 2. for $\sigma_{\dot{x}^2_{n,1}} = \sigma_{\dot{y}^2_{n,1}} = \sigma_{\dot{x}^2_{n,2}} = \sigma_{\dot{y}^2_{n,2}}$ and $\Omega_1 = \Omega_2 = 1$, $N = N_m = N_s = 2$ and various values of m. It can be seen that by increasing values of m (shifting from more severe to less severe fading conditions), $N_{z_\mathcal{F}}(z_{th})$ decreases.

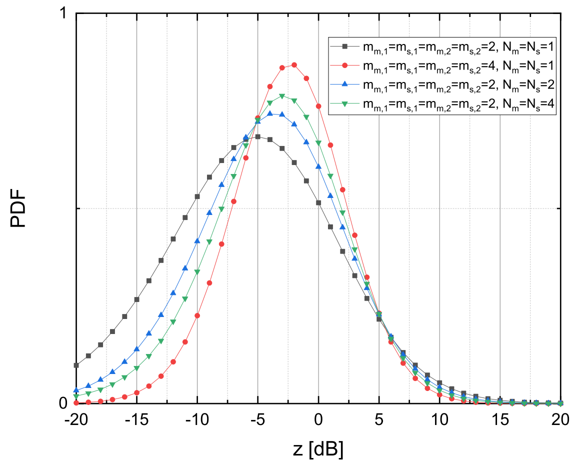

Figure 1. PDF for different m and N.

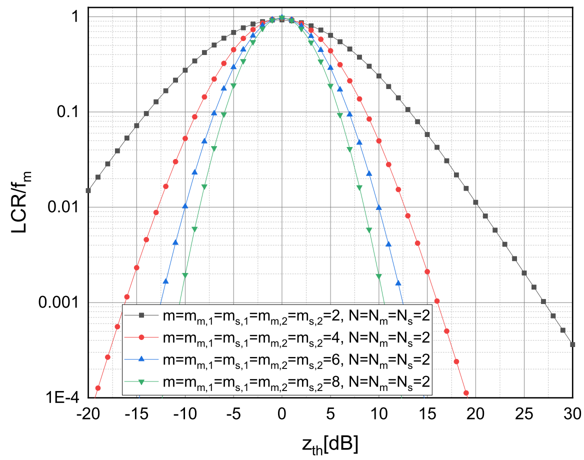

Figure 2. LCR for different m and $N = 2$.

Fig 3. shows $A_{Z_{\mathcal{F}}}(z_{th})f_m$ for $\Omega_1 = \Omega_2 = 1$ and for different m and $N = N_m = N_s$. It can be observed that by increasing values of m and N, the $A_{Z_{\mathcal{F}}}(z_{th})f_m$ decreases for lower values of z_{th} while increases for higher values of z_{th}. Moreover, the impact of N on $A_{Z_{\mathcal{F}}}(z_{th})f_m$ is more dominant for lower z_{th} while the impact of m on $A_{Z_{\mathcal{F}}}(z_{th})f_m$ is more dominant for higher z_{th}.

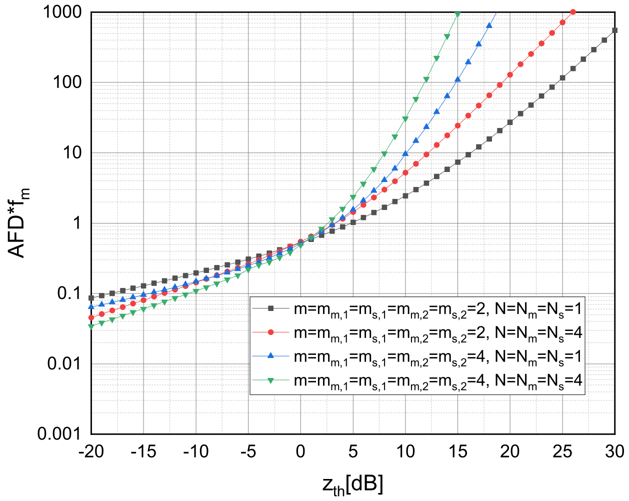

Figure 3. LCR for different m and N.

CONCLUSION

This paper investigates first and second order statistics of a ratio of two FS-F RVs in terms of $p_{Z_{\mathcal{F}}}(z_{\mathcal{F}})$, $F_{Z_{\mathcal{F}}}(z_{\mathcal{F}})$, $N_{Z_{\mathcal{F}}}(z_{th})$ and $A_{Z_{\mathcal{F}}}(z_{th})$. The system performance improvement can be achieved in less severe fading conditions (e.g., for higher values of m, LCR and AFD for lower values of z_{th} take lower values). In our future work we will consider SIR in relay and RIS systems over FS-F channels.

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