

# Stiffness as a criterion of dynamic load carrying capacity of tension-loaded bolted joints

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## ABSTRACT

The influence of the bolt size and the width of the joint members/bolt diameter ratio (relative clamped length) on the stiffness coefficient of the bolt and joint members, as well as on the ratio of these stiffness coefficients are discussed in this paper. The individual stiffness coefficients of the bolt and the joint members decrease with the increase of the relative clamped length and the decrease of the bolt size. However, the stiffness ratio of the joint members and the bolt increases with the increase of the relative clamped length. Of the considered bolts M6 ... M24, the bolted joint with the M6 bolt has the highest stiffness ratio of the joint members and the bolt, and the bolted joint with the M12 bolt has the lowest stiffness ratio. When a bolted joint is loaded with a variable working load, then the dynamic safety factor (DSF) is used for evaluating the joint load capacity. The DSF increases with a decrease in the stiffness of the bolted joint, i.e. by increasing the relative clamped length of the bolted joint. For the same conditions analyzed, the bolted joint with M6 bolt has the largest DSF, and the bolted joint with M12 bolt has the smallest DSF. The obtained result can be used by engineers to evaluate the load-carrying capacity of dynamically tension-loaded bolted joints.

## KEYWORDS

Bolted joint, Tension joint, Bolted joint stiffness, Dynamic load on tension joint, Dynamic FOS of tension joint.

## 1. INTRODUCTION

Machine parts and assemblies are connected, depending on the function, by various types of joints. The basic task of any machine parts joint is to transfer the load from one joint part to another, providing that the joint strength corresponds to the strength of the joined parts. The fixed joint of the machine parts enables the connection of machine parts and assemblies into one solid unit. Separable joints allow easy joining of machine parts into one unit, but also easy separation and subsequent re-joining without any damage. Separable connections have the widest application because they facilitate production, transportation, maintenance and repair. The most common separable connections in machine systems are bolted joints [1-3]. Designing bolted joints is a responsible engineering activity because the connection points are often the most dangerous places, where fractures or loss of hermeticity occur most often, due to insufficient strength or stiffness [4,5]. The experience of applying bolted joints shows that the joint stiffness is in most cases less than the stiffness of the individual parts that are joined, so this should be taken into account when designing and calculating bolted joints. An improved stiffness model for bolted joints is proposed in [6]. A novel approach is used to obtain a more accurate calculation of the joint stiffness, providing a more reliable prediction of the joint behaviour during the assembly and in operation. A new equivalent normal stiffness model for bolted joints considering the contact of two rough surfaces is also given in the paper [7]. The proposed equivalent normal stiffness

model could be used to determine the appropriate preload for different bolt sizes. In this paper, the effect of the relative clamped length (joint members width-to-bolt nominal diameter ratio) on the tension-loaded bolted joint stiffness and dynamic safety factor (DSF) is analyzed. The results show significant influence of bolted joint geometry on the bolted joint load carrying capacity.

## 2. TENSION-LOADED BOLTED JOINTS

During the bolt joint tightening, due to the axial movement of the nut, its distance from the bolt head decreases, until the bottom of the nut touches the plate surface (Fig. 1a). By further turning the nut with a wrench, it is tightened (the screw pair of the bolt and the nut firmly adheres and presses the plates together). The resulting axial movement of the nut to the threaded part of the bolt is made possible by elastic deformations: the elongation of the bolt shank and the compression of the plates (Fig. 1a,b). The total axial displacement of the nut at the end of tightening ( $f$ ) consists of the elastic elongation of the bolt ( $f_b$ ) and the elastic compression of the connected plates ( $f_{jm}$ ):

$$f = f_b + f_{jm} \tag{1}$$

Proportionally to the increase of these elastic displacements during tightening, the force in the bolt, i.e. in the joint members, also increases. At the end of tightening, the bolt loads the joint members with a clamping force  $F_c$ . This force loads the joint members in compression and the bolt body in tension. The following equalities can be written for the bolt and joint members displacements (elastic deformations):

$$F_c = k_b f_b = k_{jm} f_{jm}, \tag{2}$$

where  $k_b$  and  $k_{jm}$  are the stiffness coefficients of the bolt and joint members, respectively.

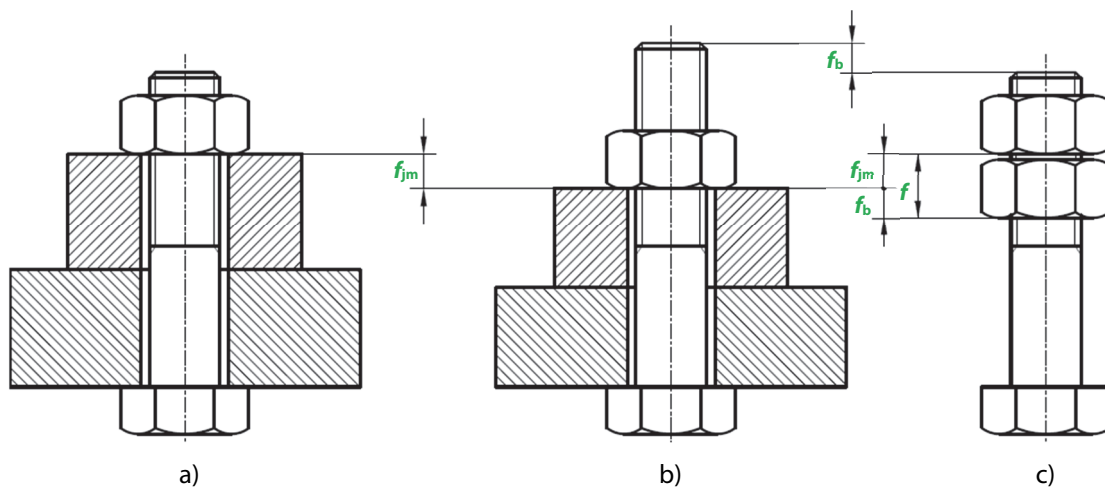


Figure 1: Bolted joint clamping

The change of forces in the bolt and joint members depending on their deformations - the bolted joint diagram is shown in Fig. 2a.

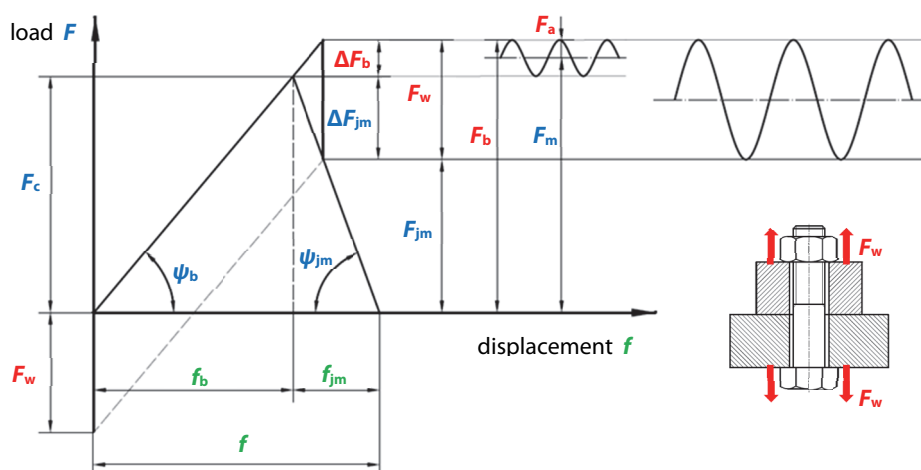


Figure 2: Bolted joint diagram

The bolted joint can be subjected to the working load only after the completed tightening. A tension-loaded bolted joint is considered here when the working load tends to separate the joined plates in the direction of the bolt axis. In doing so, it is assumed that the working load acts directly under the bolt head on one side of the joint and under the nut on the other side (Fig. 2). In this case, the effect of the external working load is transferred to the bolt without deforming the joint members. The bolt, which is supposed to prevent the separation of the joined members, is additionally loaded due to the action of the working force  $F_w$ , so it will be additionally elongated by the value  $\Delta f$ . Given that the relative position of the nut to the bolt thread has not changed, the initial compression of the plates is reduced by the same length  $\Delta f$ , so that the plates are partially relieved. At the same time, during the action of the working force, the total elongation of the bolt is  $f_b + \Delta f$ , and the compression of the plates is  $f_b - \Delta f$ . The joint working force is distributed on the bolt and the joint members so that one part of it additionally loads the bolt, and the other working force portion partially relieves the joint members. In the bolted joint diagram in Fig. 2, this is shown by the lengthening of the bolt deformation line and the shortening of the plate deformation line. The force loading the bolt is increased by the value  $\Delta F_b$ , and the force loading the joint members is decreased by  $\Delta F_{jm}$ . Based on this, it can be written:

$$F_w = F_b - F_{jm} = \Delta F_b + \Delta F_{jm} . \quad (3)$$

Figure 2 shows the cyclic working loading that changes within the limits of  $0 \dots F_w$ . The amplitude of the change of the working force is  $F_w/2$ . With a change in the working force, the working force in the tightened bolt also changes within the limits of  $0 \dots \Delta F_b$ , and the amplitude of the change in the working force in the bolt is  $\Delta F_b/2$ . This amplitude is relevant for the calculation of the dynamic load-carrying capacity of the bolt. The total load in the bolt changes from the tightening force  $F_c$  to the maximum bolt force  $F_b = F_c + \Delta F_b$ . The change in force in the bolt is much smaller than the change in the total external working load. The dependence of the working force in the bolt on its stiffness coefficient ( $k_b = \tan \psi_b < \tan \psi'_b = k'_b$ ) is shown in Fig. 3. In this example, the stiffness coefficient of the joint members is a constant value ( $k_{jm} = \tan \psi_{jm} = \text{const}$ ). The lower the bolt stiffness (i.e. the greater the elasticity), the smaller the change in force in the bolt  $\Delta F_b$ , at the same working force  $F_w$ , so that a bolt with less stiffness can be expected to have higher durability. This means that, in addition to strength, bolt stiffness is also a significant criterion for the load-carrying capacity and durability of tension-loaded bolted joints.

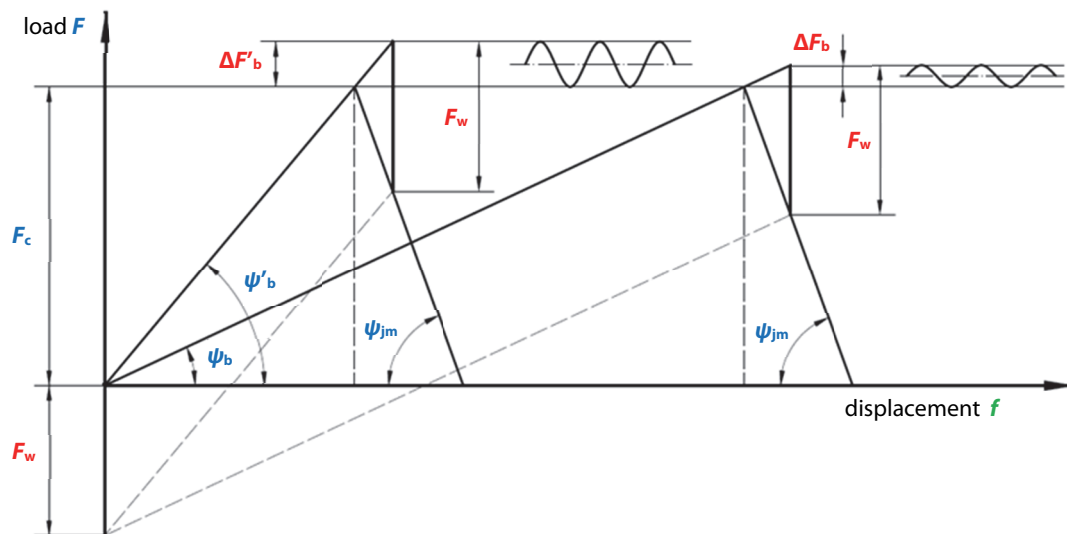


Figure 3: Diagram of bolted joints with bolts of different stiffness coefficients

### 3. STIFFNESS OF BOLTED JOINT

#### 3.1. Stiffness of bolted joint parts

Bolts and joint members deform elastically under an external working load and then return to their original state when the load is removed (like a spring). They accumulate potential energy and realize the corresponding tension force. Therefore, stiffness/elasticity is one of the most important properties of bolted joints. Less stiff (more elastic) parts can accumulate potential energy more efficiently, and therefore the stiffness of the parts that make up the bolted joint is important. The stiffness coefficients of the bolt  $k_b$  and the joint members  $k_{jm}$  represent the force, which is required to realize the deformation of the bolt  $f_b$ , i.e. the compression of the joint members  $f_{jm}$ . The dimensions of the basic structural parts of a bolt with a nut are shown in Fig. 5. The structural parts of a bolt are the head, bolt shank (threadless part) and thread part. The recommended length of the bolt threaded part  $l_t = 2d + 6$  mm (where  $d$  is nominal thread diameter) is given in [8], for bolts with length  $l < 125$  mm.

The stiffness coefficient of the bolt with variable cross-section, in addition to the cross-sectional area and the length of the threaded and non-threaded part of the bolt shank ( $k_s$ ), is also affected by the stiffness of the bolt head ( $k_h$ ) and the stiffness of the part of the bolt in connection with the nut ( $k_n$ ). The bolt stiffness coefficient is calculated by the formula:

$$\frac{1}{k_b} = \frac{1}{k_s} + \frac{1}{k_h} + \frac{1}{k_n} \tag{4}$$

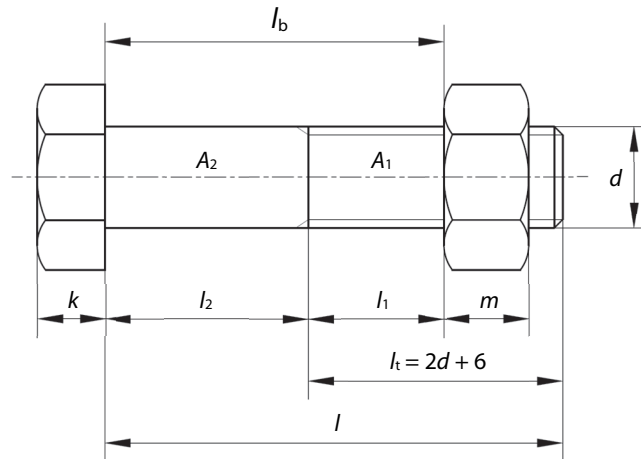


Figure 5: Bolt with hexagonal head and nut

The inverse value of the stiffness coefficient of the bolt shank (Fig. 5) is given by the expression:

$$\frac{1}{k_s} = \frac{1}{E} \sum_{i=1}^2 \frac{l_i}{A_i} = \frac{1}{E} \left( \frac{l_1}{A_1} + \frac{l_2}{A_2} \right), \tag{5}$$

where  $l_i$  is the length of the bolt part with constant cross-sectional area  $A_i$ ;  $E$  is the modulus of bolt material elasticity.

The area of the threaded part of the bolt (Fig. 5) is:

$$A_1 = \frac{\pi}{4} \left( \frac{d_2 + d_3}{2} \right)^2, \tag{6}$$

where  $d_2$  is the pitch diameter of the thread;  $d_3$  is the minor diameter of the thread.

The area of the non-threaded part of the bolt (Fig. 5) is:

$$A_2 = \frac{\pi}{4} d^2, \tag{7}$$

where  $d$  is the bolt shank diameter equal to the nominal thread diameter (SI.5).

The inverse value of the stiffness coefficient of the bolt head can be determined using the expression [8]:

$$\frac{1}{k_h} = \frac{0.15}{Ek}, \tag{8}$$

where  $k$  is the height of the bolt head (standard designation [13]).

The inverse value of the stiffness coefficient of the bolt part in contact with a nut of the same material [8]:

$$\frac{1}{k_n} = \frac{0.8}{Ed}. \tag{9}$$

The value  $1/k_n$  does not depend on the nut height  $m$  (Fig. 5). Only the first few threads carry the load, so only their influence is included. The remaining part of the bolt, in and out of the nut, does not participate in deformation and does not contribute to reducing or increasing the bolt stiffness. In further analyses, the cases of the ratio  $\zeta = l_b/d = \{2.5; 3.0; 3.5; 4.0; 4.5; 5.0; 5.5\}$ , where  $l_b$  is the distance between the bolt head and the nut, i.e. total width of joint members. The quantity  $\zeta = l_b/d$  has the physical meaning of the relative clamped length of the bolted joint. The stiffness of the joined parts is proportional to the volume of the joint members, in which elastic deformations occurred due to the tightening force in the bolt. Starting from the nut on one side and from the bolt head on the other side,

the area of influence of the deformations expands in the form of a cone with angle  $\alpha$  (Fig. 6). It was experimentally established that  $\tan\alpha = 0.4 \dots 0.5$ . In the following analyses, the mean value  $\text{tg}\alpha = 0.45$ , which corresponds to the cone angle of  $24.2^\circ$ , will be used.

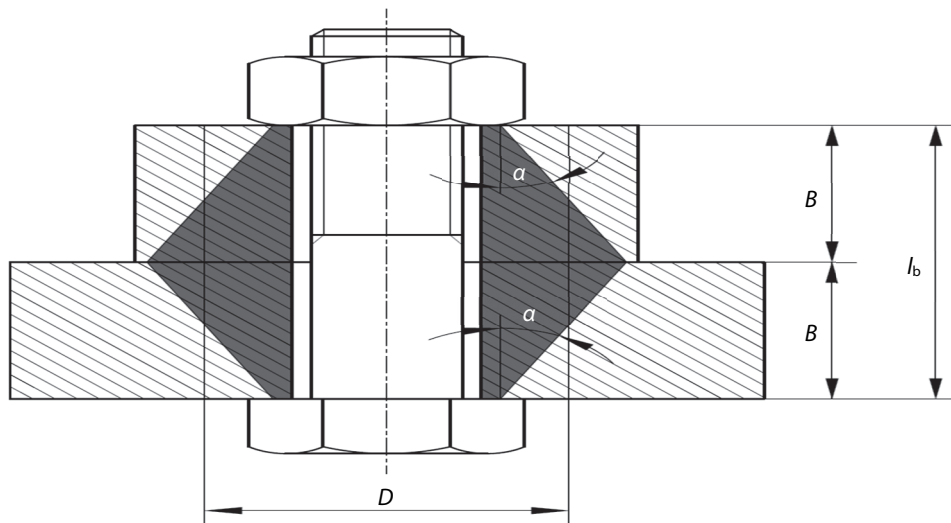


Figure 6: Clamped plates with equal width  $B$

The stiffness coefficient of joint members of equal width (Fig. 5 and 6) is determined based on the expression [8]:

$$k_{jm} = \frac{E\pi d_h \tan\alpha}{4.6 \log \frac{(s+d_h)(s+l_b \tan\alpha - d_h)}{(s-d_h)(s+l_b \tan\alpha + d_h)}} = \frac{E\pi d_h \tan\alpha}{4.6 \log \frac{(s+d_h)(s+\zeta d \tan\alpha - d_h)}{(s-d_h)(s+\zeta d \tan\alpha + d_h)}}, \quad (10)$$

where  $E$  – modulus of elasticity of the joint members (in this paper, assumed to be the same as for the bolt and nut);  $s$  – width between flats;  $d_h$  – diameter of clearance hole;  $\zeta = l_b/d$  – relative clamped length.

If the width of the joint members is smaller than the diameter of the hole clearance ( $l_b < d_h$ ), the cones can be replaced by an equivalent cylinder of diameter  $D$  (Fig. 6). In this case, the stiffness coefficient of the joint members is:

$$k_{jm} = \frac{E_{jm} A}{l_b}, \quad (11)$$

where  $A = \frac{\pi}{4}(D^2 - d_h^2)$ ,  $D = s + \frac{l_b}{2} \tan\alpha = s + \frac{\zeta d}{2} \tan\alpha$ . (12)

In this paper, it is assumed that  $l_b > d_h$ , so (10) will be used to calculate the stiffness coefficient of the joint members.

### 3.2. Stiffness of the bolted joint

The stiffness of the bolted joint as an assembly consisting of joint members, bolt and nut, is defined by the bolted joint stiffness factor. It is an important parameter, which determines how bolts and joint members accept external loads, how they react to load changes, how they react to temperature changes, etc. [1]. The bolted joint stiffness factor is a very suitable quantity for evaluating the load-carrying capacity of dynamically loaded bolted joints, from the aspect of the stiffness of the bolt and the joint members individually, and from the aspect of bolted joint as an assembly. The bolted joint stiffness factor is the ratio of the bolt stiffness coefficient and the sum of the stiffness coefficients of the bolt and the joint members:

$$\Phi = \frac{k_b}{k_b + k_{jm}} = \frac{1}{1 + \frac{k_{jm}}{k_b}} = \frac{1}{1 + \xi_k}, \quad (13)$$

where  $\xi_k$  is the relative stiffness of bolted joint parts (joint members and bolt (in practice it is mostly greater than 1,  $\xi_k = k_{jm}/k_b > 1$ , because often the stiffness of the joint members is greater than the stiffness of the bolt).

### 4. JOINT STIFFNESS ANALYSIS

#### 4.1. Bolted joint parts

For the analysis of the influencing factors of the bolted joint stiffness, bolts with a metric thread of a coarse pitch and with a hexagonal head and a nut of size M6 ... M24 were chosen (Table 1). Geometrical data were taken from appropriate standards [9-13]. Bolts, nuts and joint members are made of the same material, steel, with the modulus of elasticity  $E = 2.1 \cdot 10^5 \text{ N/mm}^2$ . The calculated quantities from expressions (4) ... (9) are given in Table 2.

Table 1: Geometrical parameters of thread, bolts, nuts and associated bolt clearance holes [9-13]

Standard metric thread	M6	M8	M10	M12	M14	M16	M18	M20	M22	M24
Minor diameter $d_3$ (mm)	4.917	6.647	8.376	10.106	11.835	13.835	15.294	17.294	19.294	20.752
Pitch diameter $d_2$ (mm)	5.350	7.188	9.026	10.863	12.701	14.701	16.376	18.376	20.376	22.051
Pitch $P$ (mm)	1.00	1.25	1.50	1.75	2.00	2.00	2.50	2.50	2.50	3.00
Height of the head $k$ (mm)	4.0	5.3	6.4	7.5	8.8	10.0	11.5	12.5	14.0	15.0
Height of the nut $m$ (mm)	4.90	6.44	8.04	10.37	12.10	14.10	15.10	16.9	18.10	20.20
Width across flats $s$ (mm)	10	13	17	19	22	24	27	30	32	36
Clearance hole diameter $d_h$ (mm)	6.6	9.0	11.0	13.5	15.5	17.5	20.0	22.0	24.0	26.0

Table 2: Bolt stiffness determination

Standard metric thread	M6	M8	M10	M12	M14	M16	M18	M20	M22	M24		
$A_1$ (mm <sup>2</sup> )	20.697	37.583	59.460	86.335	118.205	159.888	196.939	249.825	308.997	359.731		
$A_2$ (mm <sup>2</sup> )	28.274	50.265	78.540	113.097	153.938	201.062	254.469	314.159	380.133	452.389		
$A_3$ (mm <sup>2</sup> )	18.988	34.700	55.101	80.214	110.009	150.331	183.710	234.899	292.371	338.228		
$l_1$ (mm)	8.10	10.56	12.96	14.63	16.90	18.90	21.90	24.10	26.90	28.80		
$l_2$ (mm)	$\zeta$ ( $l_b/d$ )	2.5	6.90	9.44	12.04	15.37	18.10	21.10	23.10	25.90	28.10	31.20
		3.0	9.90	13.44	17.04	21.37	25.10	29.10	32.10	35.90	39.10	43.20
		3.5	12.90	17.44	22.04	27.37	32.10	37.10	41.10	45.90	50.10	55.20
		4.0	15.90	21.44	27.04	33.37	39.10	45.10	50.10	55.90	61.10	67.20
		4.5	18.90	25.44	32.04	39.37	46.10	53.10	59.10	65.90	72.10	79.20
		5.0	21.90	29.44	37.04	45.37	53.10	61.10	68.10	75.90	83.10	91.20
		5.5	24.90	33.44	42.04	51.37	60.10	69.10	77.10	85.90	94.10	103.20
$k_s$ ( $10^5 \text{ N/mm}$ )	$\zeta$ ( $l_b/d$ )	2.5	3.305	4.480	5.656	6.877	8.060	9.411	1.040	1.174	1.305	1.409
		3.0	2.832	3.830	4.828	5.859	6.862	7.987	8.848	9.965	1.106	1.196
		3.5	2.478	3.344	4.212	5.104	5.974	6.937	7.700	8.657	9.596	1.039
		4.0	2.202	2.968	3.735	4.521	5.290	6.131	6.816	7.653	8.475	9.186
		4.5	1.982	2.668	3.355	4.057	4.746	5.493	6.114	6.858	7.589	8.231
		5.0	1.801	2.423	3.045	3.680	4.304	4.975	5.544	6.212	6.87	7.456
		5.5	1.651	2.219	2.788	3.367	3.937	4.547	5.070	5.677	6.276	6.814
$k_h$ ( $10^5 \text{ N/mm}$ )	56.0	74.2	89.6	105.0	123.2	140.0	161.0	175.0	196.0	210.0		
$k_n$ ( $10^5 \text{ N/mm}$ )	15.75	21.00	26.25	31.50	36.75	42.00	47.25	52.50	57.75	63.00		
$k_b$ ( $10^5 \text{ N/mm}$ )	$\zeta$ ( $l_b/d$ )	2.5	2.605	3.517	4.424	5.357	6.274	7.288	8.093	9.094	10.090	10.920
		3.0	2.302	3.103	3.901	4.718	5.523	6.404	7.123	7.993	8.861	9.594
		3.5	2.062	2.777	3.488	4.216	4.933	5.711	6.360	7.129	7.897	8.557
		4.0	1.867	2.513	3.155	3.810	4.457	5.153	5.744	6.434	7.122	7.722
		4.5	1.706	2.294	2.879	3.475	4.065	4.695	5.238	5.862	6.485	7.036
		5.0	1.571	2.111	2.648	3.195	3.736	4.311	4.813	5.384	5.953	6.462
		5.5	1.455	1.954	2.451	2.956	3.456	3.986	4.452	4.978	5.502	5.974

Based on the data from Table 2, diagrams of the dependence of the bolt stiffness coefficient  $k_b$  on the relative clamped length for different bolt sizes were obtained (Fig. 7).

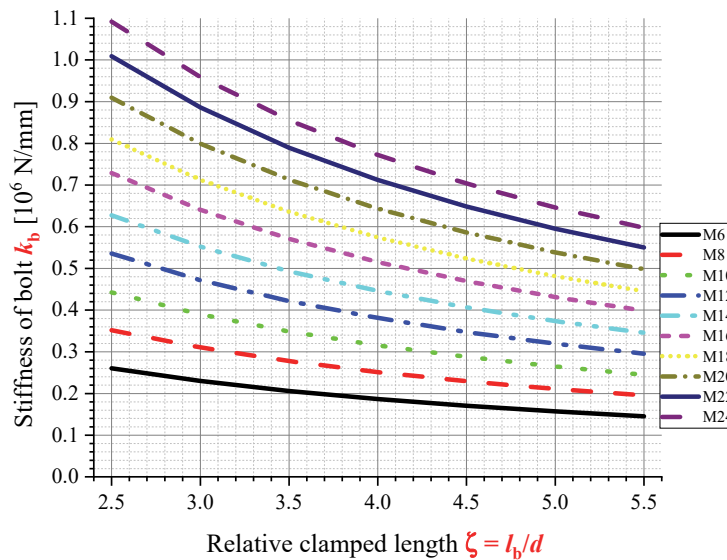


Figure 7: Bolt stiffness vs. relative clamped length and bolt size

Based on the diagram in Figure 7, it can be concluded that the bolt stiffness decreases with the increase of the relative clamped length. With an increase in the nominal thread diameter 4 times, from M6 to M24, the stiffness increases a little more than four times (4.1 ... 4.3), regardless of the relative clamped length  $l_b/d$ . The relative clamped length increases with an increase in the width of the joint members and/or a decrease in the bolt thread's nominal diameter. As the bolt size increases, the stiffness of the bolt increases, as well (Fig. 7). When changing the relative clamped length  $\zeta = l_b/d = 2.5 \dots 5.5$ , the gradient of the change in stiffness of the M24 bolt is  $0.50 \cdot 10^6$  N/mm, and that of the M6 bolt is  $0.12 \cdot 10^6$  N/mm. This means that larger bolts are more sensitive to the change in stiffness with a change in the relative clamped length of the bolted joint. Diagrams of the influence of the relative clamped length on the stiffness of the joint members, obtained using the data from Table 1 and expression (10), are shown in Figure 8. The stiffness of the joint members decreases with an increase in the relative clamped length and with a decrease in the bolt size. With a change of the nominal bolt diameter from M6 to M24, the stiffness increases about 3.2 times, regardless of the relative clamped length  $l_b/d$ . When changing the relative clamped length  $\zeta = l_b/d = 2.5 \dots 5.5$ , the gradient of the change in stiffness of the plates joined by the M24 bolt is  $1.05 \cdot 10^6$  N/mm, and by the M6 bolt, it is  $0.35 \cdot 10^6$  N/mm. This means that bolted joint assembled with the M24 bolt is three times more sensitive to the change in stiffness with a change in the relative clamped length than bolted joint assembled with the M6 bolt.

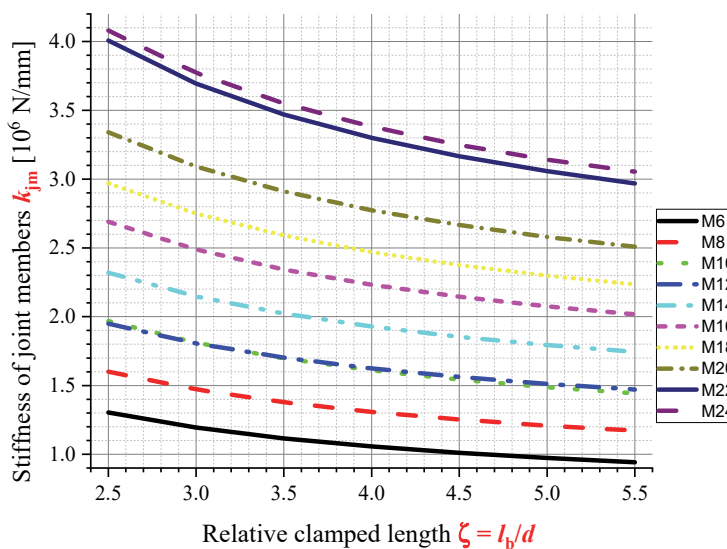


Figure 8: Joint members' stiffness vs. relative clamped length and bolt size

#### 4.2. Bolted joint

The relative stiffness of the joint members and the bolt  $\xi_k = k_{jm}/k_b$  depending on the relative clamped length and the bolt size is shown in Fig. 9. According to the diagrams in Fig. 7 and 8, the stiffness of the bolt and the stiffness of the joint members decrease with an increase in the relative clamped length and with a decrease in bolt diameter.

However, the stiffness ratio of the joint members and the bolt, i.e. the relative stiffness increases with the increase in the relative clamped length of the bolted joint.

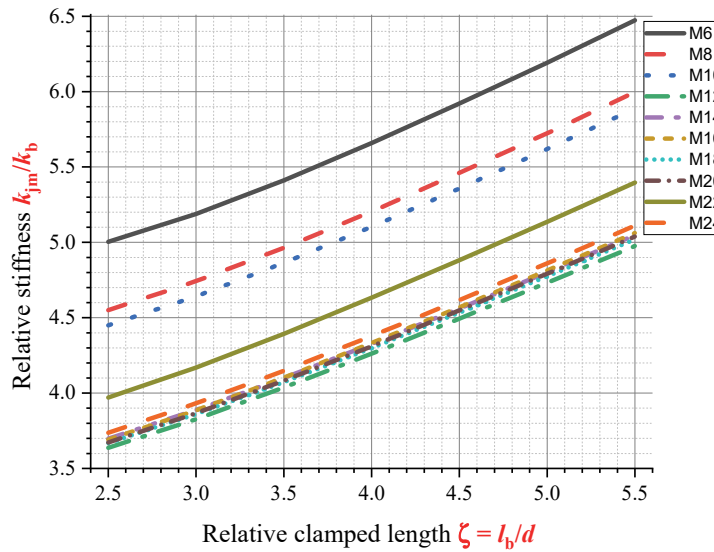


Figure 9: Relative joint members-bolt stiffness vs. relative clamped length and bolt size

For all considered bolt sizes, the bolted joint with the M6 bolt has the highest relative stiffness, and the M12 bolt has the lowest. Bolted joints with bolts M12, M14, M16, M18, M20 and M24 have approximately the same relative stiffness (lines close to the lowest line of the M12 bolt, Fig. 9). The functions in Fig. 9 are approximately linear and parallel, i.e. changes in relative stiffness with change in relative clamped length have approximately the same gradient for all bolt sizes. For all considered cases, the stiffness of the joint members is greater than the stiffness of the bolts 3.6 to 6.5 times. The dependence of the bolted joint stiffness factor  $\Phi$  on the relative clamped length is shown in Figure 10. The stiffness factor decreases with the increase of the relative clamped length. The smallest bolted joint stiffness factor has a bolted joint assembled with an M6 bolt. The highest bolted stiffness factor has a bolted joint assembled with an M12 bolt. The dependences are approximately parallel, and for all considered cases, the stiffness factor of the bolted joint is within the limits of 0.134 ... 0.216.

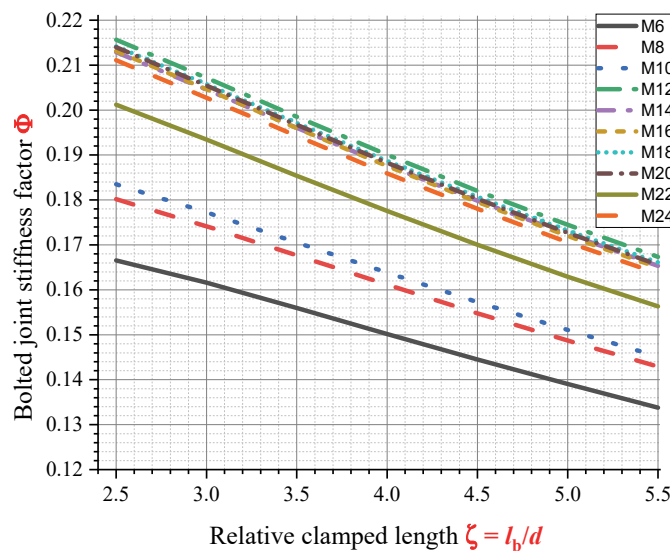


Figure 10: Bolted joint stiffness factor vs. relative clamped length and bolt size

### 5. LOAD CARRYING CAPACITY OF BOLT IN BOLTED JOINT

The load-carrying capacity criterion of the bolt is the safety factor (SF). When a bolted joint is subjected to a cyclic axial load, the load-carrying capacity is evaluated based on the dynamic safety factor (DSF). The DSF of the bolt is defined as the ratio of the amplitudes of the ultimate ( $\sigma_A$ ) and working stress ( $\sigma_a$ ) of the bolt:

$$S_D = \frac{\sigma_A}{\sigma_a} \tag{14}$$



Working stress is tension stress:

$$\sigma_a = \frac{\Delta F_a}{A_3}, \tag{15}$$

where  $\Delta F_a = \Delta F_b/2$  is the amplitude of working load in the bolt (Fig. 2 and 3);  $A_3$ – the smallest cross-sectional area with the bolt thread minor diameter  $d_3$  (Table 1 and 2).

According to Fig. 2, the change of bolt load, under the working load  $F_w$  is [1, 6, 8]:

$$\Delta F_b = \frac{k_b}{k_b + k_{jm}} F_w. \tag{16}$$

Taking into account (14), the expression (17) can be written in the form:

$$\Delta F_b = \frac{1}{1 + \frac{k_{jm}}{k_b}} F_w = \frac{1}{1 + \xi_k} F_w = \Phi F_w. \tag{17}$$

Substituting (18) into (16) and (15), we get:

$$S_D = \frac{2\sigma_A A_3}{\Phi F_w}. \tag{18}$$

The product  $2\sigma_A A_3 = 2F_A = F_D$  has the physical meaning of the ultimate dynamic load, at which the bolt would be failed by fatigue. The ultimate dynamic load refers to the bolt out of the bolted joint and depends on the bolt material properties, surface finish and nominal diameter. Therefore, (18) respecting (13) can be written in the form

$$S_D = \frac{1}{\Phi} \cdot \frac{F_D}{F_w} = (1 + \xi_k) \frac{F_D}{F_w} = \left(1 + \frac{k_{jm}}{k_b}\right) \frac{F_D}{F_w}. \tag{19}$$

The ultimate dynamic load is the constant of every single bolt. Accordingly, its intensity can be considered relatively, to the working load  $F_w$ , to analyze the influence of the stiffness factor on the bolt DSF. It is important to note that the working load can be higher than the ultimate dynamic load because part of the working load is taken over by the joint members. Based on the diagram in Fig. 10, it can be concluded that for bolts M6 ... M24 and the relative clamped length of the  $l_b/d = 2.5 \dots 5.5$ , the stiffness factor is within the limits of  $\Phi = 0.134 \dots 0.216$ . Therefore, in the analysis of the DSF, the range  $\Phi = 0.10 \dots 0.22$  will be considered. The diagram of the DSF depending on the relative clamped length and the bolt size, when the working load is equal to the bolt's ultimate dynamic load, is shown in Fig. 11. Although the working load is equal to the ultimate load, the DSF has a satisfactory value. For all cases of bolt size and relative clamped length considered in this paper, the DSF has values from 4.65 to 7.5. If the ratio of the bolt's ultimate dynamic load to the working load of the bolted joint is  $F_D/F_w \neq 1$ , the DSF values read from the diagram in Fig. 11 for specific bolt size and relative clamped length just need to be multiplied by the known  $F_D/F_w$  ratio to obtain the actual DSF value. If  $F_D/F_w < 1$ , the DSF will be less than the values in the plots in Fig. 11.

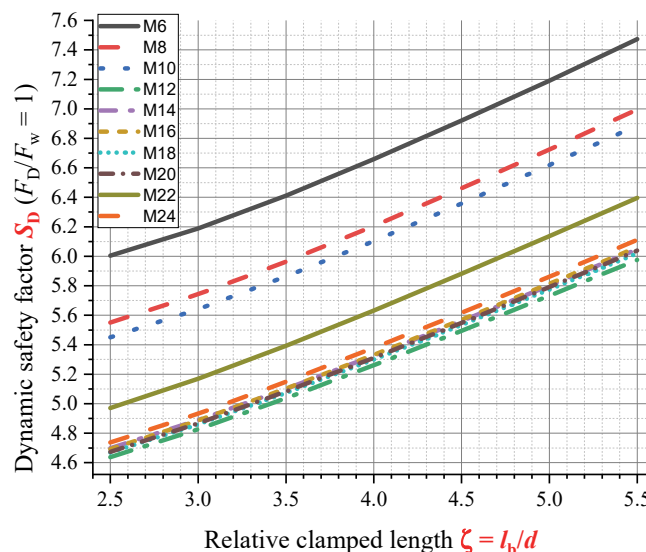


Figure 11: DSF vs. relative clamped length and bolt size

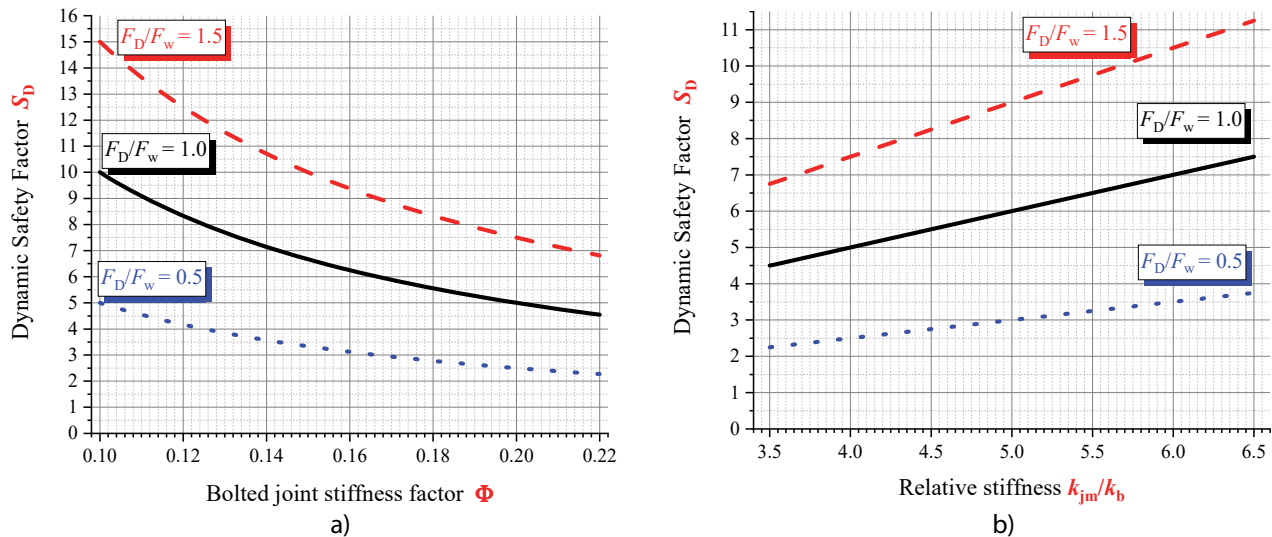


Figure 12: DSF vs. bolted joint stiffness factor (a) and relative stiffness (b) for different ultimate dynamic load of bolt

The dependence of the DSF on the stiffness factor on different ultimate/working force ratios is shown in Fig. 12a, and on the relative stiffness of the joint parts - in Fig. 12b. The values  $F_D/F_w = \{0.5, 1.0, 1.5\}$  were considered for the ratio of ultimate dynamic load and working load. The DSF decreases with an increase in the stiffness factor and with a decrease in the relative stiffness of the joint parts. With the increase of the ultimate dynamic load to the working load, the sensitivity of the bolted joint to the change of the DSF with the change of the stiffness factor and relative stiffness is greater (red lines in Fig. 12). The diagrams in Fig. 12 can be a useful aid to designers to be able to determine the DSF for known geometrical parameters of the bolted joint and the bolt material properties.

## 6. CONCLUSION

This paper shows the way how and how much the dynamic load-carrying capacity of tension-loaded bolted joints depends on the geometric parameters of the bolted joint parts (bolts and joined plates), as well as on the stiffness of the bolted joint as a whole. The dynamic safety factor (DSF) increases with increasing length and/or decreasing bolt diameter. The more elastic the bolt, the better it can withstand cyclic loads. The analysis showed that in the case of less stiffness of the bolted joint, the DSF has satisfactory good values, rather higher than 1.20 ... 2.00, which are the recommended values. Under these conditions, a desirable reduction in the safety factor can be caused by bolt material with low dynamic strength.

Further research on the influence of stiffness on the dynamic load-carrying capacity of tension-loaded bolted joints could lead to a comparative analysis of regular bolts and special elastic bolts with a reduced shank diameter (diameter smaller than the minor thread diameter), then for cases when the relative clamped length of the bolted joint  $l_b/d$  is greater than 6, the effect of temperature on the elasticity and dimensions of the bolted joint, etc.

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