PD controller design for a system of a valve controlled hydromotor

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ABSTRACT

PD controller design methodology based on D-decomposition is considered in this paper. The method consists in tuning two parameters of a controller: proportional (P) and differential (D). The method was applied to a hydromotor controlled by a valve according to the position of the valve's piston. The results obtained in this paper show the good performance of the automatic control system in comparison with other methods of controller design and other controllers designed with the same method on the valve controlled hydromotor system.

KEYWORDS

PD controller, D-decomposition, System relative stability, Hydro-motor, Valve controlled

1. INTRODUCTION

In the paper, a PD controller design methodology based on D-D decomposition was developed. The method is based on determining two parameters of the controller: proportional (P) and differential (D), while the closed loop of the automatic control system takes on the predefined design requirements. The main project requirement in this paper is related to the degree of relative stability of the closed circuit of the automatic control system.

The application of the method with this tuned controller parameters was carried out on the valve controlled hydromotor according to the position of the valve's piston. The application of the PD controller to such a system arose from the idea that the process itself is integral. This effect of the process enables the elimination of statistical error in the observed Automatic Control System.

The actual application of PD controllers in industry is rare compared to PI and PID regulators whose representation in industry is estimated at more than 90% of all applied controllers [1-2]. However, in the process we are considering, PD controllers can be successfully applied, because their main drawback, the existence of a statistical error, will be removed here by the very nature of the process we want to control.

The PD controller design method is based on D-decomposition method developed by Neimark and upgraded by Siljak [3]. The methodology is reflected in tuning of two controller parameters, so that the entire closed loop of the automatic control system takes on a preset desired degree of relative stability of the system. The method is very simple and easy to implement in industrial practice. This gives it a huge advantage compared to modern methods that require the order of the controller to be at least equal to the order of the process [4-6, 12]. This makes these methods

of tuning of controller parameters difficult to apply in industrial practice [7-9]. The advantage of our method is reflected in the fact that with the designed low-order controller, we can also control high-order processes, while at the same time fulfilling all the requirements set in front of the system from the aspect of its performance, both in the power and frequency domain [10].

The observed process in this paper is a valve controlled hydromotor according to the position of the valve piston displacement. The mathematical model of this system is given in the form of its transfer functions by desired displacement and disturbance [11].

2. DESIGN METHODOLOGY OF PD CONTROLLERS

PD controller is represented by the following transfer function:

$$W_R = K_p + K_d \cdot s \tag{1}$$

in which the transfer function of the process is as follows:

$$W_P = \frac{N(s)}{M(s)} = \frac{\sum_{k=0}^{m} b_k s^k}{\sum_{k=0}^{n} a_k s^k}, m \leq n \tag{2}$$
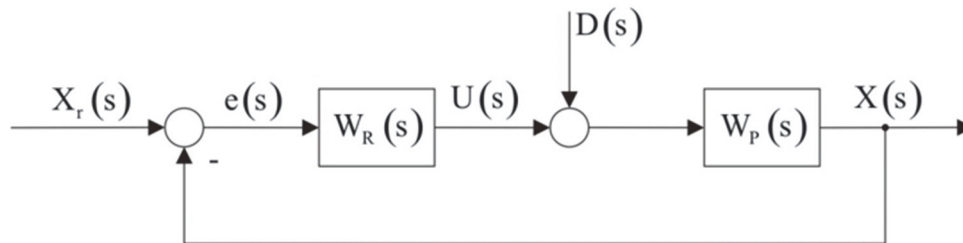

Figure 1: Closed loop control system

From Figure 1, it can be shown that the characteristic equation of the automatic control system has the following form:

$$f(s) = 1 + W_R(s) \cdot W_P(s) = 0 \tag{3}$$

$$f(s) = 1 + (K_p + K_d \cdot s) \cdot \frac{N(s)}{M(s)} \tag{4}$$

$$f(s) = M(s) + (K_d \cdot s + K_p) \cdot N(s) = 0 \tag{5}$$

$$f_1(s) = M(s) = \sum_{k=0}^{n} a_k s^k \tag{6}$$

By relating Equations (5) and (6), it can be obtained the final expression for the characteristic equation of the automatic control system in the complex domain:

$$f(s) = f_1(s) + (K_d \cdot s + K_p) \cdot N(s) = 0 \tag{7}$$

Taking into account Equation (7), it should be expressed the complex number s in the appropriate form and use it to determine the relationship between the degree of damping ξ and the variable controller parameters, Kp and Ki, included in the characteristic equation (7) for the control system. In the "s"-plane, the area below the threshold line ξ=const. (Figure 2), is mapped in the area of the corresponding damping coefficient shown by the curve ξ=const, in the plane of the controller tuning parameters (Kp, Kd).
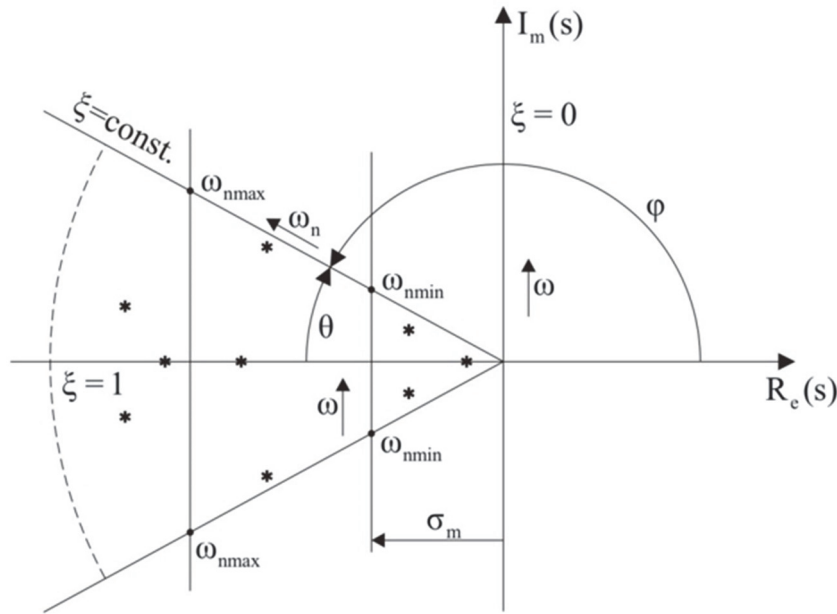
Figure 2: Frequency plane with marked area to map to parameter plane

Since:

$$s = -\omega_n \xi + j\omega_n \sqrt{1-\xi^2}, 0 \leq \xi \leq 1 \tag{8}$$

If s is given by equation (8), then s^k is expressed by the following equation:

$$s^k = \omega_n^k \left(T_k(-\xi) + j\sqrt{1-\xi^2} U_k(-\xi) \right) \tag{9}$$

or:

$$s^k = \omega_n^k \left((-1)^k T_k(\xi) + j\sqrt{1-\xi^2}(-1)^{k+1} U_k(\xi) \right) \tag{10}$$

where T_k and U_k represent Chebyshev functions of the first and second kind for which the following recurrent equations hold:

$$T_{k+1}(\xi) = 2\xi T_k(\xi) - T_{k-1}(\xi) \tag{11}$$

$$U_{k+1}(\xi) = 2\xi U_k(\xi) - U_{k-1}(\xi) \tag{12}$$

$$T_0 = 1, T_1 = \xi, U_0 = 0, U_1 = 1 \tag{13}$$

By connecting (7) and (8) it can be obtained:

$$f_1(\xi, \omega_n) + \left[K_d \left(-\xi\omega_n + j\omega_n \sqrt{1-\xi^2} \right) + K_p \right] N(\xi, \omega_n) = 0 \tag{14}$$

Now, by linking equations (6) and (10) it can be obtained:

$$f_1(\xi, \omega_n) = \alpha(\xi, \omega_n) + j\beta(\xi, \omega_n) \tag{15}$$

in which:

$$\alpha(\xi, \omega_n) = \sum_{k=0}^{n} a_k (-1)^k \omega_n^k T_k(\xi) \tag{16}$$

$$\beta(\xi, \omega_n) = \sqrt{1-\xi^2} \sum_{k=0}^{n} a_k (-1)^{k+1} \omega_n^k U_k(\xi) \tag{17}$$

The term $N(\xi, \omega_n)$ can be expressed in the following ways:

$$N(\xi, \omega_n) = \gamma(\xi, \omega_n) + j\delta(\xi, \omega_n) \tag{18}$$

$$N(\xi, \omega_n) = \sum_{k=0}^{m} b_k \omega_n^k \left((-1)^k T_k(\xi) + j\sqrt{1-\xi^2}(-1)^{k+1} U_k(\xi) \right) \tag{19}$$

Equating the equations (18) and (19), and separating the real and imaginary part, we can get:

$$\gamma(\xi,\omega_n) = \sum_{k=0}^{m} b_k \omega_n^k (-1)^k T_k(\xi) \tag{20}$$

$$\delta(\xi,\omega_n) = \sqrt{1-\xi^2} \sum_{k=0}^{m} b_k \omega_n^k (-1)^{k+1} U_k(\xi) \tag{21}$$

By linking equations (14) and (15)-(21), and separating the real and imaginary part, the following system of equations can be obtained:

$$K_d\left[\xi\omega_n\gamma(\xi,\omega_n)+\sqrt{1-\xi^2}\,\omega_n\delta(\xi,\omega_n)\right]-K_p\gamma(\xi,\omega_n) = \alpha(\xi,\omega_n) \tag{22}$$

$$K_d\left[\xi\omega_n\delta(\xi,\omega_n)-\sqrt{1-\xi^2}\,\omega_n\gamma(\xi,\omega_n)\right]-K_p\delta(\xi,\omega_n) = \beta(\xi,\omega_n) \tag{23}$$

Solving the system of equations (22)-(23) at $\omega_n{\neq}0$, $0{\leq}\xi{<}1$, they are obtained expressions for gains K_p and K_d of the PD controller. The singular straight lines are determined for $\omega_n{=}0$ and $\omega_n{=}\infty$ from the above system of equations and described by equations:

$$K_p(\xi,0) = 0 \tag{24}$$

$$K_d(\xi,\infty) = 0 \tag{25}$$

3. USE OF THE PROPOSED METHOD TO THE PROCESS OF THE HYDROMOTOR CONTROLLED BY VALVE

Using the Matlab software package, a program was written to define PD controller parameters, based on the system of equations (22)-(23) and singular straight lines (24)-(25). This program automatically plots the curve of dependence Kd as a function of the proportional gain Kp in the parameter plane for the required damping degree of closed-loop automatic control system. To show the proposed PD controller design method, hydraulic system where the motor is controlled by a directional valve, was considered. Figure 3 shows this hydraulic system.
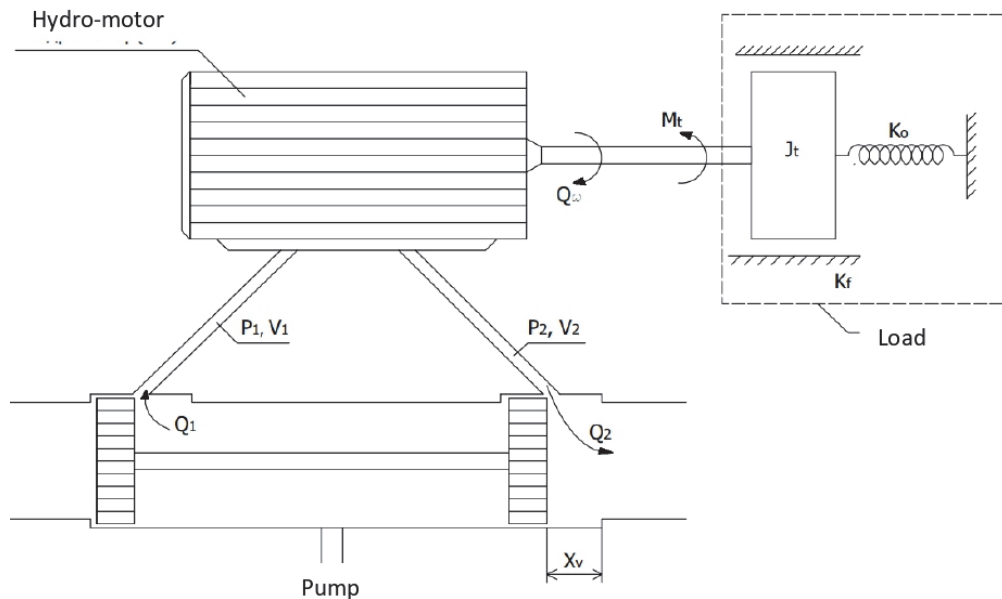

Figure 3: Hydromotor with load model

The transfer function of this system in relation to the position of the directional valve piston is given by equation (26), and in relation to the load by the equation (27).

$$Wx_v(s) = \frac{\theta_m(s)}{X_v(s)} = \frac{\dfrac{K_{Qx}}{q_m}}{s\left[\dfrac{s^2}{\omega_h^2}+\dfrac{2\delta_h}{\omega_h}s+1\right]} \tag{26}$$

$$W_M(s) = \frac{\theta_m(s)}{M_t(s)} = \frac{-\dfrac{K_{ce}}{q_m^2}\left[1+\dfrac{V_t}{4BK_{ce}}s\right]}{s\left[\dfrac{s^2}{\omega_h^2}+\dfrac{2\delta_h}{\omega_h}s+1\right]} \tag{27}$$

Where are, considering the coeficient as detalied below.

θ_m - angle of rotation on the shaft of the hydromotor,
K_{qx} - coefficient flow rate - displacement,
q_m - specific volume of the hydromotor,
X_v - displacement of the directional valve piston,
M_t - hydromotor torque,
K_{ce} - leakage coefficient on the hydromotor,
V_t - total volume of the hydromotor,
B - compressibility modulus of the hydromotor,
ω_h - undamped frequency of hydromotor,
δ_h - damping coefficient of part of the system,
Q_m - actual flow rate of the hydromotor,
n_m - number of revolutions on the hydromotor shaft.

The identification of the parameters in equation (26) was performed on the basis of the following data.

$Q_m = 60$ l/min
$n_m = 1200$ o/min
$q_m = 50$ cm^3
$\omega_h = 100$ rad/s
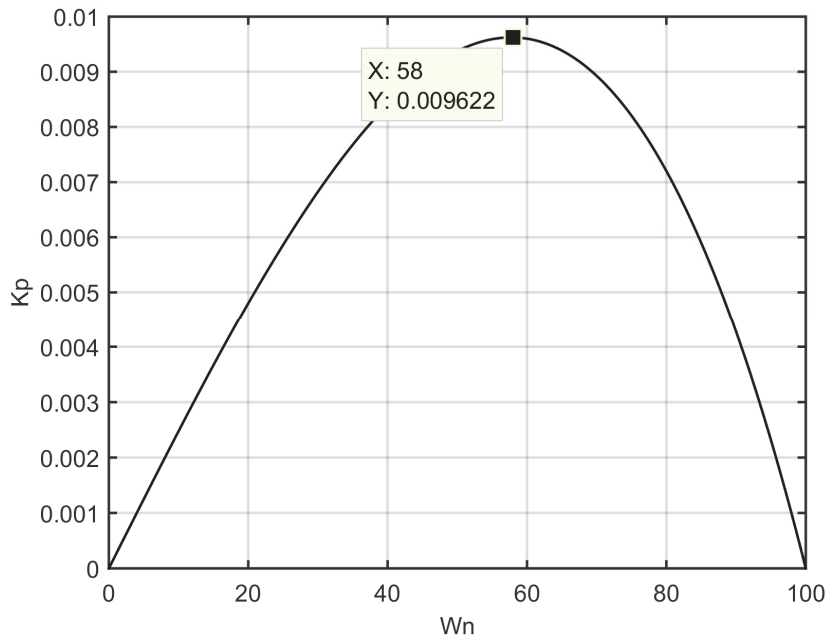$\delta_h = 0.5$
$K_{qx} = 0.2$ m^2/s

Figure 4: Proportional gain change Kp

As it can be seen, equation (26) represents a mathematical model of the hydromotor-directional valve system by the position of the directional valve piston. Figure 4. shows the dependence of the proportional gain change on the undamped frequency. Based on it, the value of the undamped frequency range, necessary for the realization of the written program in software Matlab is being chosen. After choosing the right frequency range, the program automatically draws a parametric Kd - Kp plane, based on which the values of the regulator parameters are simply read. The parametric Kd - Kp plane is shown in Figure 5. By choosing the maximum values of proportional gain and corresponding differential gain for the required degree of relative stability ξ = 0.5, the parameters of the PD regulator are completely determined.

The results obtained by the proposed methodology for the described system are presented in Figure 6. This figure shows the response of the system according to the reference, for the required degree of relative stability ξ=0.5 of the closed loop.
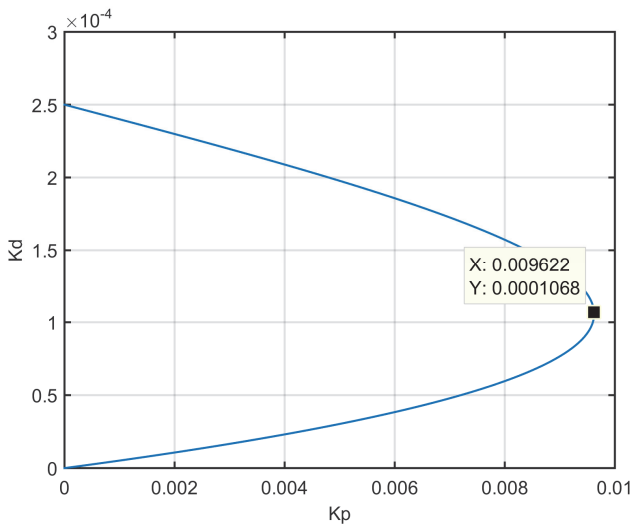
Figure 5: Parameter plane for closed loop system with transfer function W_{xv} for different values of the damping coefficients
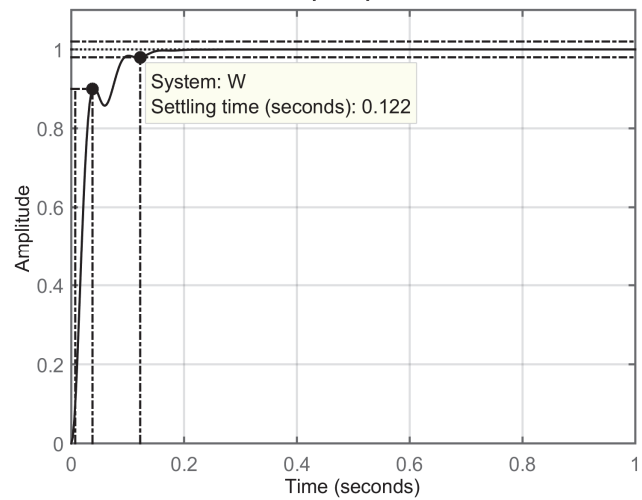

Figure 6: Automatic control system response for transfer function $W_{xv}(s)$ for the value of the damping degree $\xi = 0.5$

Figure 7 shows the comparative response of the system for the designed PD controller and the same system when it is controlled by the PI controller [19], with the same relative stability coefficient $\xi = 0.5$. The difference in performance is evident from Figure 7, which shows the superiority of the proposed PD controller over the PI controller, for the same system, under the same conditions. There is no overshoot with the PD controller, the settling time is better by 179 ms, and even the system is slightly faster.

Figure 8 shows the comparison of system responses when the proposed PD controller and the Ziegler-Nichols PI controller are applied to it. It can also be observed significantly better performances of the system response when it is controlled by the proposed PD controller.
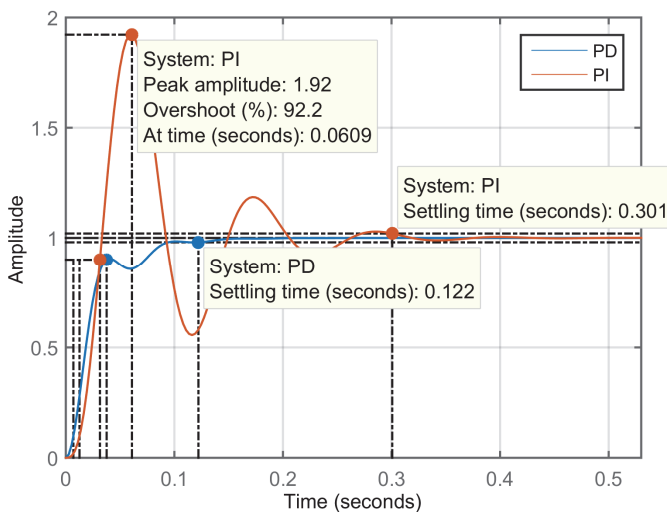

Figure 7: Comparative display of the system response for the designed PD controller with the PI controller $\xi = 0.5$
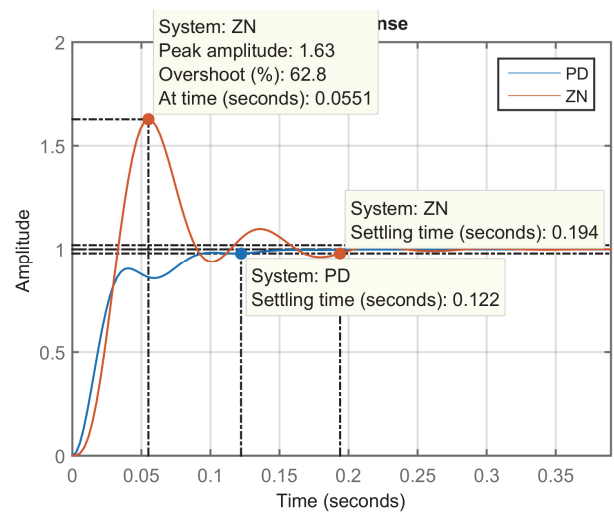

Figure 8: Comparative display of the system response for the designed PD controller with the Ziegler-Nichols PI controller

Figure 9 shows a comparison of the system responses when it is controlled with all three described controllers. Figure 10. shows a comparative view of the amplitude and phase frequency characteristics of the system when it is controlled with these three controllers. From the described picture, it can be seen that the proposed PD controller gives the highest stability margin of even 86 degrees.

Figure 10. presents a comparative view of the system response when it is controlled by the PID Ziegler-Nichols controller and proposed PD controller. The proposed controller still gives better performances. With a 46% overshoot, the settling time is 34 ms less, with almost the same response speed.
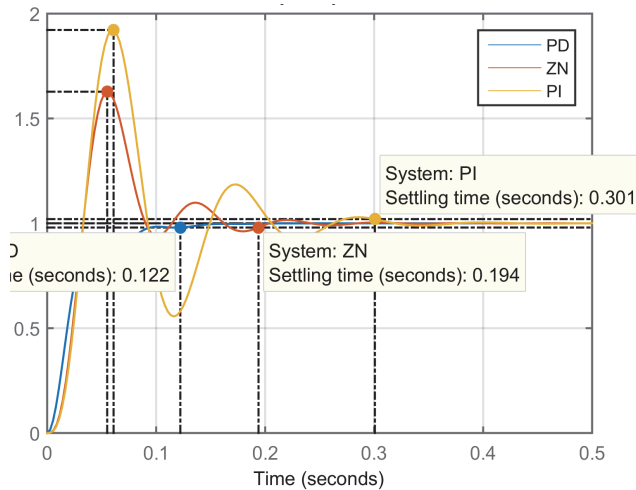
Figure 9: Comparative display of system response for designed PD controller with Ziegler-Nichols PI controller and PI controller
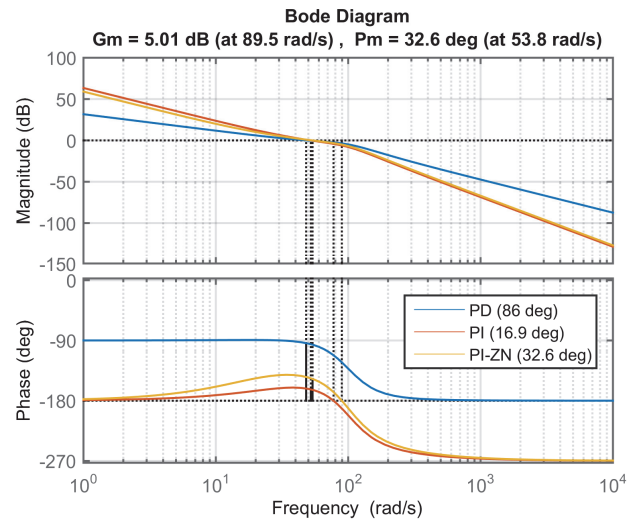

Figure 10: Comparative view of amplitude and phase frequent characteristics
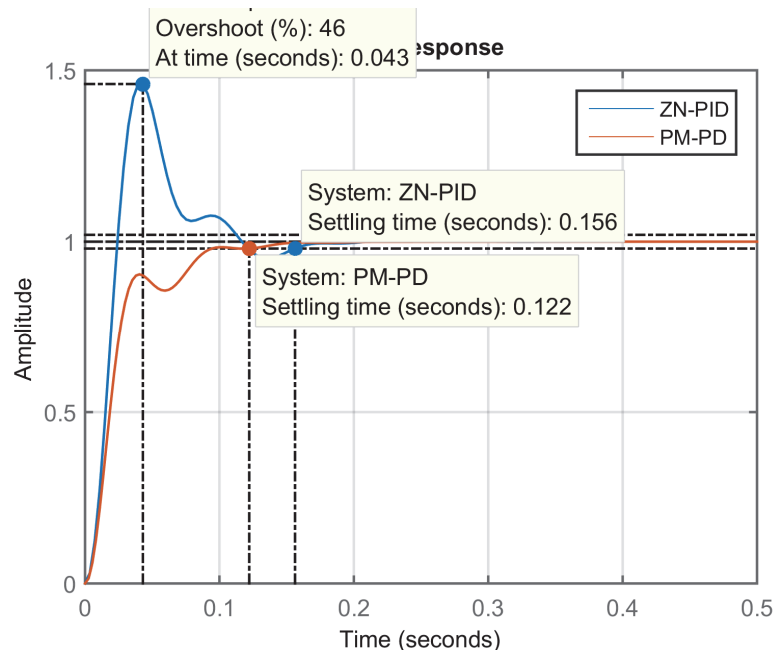

Figure 11: Comparative view of the system response for the designed PD controller with the Ziegler-Nichols PID controller

Table 1. shows the performance of the designed PD controller with other controllers that were used in this paper. They are PI controllers designed by D - decomposition, PI and PID Ziegler-Nichols controllers. In addition to performances, all the parameters of the designed controllers are given in the mentioned table.

Table 1: Comparative display of the performances of the designed PD controller with other controllers

	PM-PD	PI-DD	PI-ZN	PID-ZN
Damping degree ξ	0.5	0.5	-	-
Proportional gain K_p	0.009622	0.009281	0.01125	0.015
Differential gain K_d	0.0001068	-	-	0.0001181
Integral gain K_i	-	0.3704	0.2232	0.4762
Overshoot (reference) [%]	0	92.2	62.8	46
Settling time (reference) t_s [s]	0.122	0.301	0.194	0.156
Phase margin φ_m [°]	86	16.9	32.6	41.8

The application of the PD controller design methodology, discussed in this paper, on a hydromotor controlled valve system shows its advantages over other methods and other controllers applied to these types of processes.

4. CONCLUSIONS

In this paper, it has been shown that the PD controller, obtained by the present method provides a lot of good results, and its advantage is simplicity of the method.

The presented results show the superiority of the proposed PD controller in relation to other controllers designed using the same method. Also, the proposed controller is better than the controllers obtained according other methods. Comparisons with Ziegler-Nichols PI and PID controllers are shown here for illustration. The superiority of the proposed controller in relation to other controllers designed for the considered system is also in the very nature of the described process.

In future research, the Authors plan to show the advantages of the proposed methodology on all integral processes, both those without time delay and those with a relatively long time delay.

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