APPLICATION OF THE THRESHOLD VALUE IN THE CREDIT MULTIPLIER EVALUATION

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Abstract: Multiplication of credit funds has always been a basic assumption in the work of credit organizations. In modern conditions, they occur with banks, funds and other institutions that deal with lending. The paper examines the problem of interdependence between the goals of the central bank and the commercial bank. The main goal of the central bank is to maintain price stability. However, if the country has a high share of foreign debt in the gross domestic product, the activities of the central bank are redirected to discouraging the growth of borrowing by banking institutions abroad. In order to prevent such borrowing, the central bank uses several instruments of monetary policy, among which the limit value in the rating of the credit multiplier stands out. The importance of the threshold value in the process of multiplying the value that is lent contributes to preserving the relationship between liquidity and profitability in the appropriate proportion. The lack of funds leads to illiquidity and its consequences, and in modern conditions, unprofitability also leads to the same outcome. In this paper, we will present the application of the threshold value in creating a bank credit portfolio.

Keywords: threshold value, credit multiplier, central bank, banking institutions, lending.

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PRIMENA GRANIČNE VREDNOSTI U OCENI KREDITNOG MULTIPLIKATORA

Sažetak: Multiplikacija kreditnih sredstava oduvek je bila osnovna pretpostavka u radu kreditnih organizacija. U savremenim uslovima ona se javlja kod banaka, fondova i ostalih institucija koje se bave kreditiranjem. U radu se posmatra problem međuzavisnosti ciljeva centralne banke i poslovne banake. Osnovni cilj centralne banke je održavanje stabilnosti cena, međutim ako zemlja ima visoke udele inostranog duga u bruto društvenom proizvodu, aktivnosti centralne banke se preusmeravaju na destimuliranje rasta zaduživanja bankarskih institucija u inostranstvu. Radi sprečavanja takvog zaduživanja, centralna banka koristi nekoliko instrumenata monetarne politike, među kojima se posebno izdvaja granična vrednost u oceni kreditnog multiplikatora. Značaj granične vrednosti u procesu multipliciranja vrednosti koja se kreditira doprinosi očuvanju relacije likvidnosti i profitabilnosti u odgovarajućoj srazmeri. Nedostatak sredstava dovodi do nelikvidnosti i posledica koje sa sobom nosi, a u savremenim uslovima takođe istom ishodu vodi i neprofitabilnost. S tim u vezi u radu ćemo prikazati primenu granične vrednosti u kreiranju kreditnog portfolija, sa aspekta banke kao kreditne ustanove.

Ključne reči: granična vrednost, kreditni multiplikator, centralna banka, bankarske institucije, kreditiranje.

1. INTRODUCTION

Economic analysis of the reality of social phenomena that develop based on economic activities, primarily from credit operations, occupies a significant place in today's developed market economy, following all the prerequisites set by the law of value (Ilić & Tasić, 2021).

The research of lending phenomena from the perspective of mathematical analysis and the application of quantitative analysis is gaining more and more importance with the introduction of application programs for obtaining certain values in real-time (Cicmil, Jakšić, & Đaković, 2023). In part related to managerial decision-making and even the creation of economic flows, mathematical analysis takes an important place in explaining and presenting some new modalities in further development. Precisely for this reason, based on the analysis of the limit value of the series or rows, the analysis of the credit multiplier was arrived at as a basic assumption in the formation of the credit portfolio of each credit institution (Cicmil, Đaković, & Jakšić, 2022). Following the dominant growth in the profit of credit institutions, it can be assumed that the selection of individual portfolios in a limited set of funds is reached (Pantić, Mikulić, & Leković, 2022; Ivanov & Ristić, 2020). Precisely with such
limitations, mathematical analysis can produce certain results with binding elements in their conditional assumptions.

Credit as a relationship between the applicant and the provider (Stankov & Roganović, 2022) most often appears in bilateral relations, although today, it can be found as a multilateral relationship. The relationship between the provider and the applicant cannot be violated (Milanović, 2023). In this regard, the threshold value contains the solution in the part that refers to the degree of profit increase due to the selection of a certain portfolio.

This paper applied the threshold value method to the institution's credit portfolio to arrive at profitability indicators. Investigating ways of applying the method itself, we came to the limiting factors in the form of portfolio analysis precision, unpredictable market restrictions and the incompleteness of the factors themselves, which in the methodological sense is not reflected in the relevant relationships.

2. RESEARCH METHODS

An array or sequence is an ordered set of objects.

\[ a_1, a_2, ..., a_n, ... \]  

The members of the array can be represented by the points of a line. For example, the following figure shows the \( a_n = \frac{1}{n} \) string (Belás, Mišanková, Schönfeld, & Gavurova, 2017).

![Figure 1. Representation of a sequence on a number line](image)


The elements of that ordered set, i.e. the members of the sequence, can be any mathematical object. We will work exclusively with real numbers here since we have previously limited ourselves to the real domain.

Any set of real numbers can be ordered in some way, e.g. by size. But this does not mean that every such set (of real numbers) can be written as a sequence.
Let’s explain it using the example of a set of real numbers lying in between 0 and 1, i.e., in the interval (0,1). This set is sorted by size. This means that for each pair of numbers $b$ and $c$ from that set, we can say whether $b < c$ or $b = c$ or $b > c$. However, that set is not a sequence. Although it is easily fully ordered, we cannot write it as a string. We cannot say which is the hundredth or thousandth or even which is its second element. However, it is important for the array. The ability to assign a natural number to each member of the sequence is an essential characteristic of the sequence. If we limit ourselves to finite sets of real numbers, then the matter is simple (Krstić, Savić, & Kostić, 2023). For example, any finite ordered set of real numbers $\left(\frac{1}{2}, 3, \frac{5}{3}, 2, \frac{4}{7}\right)$ is a sequence.

What cannot be done with real numbers can be done with rational numbers. The famous German mathematician G. Cantor showed how we can string together all rational numbers. If we extract from the set of real numbers in the interval (0,1) all real fractions, i.e. rational numbers, then we have a set that can be arranged and written in the form of a sequence:

\[
\frac{1}{2}; \frac{1}{3}; \frac{1}{4}; \frac{2}{3}; \frac{1}{5}; \frac{2}{4}; \frac{3}{5}; \frac{1}{6}; \frac{4}{5}; \frac{2}{7}; \frac{3}{7}; \frac{5}{7}; \ldots.
\] (2)

In this sequence, the numbers are not ordered by size. In the first place comes the fraction, for which the sum of the numerator and denominator is equal to 3. The size of that sum orders other fractions. Within each group of fractions that have the same sum of numerator and denominator, the order is determined by the size of the numerator.

For example, $\frac{2}{5}$ is in front of $\frac{3}{4}$, because the numerator of the first fraction is smaller. For this reason, the selection method of this sequence from the set of real numbers (0,1) is not important. Namely, the choice can be made in another way.

All real fractions must be found in that sequence, i.e. the entire set of proper fractions can be exhausted by that sequence (Savić, Fabjan, & Trnavac, 2021). Therefore, one should always distinguish the possibility of formation from the way of formation of a series.

In sequence (2), no fraction can be shortened. That’s why no member of that sequence appears in it twice. If an unlimited number of repetitions is allowed, a sequence is reached

\[
\frac{1}{2}; \frac{1}{3}; \frac{1}{4}; \frac{2}{3}; \frac{1}{5}; \frac{2}{4}; \frac{3}{5}; \frac{1}{6}; \frac{4}{5}; \frac{2}{7}; \frac{3}{7}; \frac{5}{7}; \ldots.
\] (3)
In this sequence of proper fractions, it is easy to determine the position of each fraction \( \frac{p}{q} \). Its real number or index is calculated according to this formula (Dubauskas, 2012):

\[
\text{Tr}_n = \left( \frac{p + q}{2} \right)^2 - (q - \rho),
\]

\[
\rho = \begin{cases} 
1 & \text{za } p + q = 2k \\
3 & \text{za } p + q = 2k - 1 \\
4 & \text{za } k = 2, 3, \ldots
\end{cases}
\]

For example, the index of the fraction \( \frac{3}{5} \) in the sequence (3) is equal to \( \left( \frac{3+5}{2} \right)^2 - (5-1) = 12 \). In other words, the fraction \( \frac{3}{5} \) is in the 12th place in the sequence (3). Let us now take the fraction \( \frac{2}{5} \) from the odd fifth group of the sequence (3). Its index is equal to \( \left( \frac{2+5}{2} \right)^2 - \left( \frac{5-3}{4} \right) = 8 \), i.e. \( \frac{2}{5} \) is the 8th sequence member (3).

The sequence is given if we know its general term \( a_n \). For example, in a series of degrees: 
\( 3, 9, 27, \ldots, 3^n, \ldots \) (4)

the general term is \( a_n = 3^n \). The entire sequence (4) can be generated from it, replacing \( n \) with the values of the natural numbers.

Unlike sequence (4), which grows indefinitely, members of the following three sequences:

\[
\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots, \frac{1}{3^n}, \ldots
\]

\[
-\frac{1}{3}, -\frac{1}{9}, -\frac{1}{27}, \ldots, -\frac{1}{3^n}, \ldots
\]
approach one number, tend towards 0. The first series approaches zero on the right, on the side of positive numbers. The second series tends to the same limit on the left, on the side of negative numbers, while the members of the third series oscillate around 0. We say that 0 is the limit value of those three sequences. That is, these sequences converge towards zero. It is marked symbolically with the label "lim" that comes from the Latin word *limes*, which means border as in the following formula (Korauš, Simionescu, Bilan, & Schönfeld, 2017):

\[
\lim_{n \to \infty} \frac{1}{3^n} = \lim_{n \to \infty} \left( -\frac{1}{3^n} \right) = \lim_{n \to \infty} (-1)^n \frac{1}{3^n} = 0.
\]

The limit value of a sequence of numbers is defined as follows: the finite number \(a\) is the limit value or limes of the sequence \((a_n)\) if, for an arbitrarily small, positive number \(\varepsilon\), a large enough number \(n_0\) can be found such that

\[|a - a_n| < \varepsilon\]

when \(n \geq n_0\).

For example, in the case of the sequence (7) \(\varepsilon = \frac{1}{3^{10}}\), it is enough to take \(n_0 = 11\), to be

\[|0 - (-3^{-11})| = 3^{-11} < \frac{1}{3^{10}} = 3^{-10}.
\]

This means that all sequence members (7) after the tenth, both negative in odd places and positive in even places, fall into the interval \(\left(-\frac{1}{3^{10}}, \frac{1}{3^{10}}\right)\).

The sequence (4) is also convergent in a broader sense, although it does not approach a certain finite number. It converges to \(\infty\), or symbolically \(\lim_{n \to \infty} 3^n = \infty\).

Now consider this sequence:

\[
\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \ldots, (-1)^n \frac{1}{3^n}, \ldots
\]

The members of this sequence do not tend towards one number. The members in the first, third, and every subsequent odd place by themselves tend toward 0 while the sequence members in the even places tend toward 2. Therefore, the members of this series diverge. Some of them
diverge more towards 0, and others towards 2. The sequence (2.8) is divergent. It has no limit values.

3. ROWS

An infinite series is an expression of form (Stasytytė & Aleksienė, 2015)

\[ u_1 + u_2 + \ldots + u_n + \ldots, \quad \text{i.e.} \sum_{i=1}^{\infty} u_i \]  

(9)

where \( u_i \ (i=1,2,\ldots,n,\ldots) \), are terms of the series, numbers or functions or some other objects of mathematics. A member \( u_n \) is called a general member of the order. Finite rows \( s_n = \sum_{i=1}^{\infty} u_i \ (n=1,2,\ldots) \) are called partial or partial sums of rows (9). Therefore, the infinite row (9) corresponds to a series of partial sums.

\[ s_1, s_2, \ldots, s_n, \ldots \]  

(10)

If that sequence has a limit value \( s \), i.e. if there is \( \lim_{n \to \infty} s_n = s \), then it is said that the series (9) is convergent and has a sum of \( s \). Otherwise, row (9) is divergent and has no sum. Divergent sequences include all infinite sequences such that \( s_n \to +\infty \) or \( s_n \to -\infty \) when \( n \to \infty \). If row (9) converges, then \( u_n \to 0 \) when \( n \to \infty \). The reverse does not apply, i.e. from \( u_n \to 0 \), when \( n \to \infty \), it does not follow that the series (9) is convergent. For example, order

\[
1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n} + \ldots
\]

(11)

is divergent because \( s_n \to \infty \) when \( n \to \infty \), although the members of that order, the reciprocals of the natural numbers tend to zero. Therefore, the above condition is necessary but not sufficient for the convergence of order (9).

A necessary and sufficient criterion for the convergence of an order is the Cauchy criterion, which reads:

For the series (9) to converge, it is necessary and sufficient that each, no matter how small \( \varepsilon > 0 \) a member \( s_n \) of the series (10) can be found so that for an arbitrary natural number \( k \), the inequality is satisfied (Tamulevičienė, 2016)
\[ |s_{n+k} - s_n| < \varepsilon. \] (12)

Since \( s_{n+k} - s_n = u_{n+1} + u_{n+2} + \ldots + u_{n+k} \) we can replace inequality (12) with (Auerbach & Yuriy, 2012)

\[ |u_{n+1} + u_{n+2} + \ldots + u_{n+k}| < \varepsilon. \] (13)

This condition does not satisfy line (11). To show that, let us assume that \( \varepsilon = 10^{-6} \). Now, we need to find such \( s_n \) that inequality (12) is satisfied, that is, such \( u_n \) that inequality (13) is satisfied for an arbitrarily large natural number \( k \). But that is not possible. It is not possible to find, even for a very large \( n \) such a member of the order \( u_n \), that the "piece" of the order \( u_{n+1} + u_{n+2} + \ldots + u_{n+k} \) is smaller than one millionth, i.e. from the \( \varepsilon = 10^{-6} \). For example, \( n = 10^6 \) it is enough to take \( k = 2 \) to show that condition (13) is not satisfied. Namely, that's when:

\[ \frac{1}{1000001} + \frac{1}{1000002} > 10^{-6}. \]

When it is determined that a series is convergent, the question arises: How fast does it converge? This question is particularly important for the implementation of a certain order. It is usually taken that the sequence converges quickly if its remainder \( s - s_n = u_{n+1} + u_{n+2} + \ldots \) tends to zero when \( n \to \infty \) similar to the rest of the geometric sequence: \( a + au + au^2 + \ldots + au^n + \ldots \), where \( a > 0 \) and \( |u| < 1 \) constants. Suma tog poznatog reda jednaka je

\[ s = \lim_{n \to \infty} s_n = \lim_{n \to \infty} (a + au + au^2 + \ldots + au^n) = \lim_{n \to \infty} \frac{a(1-u^n)}{1-u} = \frac{a}{1-u} \] (Baum et al., 2012), and the rest \( r_n = s - s_n = \frac{a}{1-u} - \frac{a(1-u^n)}{1-u} = \frac{au^n}{1-u} \).

For example, if the row is compared

\[ 1 + \frac{1}{2^2} + \frac{1}{3^2} + \ldots + \frac{1}{n^2} + \ldots, \] (14)

which is convergent, with geometric order

\[ 1 + \frac{1}{2^2} + \frac{1}{2^2} + \ldots + \frac{1}{2^n} + \ldots, \] (15)

it can be seen that the order of the squares of the reciprocal values of the natural numbers, i.e. series (14), converges more slowly than geometric series
(15). Namely, it is enough to compare the general members of those two lines. It is obvious that \( \frac{1}{n^2} > \frac{1}{2^{n-1}} \), for \( n \geq 7 \), since then \( n^2 < 2^{n-1} \).

4. ECONOMIC APPLICATION OF MULTIPLIER

A commercial bank cannot grant more loans than it has assets (Gilchrist & Egon, 2012). Moreover, the bank may not even grant as many loans as it has collected fund, but must keep some of the collected funds in liquid form (in cash or on deposit with the National Bank) in case some investors want to withdraw their deposits. For example, if the bank receives 10 million dinars in deposits, it must keep 10% of the deposits in liquid form and can grant only 9 million dinars in loans.

What a single commercial bank cannot do, more commercial banks together can do. They can create loans, i.e. approve more loans than they have funds. Namely, the approved loan from one bank comes into the hands of another bank. Based on that role, it will approve a new loan, keeping 10% of the received funds as a reserve to cover deposits, etc. Thus, based on a deposit of 10 million dinars, the first bank can approve a loan of 9 million dinars, the second bank 8.1 million, the third bank 7.29 million dinars, etc. Thus, the process of loan multiplication takes place in the circle of commercial banks (Tsai, 2016).

In reality, this process is much more complex. For each unit of deposit received, there is a reserve for deposit coverage, which we will denote by \( r \) (Alpanda, Granziera, & Zubairy, 2019). Then, from each approved loan unit, one part, let's mark it with \( b \), goes to the benefit of some credit institution outside the circle of commercial banks and thus falls out of the loan multiplier process. In addition, one should also consider the part of the approved loan unit, which flows into the hands of the population as most money in circulation. Finally, there is a part \( c \) of the loan unit that falls out of the multiplication process because it goes into some special deposit or fund (Kiyotaki & Moore, 1997).

Of course, all these assumptions do not include all the limitations placed on that process in reality.

Let's assume that the first bank has an initial unit of free funds. The loan size in the first stage of the process is equal to 1. From that unit, they transfer to another business bank only \( 1 - b - s - c \). That bank keeps the reserve \( (1 - b - s - c) \cdot r \), and the rest is issued \( (1 - b - s - c) - (1 - b - s - c) \cdot r = (1 - b - s - c)(1 - r) \) in the form of loans. From that loan, it moves to the next, third bank (Woodford, 2011).
Application of the threshold value in the credit multiplier evaluation

\[(1-b-s-c)(1-r)-(1-b-s-c)(1-r) \cdot b - \]
\[-(1-b-s-c)(1-r) \cdot s - (1-b-s-c)(1-r) \cdot c = \]
\[= (1-b-s-c)(1-r)(1-b-s-c) = (1-b-s-c)^2 (1-r) . \]

From that, the bank keeps \((1-b-s-c)^2 \cdot (1-r)\), as a reserve, and the rest

\[ (1-b-s-c)^2 (1-r) \text{ goes further to the market, etc.} \]

The sum of all those loans approved in the \(n\) stage based on a unit of initial free funds is called the loan multiplier and is denoted by \(k_n\). Accordingly, it is

\[ k_n = 1 + (1-b-s-c)(1-r) + (1-b-s-c)^2 \cdot (1-r)^2 + \ldots + (1-b-s-c)^{n-1} (1-r)^{n-1} . \]

We see these credits are members of a geometric progression with a coefficient \((1-b-s-c)(1-r)\); where

\[ k_n = \frac{1 - (1-b-s-c)^n \cdot (1-r)^n}{1 - (1-b-s-c) \cdot (1-r)} . \]

For example, if \((1-b-s-c)(1-r) = 0.8\) then \(k_6 = 3.689; \ k_{12} = 4.656; \ k_{18} = 4.910; \ k_{24} = 4.976\), etc. This means that based on free funds of 1,000,000 RSD in stages 6, 12, 18 and 24, 3,689,000 RSD, 4,656,000 RSD, 4,910,000 RSD, and 4,976,000 RSD would be approved respectively.

Under those conditions, how many loans would be created by the end of that process? The answer is as follows:

\[ \lim_{n \to \infty} k_n = \frac{1}{1 - (1-b-s-c)(1-r)} . \]

If \((1-b-s-c)(1-r) = 0.8\), like before, then it is \(k = \frac{1}{1-0.8} = \frac{1}{0.2} = 5\).

Therefore, the initial free funds in the process of credit multiplication are fivefold.

5. CONCLUSION

Banking institutions can increase the nominal interest rate through various fees. For these reasons, data analysis using the nominal interest rate can lead to wrong conclusions, and this paper investigates the application of the limit value of the credit multiplier. Banking institutions have such a business policy that the
rate prescribed by the central bank does not negatively impact loans but only their profitability. Due to market conditions, banking institutions do not increase their interest rates on current products to avoid losing market share but rather try to introduce new products, for which they introduce various fees and thus increase the effective interest rate without changing the nominal interest rate. This activity is also considered a potential trigger for the emergence of a debt crisis. The growth of debt is also greatly influenced by banking institutions that borrow abroad, so due to insufficient accumulation of capital in the country, they place the funds in the form of various loans.

From mathematical programming models, we applied the threshold value method to the credit portfolio of the institution in order to arrive at the credit multiplier indicator. In addition, the central bank's main goal is to reduce the growth of credit placements to the population because subjects can easily borrow abroad without any intervention from the central bank. The analysis of the models presented in the paper shows that their application would be useful in the ex-ante analysis of the effect of monetary credit policy. Although assumptions that simplify the model were used, the results show a good direction in which the central bank could act when deciding to apply the limit value of the credit multiplier. Also, any additional fact related to the credit multiplier can easily be included in the program support, which could thus play an important role in regulating monetary movements. In the coming period, the central bank must improve the statistics of monitoring credit placements of banking institutions. Also, in subsequent research, changes could be introduced in the existing model for calculating the threshold value of the monetary multiplier so that this value is calculated for each institution separately according to its distribution of credit placements. The value thus calculated for each banking institution would be adjusted according to the institution's market share.

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