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Intuitive Understanding of Probability in Primary School Students: A Comparative Study of Coin Toss Problems in Serbia and the Czech Republic

Slađana Dimitrijević [1, a, *], Aleksandar Milenković [1, b], Tomas Zdrahal [2, c]
[1]University of Kragujevac, Faculty of Science, Kragujevac, Serbia
[2]Palacký University, Pedagogická fakulta, Olomouc, Czech Republic
[a]sladjana.dimitrijevic@pmf.kg.ac.rs, [b]aleksandar.milenkovic@pmf.kg.ac.rs, [c]tomas.zdrahal@upol.cz

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Abstract

With this paper, we try to emphasize the importance of teaching probability in upper grades at primary school, focusing on the need to integrate intuitive reasoning about uncertainty with structured school learning. Within this research, we conducted testing of 1086 students from Serbia ($n = 798$) and the Czech Republic ($n = 288$) aged 11.5– 14.5, where primary school students do not learn about probability as a part of the compulsory mathematics curriculum, with the aim to test intuitive understandings of basic probability concepts. The test administered to students included problems of varying complexity and difficulty related to tossing one or more coins. Within the test, we wanted to analyze the degree of students' intuition about the concepts of certain and random events, determining the probability for random events (in the elementary cases), understanding the relative frequency of some random event as an estimate of the probability of that random event, and recognizing independent events. Results of the research indicate that certain, random, and impossible events are well understood in students' intuition, as well as calculating probability in simple cases and recognizing equally likely outcomes or independent random events. Still, on the other hand, estimating the frequency of the random event (expected value for a binomial random variable) is considerably less intuitively clear to students. Considering the research results, educational institutions can reassess and update curricula to better support students' development in this important mathematical concept. Integrating basic elements of probability theory into the primary school mathematics curriculum could significantly enhance students' understanding of random phenomena and probability laws.

Keywords: students' intuition, probability, random events, estimates, independent events.

MSC2020: 97K50

1. Introduction

Intuition is an important part of acquiring and processing information in an individual's mind; the thinking process can be implicit (unconscious) or explicit (conscious). It can be said that thinking that occurs automatically or unconsciously is an indicator of thought that involves intuition. In problem-solving, intuition acts as a bridge toward finding solutions. Since mathematics is characterized by solving various types of problems, intuition plays an important role in students' mathematics learning. Intuition can be highly beneficial in problem-solving because it allows quick

[*] Corresponding author
E-mail address: sladjana.dimitrijevic@pmf.kg.ac.rs

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responses that often seem obvious and are accepted with high confidence. While this can sometimes be valid, in other cases, such behavior leads to systematic errors.

One area of mathematics where intuition can significantly contribute to the proper understanding of mathematical concepts and problem-solving is probability theory. It is important to note that basic probability terms such as "probably", "random", "certain", "uncertain", and "expected" are part of students' everyday vocabulary. However, their usual meanings are much broader and less defined than in a mathematical context. Since students are not introduced to the mathematical definitions of these terms, they tend to rely on their everyday meanings, trying to adapt them to mathematical problems that typically demand precise and unambiguous results. Consequently, it is not surprising that students often struggle to solve such problems.

In many countries, including Serbia and the Czech Republic, probability is not a part of the mandatory curriculum before high school. In Serbia specifically, students are introduced to probability only during their fourth and final year of high school, a time when they are focused on finishing their studies and making decisions about employment or university, leaving them insufficiently prepared for an in-depth exploration of the subject. Under these circumstances, students often barely acquire any knowledge of probability during their pre-university education. Considering the widespread applications of probability in everyday life and across various fields of science - such that there is scarcely a scientific discipline untouched by its principles - the questions arise: Can students, guided by their common sense and intuition, solve basic probability problems, or do they require structured support in specific areas of elementary probability concepts? Should these topics be integrated into the primary school mathematics curriculum? This paper seeks to contribute to addressing these important questions.

2. Theoretical framework

Intuition is described as a personal and immediate perception of an idea that seems obvious to those who possess it. It is usually the product of an individual's mental development and, concerning certain phenomena, should be carefully cultivated in students. Intuition naturally develops as a result of experiences with the stochastic nature of the environment (Milinković, 2015). Our everyday learning about "life" is largely grounded in our statistical observations, i.e., data we gather through empirical experience.

Since intuitive responses are based on students' personal experiences and their formal education, the tendency to give intuitive answers increases with the student's age and level of education, which can result in a rise in specific reasoning errors (Morsanyi & Szücs, 2015).

Intuition is a person's ability to understand something without relying on the theory behind it or standard rules (reasoning and intellect) (Tieszen, 2015). Since understanding the task is the first step in solving mathematical problems, intuition is useful for improving problem-solving abilities, as emphasized by Wijaya et al. (2021), among others. Studies have shown that intuitive thinking is widely used for understanding concepts, solving problems, engaging in creative thinking, and higher-order thinking in mathematics learning (Puspitasari et al., 2018). The basic concept of intuitive thinking involves using prior basic knowledge as a foundation for understanding the material being learned and then independently finding the solution (Tieszen, 2015). In mathematics, intuitive thinking is a cognitive process that generates ideas as a choice of strategy for decision-making in problem-solving through spontaneous responses (Fischbein, 1999).

Intuitive thinking can contribute to improving the solving of mathematical problems in areas such as numbers, geometry, algebra, functions, and calculus (Suwarto et al., 2023). According to these authors, "the process of intuitive thinking is produced by students having high levels of confidence, justification not always being the same as an intuitive response, and students rejecting intuitive answers" (Suwarto et al., 2023, p.2).

Researchers have discovered the role of intuitive thinking in solving mathematical problems through various studies using experimental, correlational, and qualitative methods. These studies' findings suggest that intuitive thinking can help students understand numbers and probability and that intuition serves as a mediator in mathematics education (Barahmand, 2019; Cengiz et al., 2018; Furlan et al., 2016).

When it comes to the importance of students following their intuition while acquiring new knowledge in mathematical analysis, research conducted by Roh & Lee confirmed that relying on students' intuition when introducing abstract concepts leads to better students' understanding (Roh & Lee, 2017).

The results of a comparative analysis conducted by van der Ham et al. (2017) with students from the Netherlands and Senegal indicate that intuition is universal, meaning that sociocultural and geographical conditions do not influence it. However, there are individual differences in how intuition functions in students' problem-solving, specifically in this case, within the field of 3D geometry (Van der Ham et al., 2017).

Zamora (2014) discusses intuition in probability and distinguishes between two types of intuition: primary intuition, which is mainly acquired through experience, and secondary intuition, which is developed through education, usually in a school setting. Milinković (2015) claims that in the case of intuition about the frequency of events, we tend to make judgments based on previous time-limited experience, which we estimate to be adequate.

Students often base their decisions on intuition, but their intuition is usually inadequate when the concept of chance is involved. However, according to Fischbein, as cited by Alvarado et al. (2018), students' confidence in their intuition should not be ignored; it must be developed in regular teaching so that students can use it in decision-making throughout their lives. Without formal education, children cannot reach an operational concept of probability, and students' intuition about probability is formed by the age of 14 or 15 (Fischbein et al., 1970). In several studies, the authors state that it is necessary to introduce probability concepts at the pre-university level through new methodological approaches in order to stimulate probabilistic reasoning in students (Batanero, 2006; Ramírez-Contreras et al., 2023). Moreover, the development of probabilistic intuition in the early stages of students' education is crucial, as otherwise, students achieve poor results in probability when more complex concepts in probability and statistics are introduced (Batanero et al., 2021; Zamora, 2014).

Babai, Brecher, and Stavy examined the influence of intuition on solving probability tasks in their research (Babai et al., 2006). The results of their study indicate that correct answers aligned with students' intuition are significantly more prevalent and that students provided these answers in a shorter amount of time compared to correct answers in tasks where the solutions are counterintuitive. Regarding the level of mathematical education, the results suggest that the number of correct answers is positively correlated with the level of mathematical education (Babai et al., 2006).

In a previously conducted study in Serbia, Milinković (2015) showed that elementary school students possess intuition about the concept of chance. This study provides significant insights into how children naturally develop a sense of probability through everyday experiences while emphasizing the importance of prior understanding of certain mathematical concepts. Specifically, the findings indicate that mathematical concepts such as fractions and proportions are essential for properly developing intuition about probability. Children who have mastered these basic mathematical concepts have a better foundation for developing formal stochastic reasoning. Understanding fractions, proportions, and rational numbers helps them to accurately interpret probabilities and positively influence stochastic reasoning. Therefore, it is suggested that mastering these mathematical topics should precede the introduction of probability content in teaching. In other words, students should first acquire a solid knowledge of fractions, their operations, and rational numbers so that later learning about probability becomes more accessible and intuitive. This approach allows a gradual development of mathematical understanding and provides a better foundation for adopting more complex stochastic concepts in later stages of education.

3. Methodology

The aim of the research was to investigate the extent to which students of upper grades of primary school (second cycle, age 11-14) have developed the intuition to calculate the probability of random events and to recognize equally likely outcomes in case of tossing a coin as well as to what extent they intuitively understand a certain event, the independence of random events and can they give reasonable estimates of the frequency of random events in the case of repeating trials (can they recognize the expected value in the case of binomial distribution). This population of students was chosen for research because we, as many other researchers (Fischbein, 1970; Milinković, 2015), believe that initial concepts and statements about probability should be introduced and developed alongside the idea of fractions and proportionality, the study of which is part of the curriculum of the second cycle of primary school. Additionally, the objective included comparing the responses and success of students from two countries, the Republic of Serbia and the Czech Republic, where probability content is not part of the mandatory curriculum in the upper grades of primary school. Also, we investigate the role of the age of students regarding the level of proper intuitional reasoning about considered basic probability problems. The research objectives arise from the aim of the study — to assess the level of intuition development for:

1. calculating the probability in the most straightforward random experiment—tossing one regular coin;
2. recognizing the difference between a certain event and random events;
3. reasonable estimating of frequency of random events in the case of repeating trials (recognize the expected value in the case of binomial distribution);
4. recognizing independent events.

The data analyzed in this paper was collected as part of a broader research project on students' intuitive understanding of probability. During the study, students provided personal information about themselves (gender, grade, mark from mathematics, if they have any experience with games that involve rolling dice) and answered 21 questions (problems) in a written questionnaire with a time limit of 45 minutes. There were fifteen multiple-choice questions and six open-ended tasks. Five questions were about tossing coins, and sixteen were about rolling dice. Students were voluntarily and anonymously engaged in problem-solving activities during school class with their teachers present. In the beginning, teachers provided short explanations regarding the purpose of data collection—to create a dataset for scientific research. The data collection occurred in schools in the Republic of Serbia and the Czech Republic during 2022 and 2023.

The research sample consists of 538 boys and 529 girls from the higher primary school grades in the Republic of Serbia and the Czech Republic. Students in Serbia start primary school at the age of 6.5 to 7.5 and have eight grades, while students in the Czech Republic start school when they are one year younger and attend nine grades. So, for students in the Republic of Serbia, the second education cycle begins when they enter the 5th grade, while their peers from the Czech Republic attend the 6th grade. Therefore, in tables, we will write the average age of students instead of grade numbers. The sample is described in Table 1.

Table 1. Distribution of students by age and country

Country/Age	11.5 years	12.5 years	13.5 years	14.5 years	Total
Republic of Serbia	191 (23.9%)	202 (25.3%)	191 (24%)	214 (26.8%)	798 (100%)
Czech Republic	69 (24%)	81 (28.1%)	74 (25.7%)	64 (22.2%)	288 (100%)
Total	260 (23.9%)	283 (26.1%)	265 (24.4%)	278 (25.6%)	1086 (100%)

The statistical data analysis was conducted using the IBM SPSS Statistics 20 software package. The Chi-square test of independence and binary logistic regression were used to examine whether there is an association between variables.

4. Results and Discussion

First, we emphasize that, as was naturally expected, the great majority of students, 97.15% of them, answered that they have experience with games that involve rolling dice. So, they have some experience to build on intuition.

In this paper, we analyze the results for five problems that deal with tossing coins.

With the first research objective, by Problem 1, we aimed to determine the extent to which students have developed intuition for determining the probability of a simple random event (one of two equally likely outcomes). In the text of the problems, we use the word "chance" because, in everyday language, it is a synonym for the word "probability", and it is used more frequently.

Problem 1. *When tossing a coin, a head or a tail may fall. What is the chance that the tail will fall?*

It is important to emphasize that the great majority of students see chance as a number. 79.44% of students gave a number as an answer, 10.04% gave a descriptive answer (the chance is small/big and similar), and only 4.67% believed it was impossible to determine the chance. Other students (5.85%) did not answer or wrote that they did not know the answer. Table 2 presents the number and percentage of students from both countries and for each grade who provided the correct numerical answer when solving Problem 1. The percentages were calculated based on the total number of students of the same age, first for each country separately and then for both.

Table 2. The number and the percentage of correct numerical answers for Problem 1.

Country/Age	11.5 years	12.5 years	13.5 years	14.5 years	Total
Republic of Serbia	108 (56.5%)	134 (66.3%)	121 (63.3%)	152 (71.0%)	515 (64.5%)
Czech Republic	28 (40.5%)	49 (60.5%)	43 (58.1%)	37 (57.8%)	157 (54.5%)
Total	136 (52.3%)	183 (64.6%)	164 (61.9%)	189 (68.0%)	672 (61.8%)

Results show that there are statistically significant but slight differences in the percentage of students from the two countries who provided correct numeric answers to the first task ($\chi^2 = 15.42, df = 1, p = 0.001 < 0.05, Phi = 0.12$). Of the 798 students from Serbia, nearly two-thirds (64.5%) answered the question correctly, while among the 288 students from the Czech Republic, 10% fewer students provided a correct numerical answer (157 students, or 54.5%). In the future, it would be good to examine the reasons for the difference further or to test this again with representative samples of similar size from both countries. Since the biggest difference is in the group of the youngest students, one hypothesis could be that the reason is different levels of knowledge about fractions caused by minor differences in teaching mathematics programs.

When we analyzed whether students' age significantly impacted the accuracy of their responses to this question, results showed a significant but weak association between age and accuracy for students from Serbia ($\chi^2 = 9.01, df = 3, p = 0.003 < 0.05, Phi = 0.11$). However, for students from the Czech Republic, this association was not confirmed ($\chi^2 = 7.24, df = 3, p = 0.065$), although it approached significance. While the oldest Serbian students demonstrated the highest accuracy, the percentage of correct numeric answers does not consistently increase with age, indicating the need for further research. The binary logistic models (see Appendix 1.1) suggest that there is no significant difference between the results of 11.5-year-old and 13.5-year-old students and between the results of 12.5-year-old and 14.5-year-old students in Serbia. This trend may also relate to the mathematics curriculum, as students around age 12.5 focus on fractions, rational numbers, and proportions.

With the second research objective, Problem 2, we examined the students' intuition for recognizing a certain and random event.

Problem 2. *If a coin is tossed 2 times, the tail will DEFINITELY fall:*

a) Twice; b) Once; c) Never; d) Cannot be determined.

The majority of students who answered recognized that the frequency of a random event's occurrence in repeated trials cannot be precisely determined with certainty (Fig. 1).

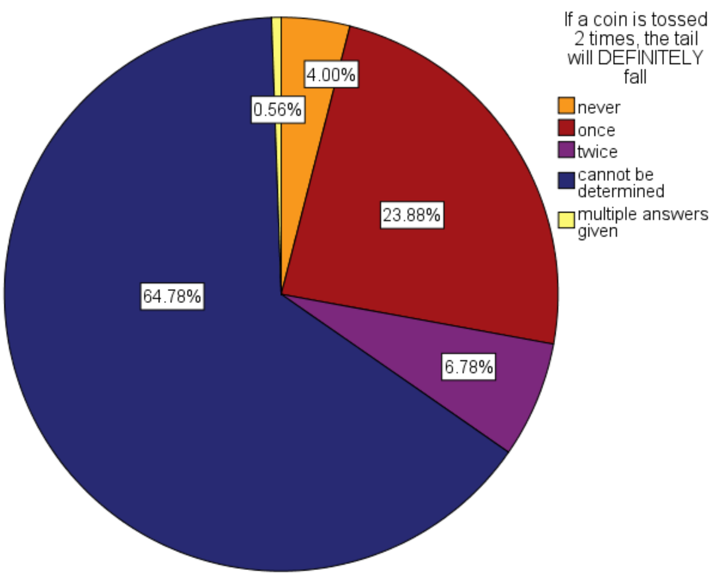

Fig. 1. Students' answers to Problem 2.

The number of correct answers and appropriate percentages for students based on their age from Serbia and the Czech Republic, as well as for the entire sample, is presented in Table 3.

Table 3. The number and the percentage of correct answers for Problem 2.

Country/Age	11.5 years	12.5 years	13.5 years	14.5 years	Total
Republic of Serbia	107 (56.0%)	137 (67.8%)	103 (53.9%)	163 (76.2%)	510 (63.9%)
Czech Republic	31 (44.9%)	58 (72.5%)	51 (68.9%)	47 (73.4%)	187 (65.2%)
Total	138 (53.1%)	195 (69.1%)	154 (58.1%)	210 (75.5%)	697 (64.2%)

If we compare the percentages of students from Serbia and from the Czech Republic who answered this task correctly, we see that these numbers are pretty similar in Serbia and in the Czech Republic ($\chi^2 = 0.143, df = 1, p = 0.705$). Additionally, results indicate that the student's answers are in weak relation to the student's age ($\chi^2 = 36.48, df = 3, p < 0.0005, Phi = 0.18$). Similarly to Problem 1, but now for the whole sample, the binary logistic model (see Appendix 1.2) suggests that there is no significant difference between the results of 11.5-year-old and 13.5-year-old students and between the results of 12.5-year-old and 14.5-year-old students.

We also aimed to examine students' ability to estimate the frequency of a random event across repeated trials, precisely to determine the mathematical expectation of a binomial random variable that represents the number of tails obtained when tossing a coin. To this end, we presented Problem 3 and Problem 4 to the students, with different numbers of trials and different types of tasks.

Problem 3. *If a coin is tossed 2 times, how many times SHOULD you EXPECT the tail to fall?*
a) Twice; b) Once; c) Never; d) Cannot be determined.

Table 4 presents the number and percentage of students from both countries and for each grade who provided the correct numerical answer when solving Problem 3.

Table 4. The number and the percentage of the correct numerical answers for Problem 3.

Country/Age	11.5 years	12.5 years	13.5 years	14.5 years	Total
Republic of Serbia	55 (28.8%)	62 (30.7%)	76 (39.8%)	67 (31.3%)	260 (32.6%)
Czech Republic	29 (42.0%)	26 (32.1%)	27 (36.5%)	28 (43.8%)	110 (38.2%)
Total	84 (32.3%)	88 (31.1%)	103 (38.9%)	95 (34.2%)	370 (34.1%)

Comparing the percentages of students who answered the task correctly from Serbia (32.6%) and from the Czech Republic (38.2%), we found that the difference is not statistically significant for students from the two countries ($\chi^2 = 2.97, df = 1, p = 0.085$). It is noteworthy that, unlike the first two tasks, where the percentages of students who answered correctly were above half (up to two-thirds), in this case, that percentage is significantly lower, just over one-third of the students. Specifically, a careful examination of the table reveals that in neither country nor any grade, the percentage of students who solved the task correctly reached half the total number of students. Additionally, unlike the first two tasks, in this task, the relationship between the students' answers and their ages is not confirmed ($\chi^2 = 4.19, df = 3, p = 0.242$).

Notably, most students (who answered this question) indicate that they find it impossible to estimate the frequency of an event (see Fig. 2). Their responses can be interpreted as an expression of their recognition of random events, reflecting a correct understanding that it is impossible to predict with certainty how many times a particular event will occur in repeated trials. This response indicates that the question may have been misunderstood. In future research, we plan to present multiple versions of this type of question to ensure students' comprehension or, probably even better, to conduct qualitative research through student interviews to clarify their understanding.

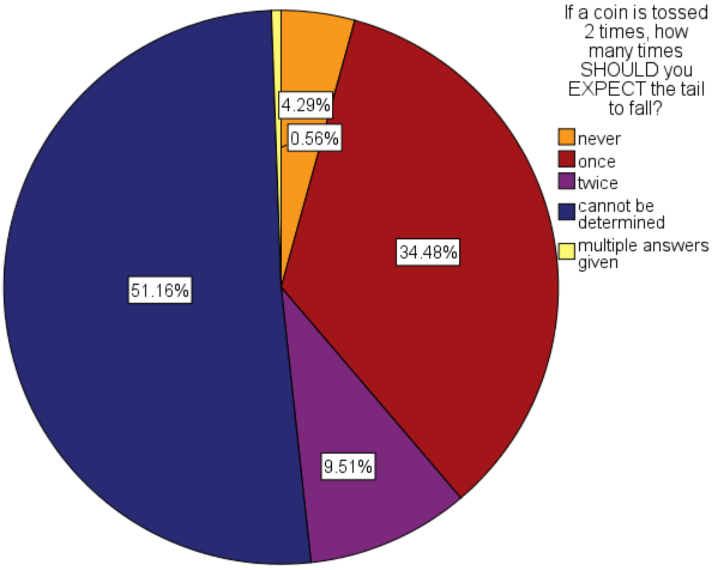

Fig. 2. Students' answers to Problem 3.

To gain a more detailed insight into the students' intuitive reasoning and to see if a greater number of repeating trials influenced it, we assigned them another similar task, this time an open-ended one.

Problem 4. *If a coin is tossed 20 times, how many times SHOULD we EXPECT the tail to fall?*

Table 5 presents the number of correct numeric answers for Problem 4 and appropriate percentages for students based on their age from Serbia and the Czech Republic, as well as for the entire sample.

Table 5. The number and the percentage of the correct numeric answers for Problem 4.

Country/Age	11.5 years	12.5 years	13.5 years	14.5 years	Total
Republic of Serbia	64 (33.5%)	70 (34.6%)	71 (37.2%)	67 (31.3%)	272 (34.1%)
Czech Republic	19 (27.5%)	21 (25.9%)	13 (17.6%)	30 (46.9%)	83 (28.8%)
Total	83 (31.9%)	91 (32.1%)	84 (31.7%)	97 (34.9%)	355 (32.7%)

In the fourth task, as in the previous one, there is no statistically significant association between students success in solving the problem and their country of origin ($\chi^2 = 2.67, df = 1, p = 0.102$). In Serbia, the percentage of correct numerical answers is 34.1%, whereas the percentage of students from the Czech Republic who solved the task correctly is 28.8%. These results further reinforce the findings from the previous task analysis, indicating that students' intuition is still insufficient for understanding and solving more complex probability problems. Also, there is no statistically significant influence of students' ages on their answers ($\chi^2 = 0.84, df = 3, p = 0.840$). However, we can note that the oldest students (9th grade) in the Czech Republic demonstrate a higher level (46.9% of correct answers) of understanding of this problem than the others.

It is interesting to emphasize that, in the case of this open-ended problem, students who provided an answer wrote a number between 0 and 20, which implies that they have a good understanding of the possible values that the variable representing the number of fallen tails can take. The most frequent (33.43% given answers) of the given numeric responses is the correct answer, indicating that the tail is expected to fall 10 times. The percentage of those who say that it is impossible to determine that number (26.24%) is lower than in the previous Problem 3. Also, 2.22% of students said that they did not know the answer (Fig. 3).

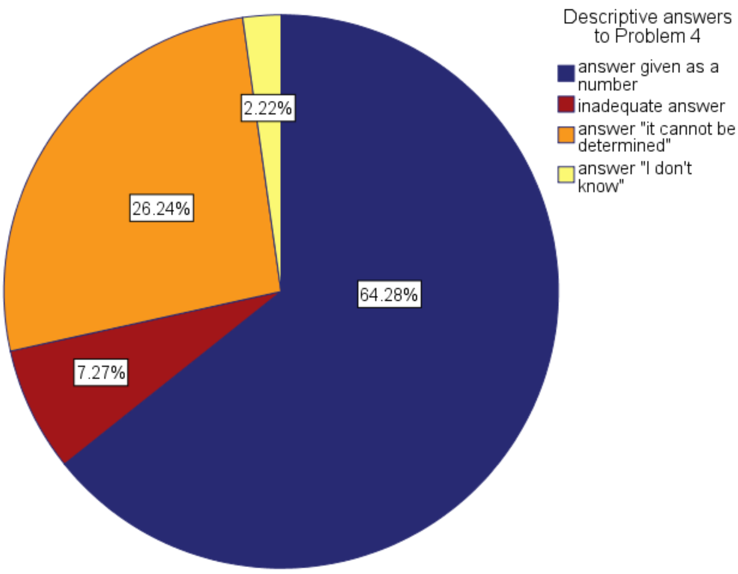

Fig. 3. Types of students' answers to Problem 4.

Combining the results of Problems 3 and 4, we established that the majority of students (71.18% of them) give consistent answers on those two problems (Table 6), ae. there is a significant modest association between answers ($\chi^2 = 133.190, df = 1, p < 0.0005, Phi = 0.352$). Therefore, we can conclude that changing the number of repeating trials does not significantly change students' reasoning. Since Problem 4 is open-ended, these consistent answers indicate that students did not randomly choose answers in Problem 3.

Table 6. The number of correct numerical and other students' answers for problems 3 and 4.

		Problem 4		
		Correct number	Other answers	Total
Problem3	Correct number	206	164	370
	Other answers	149	567	716
	Total	355	731	1086

For the final task, we aimed to examine the level of students' intuition regarding independent events. The task was as follows.

Problem 5. *Peter tossed the coin three times, and all three times, the tail fell. When Peter tosses the coin for the fourth time:*

a) The chances of a falling head are higher than a falling tail.

b) The chances of a falling tail are higher than a falling head.

c) The chances of falling are equal for the tail and the head.

Table 7 presents the number and percentage of students from both countries and for each grade who provided the correct answer when solving Problem 5.

Table 7. The number and the percentage of the correct answers for Problem 5.

Country/Age	11.5 years	12.5 years	13.5 years	14.5 years	Total
Republic of Serbia	116 (60.7%)	146 (72.3%)	126 (66.0%)	166 (77.6%)	554 (69.4%)
Czech Republic	36 (52.2%)	54 (66.7%)	50 (67.6%)	44 (68.8%)	184 (63.9%)
Total	152 (58.5%)	200 (70.7%)	176 (66.4.7%)	210 (75.5%)	738 (68.0%)

Based on the data ($\chi^2 = 2.98, df = 1, p = 0.084$) there is no statistically significant difference in the overall percentages of correct answers between students from these two countries. It is also noteworthy that many students recognized the independence of events in the case of repeating coin tossing. More than two-thirds of students (67.9%) answered the question correctly. Additionally, results ($\chi^2 = 19.35, df = 3, p < 0.0005, Phi = 0.13$) indicate that the increase in students' age is generally followed by an increase in the percentage of correct answers. The binary logistic models for the whole sample (see Appendix 1.3) suggest the same as for Problems 1 and 2 that there is no significant difference only between the results of 11.5-year-old and 13.5-year-old students and between the results of 12.5-year-old and 14.5-year-old students.

Based on the first result, we can conclude that, in various basic situations, the intuitive calculation of the probability of a random event using the classical definition of probability is well-developed among students. Although students have not officially encountered these concepts and procedures in their formal education, just over three-fifths solved the task correctly. Additionally, it was observed that students from Serbia, unlike students from the Czech Republic, showed noticeable improvement among older students in this context. These differences may be attributed to the types of tasks teachers use when introducing fractions, the interpretation of fractions as quotients, rational numbers, and proportions - key concepts for developing an intuition about probability (Milinković, 2015). Students likely recognized similarities between the tasks in this test and those they had encountered in their school environment. Of course, these assumptions should be examined in more detail in the future.

Furthermore, when examining students' success in recognizing certain and random events, the results indicate that students generally have a well-developed intuition for distinguishing between a random event that is not certain and a certain event. Specifically, nearly two out of three students in the entire sample successfully recognized that none of the events presented in the second problem were a certain event. This intuition, likely enhanced by the precision required of students when solving tasks and making reasoned conclusions, develops further with their growth and education. As a result, the oldest students make better judgments when recognizing a certain event across the entire sample and among students from Serbia and the Czech Republic separately.

Unlike calculating probabilities for a simple random event and determining whether an event is random or certain, results for estimating the frequency of outcomes in repeating trials are slightly different. Specifically, on average, only one out of three students correctly concludes that when tossing a coin twice, we expect tail to appear once. Very few students (only about 2%) perform worse when the number of coin tosses increases and when they are not given multiple-choice options. The percentage of correct numeric answers in both countries is quite similar, indicating no statistically significant difference in the number of correct numeric answers between students from Serbia and the Czech Republic. Additionally, unlike the previous two research objectives, where intuition generally improved with the increasing age of the students, there are no differences in student performance based on age (school grade). The only exception is noted in the more complex task for the sample of students from the Czech Republic, particularly among those in the final year of primary school (9th grade). The reasons for students not performing well in solving these tasks may also be found in the problem's wording, as many students stated that it is impossible to estimate the frequency of the event, and for more detailed insights, new researches are needed.

When it comes to recognizing independent events (specifically in the context of the outcomes of coin tosses given the results of several previous tosses), students also demonstrate sound reasoning based on their intuition, and no significant differences were observed in the distribution of correct and incorrect answers between students from the two countries. Again, on average, two out of three upper-grade primary school students from both countries answered this task correctly.

5. Conclusion

We can assert that our findings align with van der Ham et al. (2017) findings, reinforcing the idea that intuition about probability is universal. Notably, in only one out of five problems, students' responses were associated with their country of origin.

We can summarize that students have a well-founded intuition about certain, random, impossible, and independent random events. However, they are less accurate in calculating probability in simple cases and recognizing equally likely outcomes, which usually improve through time. On the other hand, the estimates of the frequency of random events in repeating trials, which are equivalent to determining mathematical expectation in the case of binominal distribution, are significantly less intuitively recognized by students, and their intuitive understanding of this concept does not improve as they progress through upper primary grades. Moreover, based on the research results, it can be observed that upper primary school students have a good intuition for qualitatively describing uncertain events. However, their success decreases when required to numerically represent an event's probability. This suggests that, in order to tackle more complex probability problems and to grasp the mathematical framework for representing random events, students need structured assistance through formal mathematical education. Once again, for a more detailed insight into the development of students' intuition regarding elements of probability, the study should be repeated with an even larger sample of upper primary school students, including somewhat more complex problems and qualitative research methods.

An implication that can be drawn from the results is that elements of probability should be included in the curriculum for upper primary students to prevent misunderstandings that may lead to severe misconceptions about probability concepts. Considering the research findings, educational institutions might reconsider their teaching plans and programs to better support students in understanding these important mathematical concepts by integrating basic elements of probability theory into the mathematics curriculum for upper primary grades. This approach could significantly enhance students' understanding of random phenomena and the laws of probability. Such integration would not only improve students' competencies in mathematics but also foster the development of their critical thinking and problem-solving skills and the application of mathematics in real-life situations.

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Appendix 1.1

Table 8. Binary logistic regression analysis for Serbian students' success in solving problem 1, using 11.5 years of age as the reference category.

	B	S.E.	Wald	df	Sig.	Exp(B)	95% C.I. for EXP(B)	
							Lower	Upper
Age			28.008	3	.000			
Age (12.5)	.504	.210	5.772	1	.016	1.655	1.097	2.495
Age (13.5)	-.085	.206	.169	1	.681	.919	.614	1.375
Age (14.5)	.920	.217	18.008	1	.000	2.509	1.641	3.837
Constant	.242	.146	2.756	1	.097	1.274		

Table 9. Binary logistic regression analysis for Serbian students' success in solving problem 1, using 14.5 years of age as the reference category.

	B	S.E.	Wald	df	Sig.	Exp(B)	95% C.I. for EXP(B)	
							Lower	Upper
Age			28.008	3	.000			
Age (13.5)	-.920	.217	18.008	1	.000	.399	.261	.610
Age (12.5)	-.416	.220	3.579	1	.059	.659	.428	1.015
Age (11.5)	-1.005	.216	21.555	1	.000	.366	.240	.560
Constant	1.162	.160	52.444	1	.000	3.196		

Appendix 1.2

Table 10. Binary logistic regression analysis for students' success in solving problem 2, using 11.5 years of age as the reference category.

	B	S.E.	Wald	df	Sig.	Exp(B)	95% C.I. for EXP(B)	
							Lower	Upper
Country (1)	-.079	.147	.293	1	.588	.924	.693	1.231
Age			36.193	3	.000			
Age (12.5)	.683	.179	14.526	1	.000	1.979	1.393	2.811
Age (13.5)	.203	.176	1.333	1	.248	1.225	.868	1.730
Age (14.5)	1.007	.187	29.032	1	.000	2.738	1.898	3.951
Constant	.182	.165	1.217	1	.270	1.199		

Table 11. Binary logistic regression analysis for students' success in solving problem 2, using 14.5 years of age as the reference category.

	B	S.E.	Wald	df	Sig.	Exp(B)	95% C.I. for EXP(B)	
							Lower	Upper
Country (1)	-.079	.147	.293	1	.588	.924	.693	1.231
Age			36.193	3	.000			
Age (13.5)	-1.007	.187	29.032	1	.000	.365	.253	.527
Age (12.5)	-.325	.190	2.917	1	.088	.723	.498	1.049
Age (11.5)	-.804	.187	18.459	1	.000	.447	.310	.646
Constant	1.189	.180	43.540	1	.000	3.284		

Appendix 1.3

Table 12. Binary logistic regression analysis for students' success in solving problem 5, using 11.5 years of age as the reference category.

	B	S.E.	Wald	df	Sig.	Exp(B)	95% C.I. for EXP(B)	
							Lower	Upper
Country (1)	.240	.146	2.702	1	.100	1.272	.955	1.694
Age			18.840	3	.000			
Age (12.5)	.544	.182	8.977	1	.003	1.723	1.207	2.460
Age (13.5)	.344	.181	3.610	1	.057	1.411	.989	2.013
Age (14.5)	.779	.188	17.149	1	.000	2.180	1.507	3.152
Constant	.166	.165	1.014	1	.314	1.181		

Table 13. Binary logistic regression analysis for students' success in solving problem 5, using 14.5 years of age as the reference category.

	B	S.E.	Wald	df	Sig.	Exp(B)	95% C.I. for EXP(B)	
							Lower	Upper
Country (1)	.240	.146	2.702	1	.100	1.272	.955	1.694
Age			18.840	3	.000			
Age (13.5)	-.779	.188	17.149	1	.000	.459	.317	.663
Age (12.5)	-.235	.191	1.505	1	.220	.791	.543	1.151
Age (11.5)	-.435	.191	5.176	1	.023	.647	.445	.942
Constant	.945	.177	28.382	1	.000	2.573		