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Teacher and Teaching Mathematics, Development of (Mathematical) Thinking

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Abstract

Each period of social development is marked by a corresponding pedagogy and accompanying didactics, that is, special forms of didactics. In that sense, the methodology of teaching mathematics is also determined by various influences induced by complex relations between different factors in the educational field. Teachers (within the area of teaching mathematics) are creators of educational situations (incentives) in a demanding endeavor of developing students' mathematical thinking. The assumptions for high-quality educational work in the segment of teaching and learning mathematics are contained in the initial qualifications, reception of didactic reality, ability/training for permanent improvement in practice, as well as the existence of varying/adequate forms of support service for teachers. This text aims to direct attention towards the aforementioned key elements of hallmarks of the teaching mathematics vocation in our times and to situate this vocation within the reality of the educational process, together with the peculiarities of teaching mathematics. Recognizing this special character does represent a challenge for teachers which can be resolved by connecting it with didactics related to other subjects in common educational work.

Keywords: teacher, teaching, learning, teaching mathematics, developing mathematical thinking.

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1. Introduction

Knowledge of the totality of the world and humanity, as well as specialized grasping of its components, dimensions, areas, different levels, and so on, is impossible to achieve without the artifact made and developed by man – mathematics – which is the instrument for knowing and describing the world that is at the same time preserving that same world (Stipanić, 1988). Science itself, arising from mathematics, like a pearl in a whirl of human efforts to understand the environment and perfect our beliefs that have arisen from the transcendent, is being preserved by mathematics as its meta-methodology of a sort, but also by methodology, that is, actual instruments for producing, examining and verifying theories regarding the concept and order of the world. In ancient times, encountering irrational numbers meant facing transcendence (Božić, 2002; Średinger, 2007). This primal and pre-black hole that had existed until that point in the harmonic development of understanding the Cosmos once again arose by virtue of mathematics (the development of mathematics) as an inevitable entity of human consciousness and the development of (human) thinking. Regardless of science wars, the divide between the natural and social sciences and berating the latter on the grounds of their lacking scientific aspects because they were supposedly insufficiently infused with mathematics (Flivbjerg, 2012), the contemporary colossus made up from the system of knowledge of the world and its order, is not possible without mathematics. The universality of mathematics originates not only from its metaphysical role and theoretical generality

that it achieves but also from the empirical need to sort out cadastral relations, track and perform predictions of financial streams and balances, forecast meteorological changes, determine metrics in an act of versification and alike. Hegel (2005) warns that science is not an imposed "outer formalism" of tables, schemes, and algorithms to be applied but is actually content, quality, thinking, and consciousness that makes possible speculative thought. In that sense, according to Hegel, philosophical truth surpasses mathematics as its necessary but not sufficient condition.

At the same time, Ken Robinson (2011) notes that all we create within a community is our culture; thus, creativity and culture represent the basis for human understanding. Mathematics is generated within the human community and has undergone changes; thus, it has created a culture for the community that is changing in the same way it has changed itself. Therefore, mathematics as an agent of communication is a mediator of a kind, not only in this present, currently (physically) living community, but also between all communities in the historical retrospect of the four-dimensional space-time. And as such, it is a generator of culture and civilization.

Daniel Cohen (2014) emphasizes that globalization, according to current tendencies, is a hybrid process due to the fact that the life of a community is largely occurring online (virtually), owing to the enormous influence of the Internet on all aspects of individual life – business environment, private sphere, participation in public manifestations and cultural events. This new digital society, together with the oncoming of yet another creation of contemporary man – artificial intelligence – transforms individualities that would participate in the life of the community (hybrid life, hybrid communities) and that ought to possess competencies for overcoming loneliness, that is, isolation – knowing and actively using technology, managing information in the environment, participation in the new cybernetic reality. Already in his *Open Work*, Umberto Eco (1965) lays emphasis on the change of the order of the world that is no longer *Order*, as he relates, but new *Orders* – he uses plural – and adds the need to seek open patterns and vision of the world that are accepting of *ambiguities* that are dialectically contained and not chosen arbitrarily. This is a kind of paradox of the world today, soaked as it is in a precise digital conjuncture that operates with a single meaning on the one hand but which is, on the other hand, energetically incentivized by opposites that abide in the unity of act which is however no longer bipolar in Hegelian manner (dualities – subjective and objective; or thesis and antithesis, which are surpassed by synthesis).

The importance of learning mathematics stems from here as mathematics is an institutional protégé, being conditio sine qua non for any relevant education and upbringing. Mathematics syllabi, besides the learning contents, are also comprised of recommendations for practical application, that is, methods and techniques for successful integration of the determined (set up) volume of mathematical knowledge. Therefore, mathematics as a school subject, besides a pedagogical framework (which includes didactic and methodical frameworks), also contains interconnectivity, reliance on social aspects, and directedness towards socially agreed and negotiated educational policies. The settings of the new society arise incrementally, and they continuously and imperceptibly change everyday practice determined by social currents of the spirit of the time that has come about after various post's and -isms. It is the period after the postmodern wave, postindustrial influence, and postmaterialist society. *Education, research, health, digital economy… are trademarks of this socie*ty. (Koen, 2014).

It is clear that learning mathematics in such a complex reality establishes high demands for producing programs of teaching and learning this subject (and in the last instance, the school syllabi of mathematics), and by this same process, it does so for knowing and managing the multiplicity of the educational process by the teachers, so that it would end up with the minimum amount of idiosyncrasies in understanding conditions and the entropic factors in enabling school learning (practically, mathematics). The need to learn mathematics has never before asked for such an entanglement with everyday issues, social participation by students, inclusion into the environment, and implementation together with other study programs inside a unified whole of the processes taking place in a particular school in an authentic fashion. School syllabus of mathematics

overcomes its textuality, becoming a hypertext just as Umberto Eco (2002) has said about literature, and in many forms and layers, it connects mathematical concepts with the concepts belonging to other subjects into an adapted learning context that is then differentiated or individualized for each student. Taken individually, each student is then placed into his or her own (adequate) position relative to the whole dynamics of teaching and learning a subject, in fact, learning subjects.

2. Mathematics teacher in the reality of the educational process

A teacher's position relative to the whole of the educational reality can be described by a phrase borrowed from quantum mechanics – *Heisenberg's Uncertainty Principle*, for the reason that the resultant of multiple vectors (influences), that cannot be grasped in a single moment and have to instead be approached by partial insights into activities, knowledge, skills, attitudes, relations, state of professional development, and many more factors. It is determined by many elements related to the actual institution in which the teacher is employed, the educational cycle inside of which he performs his professional activities, the level and type of his own education, relationship with different partners that he has to converse with on a daily basis, the reflection of activities that he realizes and the ability to perform course corrections of his actions, and by instructive and pedagogical overseeing that directs his work. To be sure, the position of mathematics teacher brings additional specificities relative to the context of the subject as a general, vocational, or narrowly vocational subject. In fact, the position of mathematics teacher is also determined by the actual syllabus for teaching and learning mathematics and the school syllabus for this subject.

Da Ponte (2009) emphasizes that the education of mathematics teachers is related to the development of teachers' knowledge, teaching practice, and teachers' identity. Therefore, theories that have for a long time been preoccupied with knowledge and cognitive processes have now been overcome by taking into consideration the perspective of social development. The process of teaching and learning mathematics is not dominantly and directly directed towards cognitive processes that ought to take place on the individual level, but these cognitive processes are initiated and implemented within a complex social interaction of structured activities in the learning community. Mathematics teacher does not envision the process that is dominantly directed towards inducing cognitive changes in students, but organizes the community as a framework and agent of incentivizing social exchange between participants – these social exchanges commence knowledge transfers and cognitive streams in the consciousness of participants of the learning process. The universality of this approach is reflected in the fact that the very process of educating mathematics teachers itself takes heed of social perspectives directed at multiple interactivity in the learning community.

It can be said that mathematics teachers should basically possess three kinds of knowledge (Shulman, 1986; Morris, 2001; Milenković et al., 2024): (1) knowledge pertaining to the contents of mathematics as a school subject, that is, mathematics as a science by which the school subject was generated; (2) pedagogical and didactic knowledge, that is, knowledge of methodology of teaching mathematics (specialized didactics) – knowledge of pedagogy of the mathematics as a school subject; (3) knowledge of the school curriculum and the plan (curriculum) of the actual subjects (programs, themes, and materials designed for the mathematics as a school subject).

Mathematics teachers are key figures upon whom changes in the area of mathematical education depend (Da Ponte, 1994), and thus, preparation for this calling is equally as important as being employed. This is especially true for efforts to strive towards the quality of education work and especially for the process of institutionally organized teaching of mathematics.

3. Professional development of mathematics teacher

Due to the complexity of the demands and the role of a mathematics teacher, his professional development is of special interest to us as an agent of improving teaching and learning practice.

This problem has been treated in various ways, and thus, there are approaches that allow professional development already for interns through online exchange and blogs, by gathering them into communities of reflective practitioners and by actualizing deeper levels of learning and thinking about the implemented teaching work (Boulton & Hramiak, 2012). Boulton & Hramiak (2012) also point out that teachers should learn to ponder upon their own practice even during their schooling, that it is a skill which is difficult to acquire and that represents an important aspect when performing educational work. On the other hand, what is intensified is the demand recognized in all research that students in institutionally organized education need more individual attention, which refers to the need to provide the teachers with larger support in developing competencies and progressing the teaching level so that they could respond to the millennial challenge of modern school practices (Ibrahim et al., 2019). Bakar & Mohamed (2008) have shown that teachers need to possess self-confidence in using information and communication technologies (ICT) in the process of teaching and learning, that is, to be able to integrate ICT into the teaching process that they are implementing. This is in agreement with the recent experience of the pandemic and the response of educational systems across the world to using ICT devices in overcoming the issues of sustaining the work of the educational process and managing the process of teaching and learning in various ways – directly (in controlled conditions), online, and in a hybrid format. For teachers of mathematics, given the contents of the study curriculum and the nature of the science of this subject that is directly connected to information science, there is no particular problem in using ICT; within the practical training, during the process of acquiring teaching competencies, it is necessary to train them for using software whose function is managing teaching and learning processes (using ICT equipment directly during the teaching and learning processes, sharing/exchange of didactic materials via platforms for online learning, tracking and development of learning success…).

According to the findings of research, the topic of which was related to the factors decisively influencing professional development, those that can be singled out are consistency related to knowledge and beliefs; directedness at contents and ways in which students adopt them; harmonization with state/national regulations (educational standards); possibilities for active learning by teachers themselves; and cooperation with other teachers (Desimone, 2009; Harris et al., 2012; Koster et al., 2017). Professional development of teachers is an important instrument in improving schools and making teaching and learning better; that is, it represents an important factor in raising the quality of education (Opfer & Pedder, 2011; Ma et al., 2018). For teaching mathematics, professional development is aimed at the teacher's qualifications in areas related to the knowledge of mathematical content; pedagogy (or didactics, methodology); cognitive processes that take place while studying mathematics, and a mixture of aforementioned forms of knowledge (Da Ponte, 1994). It can be said that a teacher is a complicated and complex variable due to the reason that, besides the noted general kind of knowledge, he is supposed to also possess knowledge and skills for learning and functioning in various social contexts of learning, and also because knowledge is a social construct like environment/reality in which these processes occur. (Balacheff, 1990; Da Ponte, 1994; Dreher, 2016). Within personal plans of professional development or plans of vocational development at the level of faculty councils that involve mathematics teachers, it is necessary to ensure that teachers can successfully integrate the demands of the complexity of the contexts of learning mathematics by means of programming and operational planning of teaching and learning processes (Da Ponte, 1994).

Improving teachers' competencies in the sense of professional development has, in time, become an issue of exceptional complexity. Teachers are set before challenges that arise before them every day and continuously in all their complexity and the demands of the circumstances of contemporary society that is ever-evolving, especially in technology. Thus, the described trend generates ever new opposites that need to be resolved. Mathematics teacher has no longer (and for a long time as well) been focused exclusively on individual students and their cognitive development but is instead in charge of conditions of learning of great complexity in an actual socio-cultural context (Da Ponte,

2012; Gutiérrez et al., 2016; Lin & Rowland, 2016). All things considered, teachers implement teaching in interactive surroundings, in exchange with other teachers, in different situations of demanded complementarity – internal and external, formal and informal (Da Ponte & Chapman, 2006; Lin & Rowland, 2016). The very process they envision needs to be interactive and guided for the purpose of accomplishing benefits for students by reaching adequate and projected achievements.

In the working environment, the teacher corresponds in several communicational spaces – with students, other teachers with whom he cooperates, and with parents for the purpose of enabling unhindered transfer and construction of knowledge, the flow of the learning process, reaching achievements and supporting cognitive development through mathematical concepts. By getting employment, (a mathematics) teacher becomes a member of the community oriented towards "cultural and social perspectives of learning"; that is, he joins it by virtue of the ability to communicate and act in harmony with its special norms (Lin & Rowland, 2016). That is why it becomes necessary for the teacher to improve his skills and competencies in his professional development in order to participate and involve others in the complex interaction of professional teaching and organizing and leading an encouraging learning environment. While these processes are taking place inside professional development and training, the teacher is building his teaching identity (Da Ponte, 2012). It is necessary for the teacher to be directed towards the teaching process during which learning takes place, as well as towards the central role of students participating in exchange with other students. The activities of teachers are a function of learning success, and so the teacher is directed toward the process and not his own work (Da Ponte et al., 2019). The teacher is directed more towards the sources of knowledge and learning and less towards isolated notions that need to be grasped and then related to what has already been grasped – the student goes on an adventure of examining a complex context, through which he is guided in cooperation with others from his environment (other students, teacher, expert help, sources of knowledge from the surroundings). From a practice described in this way, professional development arises because this kind of practice is achievable when cooperating with other teachers and coordinating activities of professional teaching in horizontal exchange.

Professional development is not an isolated process; it occurs as an organic process of a professional community that has become such originated from a structure organized for performing an actual activity. Thus, a trainee teacher is made into a teacher by a process of initiation and is equipped to independently participate in performing the educational work by sharing and taking personal responsibility. After that, within the professional community, he continues to exchange and reflect on his teaching practice and to participate in the mutual creation of education work as a whole of processes that take place in a learning institution (school) and whose inseparable (organic) part is teaching mathematics.

During the everyday performance of teaching practice, it is necessary to document it, that is, to project the design, the flow, and the tracking of implementation into the written format. This written format can be printed or digitized, depending on the established procedures and culture of mediation inside the institution. Each act of documenting the practice is coding knowledge so that it becomes explicit and can be used in different contexts (situations) that so demand. Adequate reflection, planning, design (preparing for direct teaching), reporting on effectivity, and documentation of teaching practice enable efficiency in professional exchange (Cameron, 2009), connecting and building a professional learning community and allowing its development. Acquiring competencies for performing this segment of work represents a challenge for a teacher as he travels towards a real understanding of his own educational practice. Producing syllabi and operational plans for their realization, as well as making preparations for the direct performance of work, is a creative process through which the teacher over and over again reconstructs and reconstitutes his own knowledge of his scientific discipline, mathematics in every new actualization of teaching, and by this he advances and develops his own cognitive capacities of an efficient teacher (Cameron, 2007).

On the national level, standards are adopted related to students' achievements (educational standards), competencies for performing the teaching profession, quality of educational work, and managing educational institutions. Understanding the purpose of these documents and the level of knowing, connecting, and using them while designing the flow of teaching and learning is important for a mathematics teacher because they direct attention to his position, establish demands, and provide success in the educational process. Standards related to teaching competencies are especially important for professional development, and they represent guidelines when assessing, evaluating, and directing a career. The whole legislation of education represents guidelines for teachers, makes decision-making and managing teaching and learning processes easier, and corresponds to the needs of developing a democratic society based on the support for active participation and citizens' initiative.

John Hattie (2012) stresses the necessity of *visible learning*; actually, by researching the process of teaching and learning, he has designed an efficient model that is supposed to produce success without diminishing the effectiveness of educational work. To reach this, it is necessary to encompass a large number of parameters (factors) that influence varying intensities in the learning flow. Separated dimensions according to which the parameters are grouped are related to students, families, schools, teachers, curriculum, and teaching. One of the most important factors that influence the success of segments of the educational process is the feedback culture that refers to students and teachers. It is necessary for students and teachers to receive timely and concrete feedback on their work with instructions for improvement. Such an approach makes tracking students' achievements and reaching standards successful and serves the function of progress. Of course, it is necessary that while designing the processes that they aim to realize, teachers need to foresee tracking parameters that contribute to the success of learning and make them a function of achieving quality of educational work.

Another societal demand is related to schools adapting to the needs of students and becoming part of educational trends. Planning and realization of school curricula needs to enable optimal development for each student so that by applying differentiation and individualization, each student should feel successful during the processes of teaching and learning. Anthony and Yasin (2019) consider it necessary to strengthen teaching competencies and self-esteem to apply inclusivity in education as a mandatory part of professional development. This would represent a part of the teacher's response to the demands of our times with the backdrop of the growing complexity of conditions in which the educational process is taking place (differences in family origins and support – socio-economic status, specific needs of students, growing use of digital technologies, rapid growth in knowledge areas, developments in approaches to grading…). Working with students that demands creating and implementing individual syllabi also necessitates, besides knowledge and experience, training in the areas of knowing various ways of showing specificity and needs for an individualized approach integrated into a wider context of differentiated teaching practice (Coates, 2012). Regardless of their level of professional development, mathematics teachers need such directedness that each individualization and production of individual syllabi require special settings that are personalized and thus mutually different. Such an approach refers to the necessity of permanent professional development, exchange of knowledge and experiences with colleagues from the direct environment (horizontal level), and attending programs whose results enable taking the perspective that could enable efficient, inclusive educational practice. Research points out that teachers are initially not prepared enough to apply educational inclusion in teaching (Hemmings & Woodcock, 2011). This is all the more reason why it is necessary for the professional development of teachers to include various forms of support while working in the area of inclusive education and applying individual educational syllabi in different contexts of teaching and learning.

4. Development of (mathematical) thinking

When Heidegger says that "once we assent to learn, we have already admitted that we are not capable of thinking" (Хајдегер, 1999, p. 101), he suggests that education actually requires, that is, supplies material, substance, for thinking. Learning and thinking, so paradoxically represented, do not oppose each other in a counterpoise, but indeed, one is a presupposition of the other. When we express it thus, we could say that learning (temporarily) cancels thinking, but that is illusory because it actually establishes and stimulates it; moreover, it makes possible knowledge that is the subject of thinking. Therefore, the central notion of didactics for a particular subject, especially the methodology of teaching mathematics, is thinking, mathematical thinking, and the area of scientific research refers to its development. Adopting mathematical knowledge, mathematical notions (concepts), constructing mathematics (or parts of mathematics), envisioning efficient ways of learning mathematics, discovering mathematics in the reality that surrounds us – material environment as well as the abstractions of mental sphere, in individual consciousness, is made possible by thinking functions and the development of thinking by the learning subject.

Organization of the learning process is inconceivable without knowing the fundamentals of the developmental approach by Piaget, the instrumental method of Vygotsky, the theory of self and actualization of self of Maslow, Bloom's taxonomy of educational goals (or results of learning), educational neuroscience and plasticity of brain and lateral thinking of De Bono, and others. These theories explain the development of thinking in a particular socio-cultural context and the possibilities of pedagogical interventions; they make it possible to understand the reactions and behavior of participating subjects and refer to the purposefulness and necessity of structuring the learning process (Damjanović, 2010; OECD, 2010; Дамјановић, 2016).

Hrnjica (2005) explains thinking as a "form of adaptive behaviour" and explains this condition with the existence of a motive that energetically supports it, and that would first and foremost be the "existence of a particular task". It is exactly teaching mathematics that dominantly rests up on solving certain tasks, the conditions of which are grasped by gathering and analyzing data – by turning it into information and reaching conclusions in a concrete adequate scheme or algorithm for a specific type of previously known (or: solved) situations (or: problems) (Дамјановић, 2016). Certainly, as teaching mathematics is a creative process, the conditions that the teacher creates are not always based on previously adopted patterns of solutions or combinations thereof (known and adopted algorithms); instead, they demand additional cognitive effort in order to take hold of a new pattern. This additional cognitive effort is conditioned by the ability of lateral thinking, which is, in fact, an inventive grasp of conditions and creating completely new thinking patterns for overcoming a problem set (De Bono, 2001). The ability to comprehend problem situations in a manner that includes multiple perspectives contributes to the development of mathematical thinking (De Bono, 2000; De Bono, 2001). Projected students' achievements during the process of teaching and learning mathematics are accomplished by grasping and solving complex structured conditions (situations, problems) that the teacher had prepared, and they are marked by the demand for critical theorizing and creative use of mental operations. According to this description, the teacher envisions a process of learning that stimulates the development of mathematical thinking and thinking in general. It could be said that constructing, structuring, and producing mathematical knowledge of a subject undergoing a learning process enables a degree of development of mathematical thinking; in fact, mathematical thinking is inevitable and "absolutely necessary" when having in mind the aspect of needs for a scientific basis for acquiring knowledge of reality (Дамјановић, 2016).

Lateral thinking belongs to higher levels of thinking according to Bloom's taxonomy because it includes analysis of conditions, synthesis of aspects and evaluation, and, in fact, making a decision

for the application of an adequate procedure that enables solving a problem, specifically, a mathematical problem.

Mathematical problems in the context of stimulating the development of mathematical thinking supersede conditions in which we operate exclusively with mathematical ideas or objects. The conditions of the reality of teaching and learning mathematics in school are produced by interrelatedness between school subjects in such a way that student first recognizes representations of mathematical ideas, then proceed to mathematically code and solve them in the mathematical environment, and finally discuss the results reached according to the initial context and relations between ideas initially set.

The previously described process of learning mathematics has been placed into a wider context of events in which the education process takes place (for example, it is institutionally defined) so that teaching mathematics is not an isolated or separated fragment of didactic action, but is organically and complementarily interwoven with a synergistic tangle with other didactically intended influences. School syllabus for teaching and learning mathematics is actually an inseparable, non-disjunctive part of a united school curriculum of a specific institution. In that sense, it demonstrates the unity between mathematics as a science and other natural and social sciences, humanities, and artistic disciplines. Thus, the development of mathematical thinking advances and dialectically contributes towards the mental development of the subject of the learning process, by which Bourdieu's habitus (or system of dispositions; producing practices) is enriched as a product of totality of intended didactic influences (Burdije, 1999; Burdije & Paseron, 2014; Bourdieu, 2014).

The role of mathematics teacher takes place in the conditions that surpass the "boundaries of a subject", in social interaction that occurs between teachers, students, teachers and students, their communities, and learning becomes a "*basic activity* of school life that reshapes social relations" (Flores, 2016, p. 191). Owing to the development and availability of ICT, we have seen a growth in the influence of mass media and possibilities of learning outside school, coexistence of different educational models has multiplied, and teacher's work has become fragmented in a multicultural and pluralistic society, accompanied by the absurdity of the rise in responsibility, bureaucracy, and public insight (Flores, 2016; Kowalczuk-Waledziak et al., 2019). School is a kind of ambient for adopting culture and, accordingly, the space for events and processes that take place according to the scenario set by the school curriculum in a social exchange. Learning as an individual and group process is intended by the school syllabus, which is actually a curriculum that has been worked out and adapted to the conditions of the specific institution (school); mathematics teacher reaches out of his subject's range towards a wider context and acts through social interactions that he initiates (provokes) by designing teaching for which he is responsible. In this way, learning mathematics and adopting mathematical concepts, that is, the development of mathematical thinking, takes place on an individual level as an organized cognitive activity in the social and cultural space of the event. This social and cultural space of the event in which educational work is still directly performed can be a class (group), school, direct environment of students, virtual space of platforms for distance learning, or other hybrid formats of didactic exchange.

Digitalization has completely overtaken education and enabled additional space for teachers and students to successfully reach accomplishments and learning results (Reimer & Moyer, 2005; Yuan et al., 2010; Дамјановић, 2016). Digital competencies for students are a subject of learning and acquisition of skills of using ICT, but they are also metaknowledge and skills by virtue of which learning success is achieved. For mathematics teachers, digital competencies allow for managing the educational process and setting up multiple communicational exchanges during the realization of teaching design.

Digitalization helps significantly with adapting the learning processes to the individual characteristics of students and, in this way, to the availability of education. From the standpoint of didactics, digitalization allows for a diversity of teaching methods and approaches to learning that are supposed to produce and provide for learning success. A design of a taught class foresees a purposeful use of ICT as teaching aids, and a mathematics teacher uses software programs even outside the regular course of teaching for the purpose of exchanging didactic materials, tracking and assessing students' achievements or direct audio-visual communication on the platform for online learning. (Reimer & Moyer, 2005; Miller & Upton, 2008; Yuan et al., 2010; Baki et al., 2011; Дамјановић, 2016).

5. Social circumstances of teaching (mathematics)

Like any other activity with importance for society, so do education, teaching, or institutionally organized teaching fall under the influence or are rather mutually dependent upon various socio-cultural trends that directly take place in space and time of habitation of a specific community gathered around a determined school curriculum.

Postmodernity has shaken up society, and under the impact of this wave, strengthened by globalization, humanity has reached the postmodern state so that developed nations as primary centers for events, followed by the world periphery, felt certain changes relative to the long-lasting vertical hierarchy in the relations of power. This transition from vertical hierarchy to horizontal relations of centre-periphery is exactly the first and crucial change. Social dynamics are changed, and the centre that is in charge of the system's functioning keeps, by permanent self-reflection, its conception in balance with conceptions from the peripheral groups that strive to impose their own socio-cultural patterns (Semprini, 2004). As regards education and school, impassioned discussions are held because, in an axiological sense, this represents a release of an individual – it enables autonomy of thinking and overcoming initial (generic) socio-cultural pattern, that is, it provides potential for free movement, "transposition", "approaching" the centre from the direction of the periphery and sometimes margin (Semprini, 2004). Therefore, it is equally important for mathematics teachers to understand social reality as they participate in making a unique school curriculum that is then implemented within the whole of the school processes, and all of this in an equally unique socio-cultural space of events in interaction with all the participants (students, teachers, parents, local authorities…). Teachers have assigned, clearly regulated responsibilities and, in one segment, make decisions related to the educational process that takes place on a daily basis.

Characteristics of today's society, culture, science, and education originally lie in *openness* and *pluralism* (plurality). These designations are the basis for the conception of any socio-cultural endeavour that is now called a project. Functionality or purpose is the initial reason for acting – *why is it (now) important?* Thus, pedagogy responds with a curriculum that is open and directed towards learning results, with a minimum of prescribed content, and didactics offers many methods and techniques of learning so that in such diverse learning, success could take place. Knowledge acquired directly is related to practice, and learning place has become the reality of everyday life. General, axiological guidelines for (mathematics) teachers could belong to a wider set of guidelines that give a mark of the times in which pedagogical work is performed (Дамјановић, 2008):

- *The question of paradigm* – historical development of scientific thought is a ceaseless change of paradigms by which enormous technological progress is achieved (Kuhn, 1970; Петровић, 2010; Bourdieu, 2014; Златановић, 2015). In that sense, pedagogy itself as a science is no longer directed

towards a unified paradigm but instead operates in a pluralism of acceptability and purposefulness, the need for building a multicultural, tolerant, participative, democratic society.

- *Eclectic or synthetic approach* – didactic diversity of learning methods and techniques as well as acquiring knowledge has a unique goal, which is to be successful. Accepting everything that contributes to students' development and possessing functional knowledge is part of an educational process supported by intelligent emotionality. This eclectic collection of actions produces a flexible, adaptable, independent, and communicable system of dispositions – a complex system that synthesizes the whole knowledge of the learning subject (*producing practices)*. Pondering made in a disjunct manner that is related to the nature of the approach to the design of the learning process is overcome by reaching desired results and becomes a conjuncture by which the intended eclectic procedure allows us to synthesize a certain habitus (in the process of *socially structured interactions*).
- *Stimulating the development of critical thinking* – the ability to identify what is worthy and logical, consider multiple perspectives, assess information, use in new and unknown conditions (of learning), predict how circumstances might develop, reflexivity of thinking, understanding and overcome ambiguity, uncertainty and stochastic nature of the world.
- *Strengthening cooperative as opposed to comparative values* – an environment in which learning is organized and structured in such a way that cognitive abilities are developed in the socio-cultural context of intended and guided interactions is stimulating and possible if relations of cooperation, exchange, and complementarity are established therein. A community in which learning is actualized is a community of cooperation – learning subjects overcome relations in which they are *next* to others and are instead *with others* in interaction.
- *Flexible management and guiding educational process* – this presupposes organizing pedagogical activities in a connected, non-contradictory, and flexible system so that by projected tracking and evaluating of realization, it becomes possible to correct and improve the quality of educational work.
- *Teacher in the role of empirical dynamic resource* – the role of the teacher is to design and map the learning process, to be an expert in learning by combining various methods and techniques of learning, and by inducing changes in students' knowledge by guided interaction on the level of the learning group. At the same time, in the context of the subject, he represents a knowledge resource, gives guidelines and instructions to students, directs them to the sources of knowledge, and oversees the realization of learning design. Teacher makes it possible for students to show their individuality, which means that they can self-realize as was noticed by *Robinson* (2011).
- *The problem of pseudo-science and counter-science* – plenty of concepts, narratives, and discourses pretend or "apply" for the ranking of scientific theory even though they do not satisfy methodological conditions. Seemingly, these "look" like academic statements, whereas, in fact, they represent a cocktail of scientific segments, belles-lettres, myth, a mixture of genres (mostly publicist and popular contents), and finally, manipulations of scientific facts. Mostly, their advocates hold academic titles; they are "misunderstood", excluded, former representatives of the scientific community belonging to a special area of knowledge or a scientific discipline that is not related to the subject on which they speak. Enormous growth and production of knowledge raise the issue of control and relatedness into a whole of understanding the world and its functioning. Another question that is raised is the one of "dealing with knowledge" as opposed to "the idea of education" (Liessmann, 2009). Counter-knowledge appears in the form of knowledge, whereas it is not really knowledge (Thompson, 2009). Media, especially the internet, together with information, spread pseudo- and counter-knowledge as a byproduct, and so teachers need to add to their activities

warning students of this, and they should also protect themselves from the consequences of this kind of contamination of public space.

- *Crisis of pedagogy – arising of antipedagogy* – respecting, accepting, and including a plurality of actions that contribute towards learning success on the side of institutional pedagogy; all agents of a global process of knowledge exchange have appeared at the doorstep of educational institutions. This general and uncontrolled educational interaction has "brought into play" those who deny institutional framework and its possibilities by now. Some of these agents create their own anti-pedagogical procedures and represent them as liberating and desirable in the process of self-actualization of subjects by education. It remains an open issue whether these anti-pedagogical impulses contribute to humanity or specific learning communities in the synergy of educational effects.

School as institutional pedagogy has the role of preventing torsion of social consciousness by fulfilling its culturological mission. Globalization promotes multiculturalism and differences in a unified space of events so that all enter the exchange from the position of their own identities. This interaction liberates individuals by education in the process of actualization and development of awareness of oneself and the environment. Pluralism of realities in this socio-cultural process that takes place with the help of institutionalized pedagogy needs to realize the function of establishing tolerance as opposed to permanent induction of confrontations. In that sense, there is a need for the pedagogy(-ies) of educational institutions to affirm relationships of coordination and cooperation as axiological dimensions of the space of exchange of knowledge and cognitive development.

6. Eclectic and synthetic in the process of development of mathematical thinking – inventory

Anthropological ambition, as noted by Claude Lévi-Strauss (2013), to comprehend the world in its *totality* and see society as a unified system whose parts are all organically connected ought to be the basic assumption for a (mathematics) teacher. Teaching, or institutionally organized and structured learning, is a part of a totality in which education itself takes place in a general interaction. The importance of this interaction lies in the fact that cognitive development, which is a self-evident fact, needs to be recognized as possible to manage and guide, for "progressing asks that people cooperate, and during the cooperations it is necessarily and gradually discovered that it is exactly the initial diversity that made the cooperation fruitful and necessary" (Levi-Stros, 2013, p. 205). The aforementioned justifies insisting on social dimensions of learning and tolerance of diversity and multiplicity, not as an existential fact but as a condition/demand of qualitative change in a general and a specific process of learning that occurs within the totality of human society.

This world is not a "postmodern inferno" as it is eclectically described by *Vladimir Tasić* (2009) in complex mosaic-like pictures, expressing a common fear about where *post-culture* is really leading us because by previously asking, in another place, for "roots of the strange confusion that we know under the name of 'postmodernism'" (Tasić, 2002, p. 220), "that the interdisciplinary conversation is, first and foremost, *possible*" (Tasić, 2002, p. 237). Admitting complexity to social sciences and describing postmodernism as a "complex network of ideas – mutually extremely weakly connected – that it is difficult to describe it except as an undefined and vague spirit of the times" (Sokal & Brikmon, 2018, p. 244). The creator (s) of the "comedy of Sokal's incident" warns that "the most serious consequences for culture come from applying relativism on social sciences" (Sokal & Brikmon, 2018, p. 241). Sokal has unmasked tendencies of using natural sciences as a "reservoir of metaphors" for social sciences and thus contributed, just as most criticizers of postmodernism and the postmodern state, towards grasping and understanding the anthropological totality of society. This noticing of various distortions in science and culture makes it possible to

direct the learning process more clearly and purposefully so that subjects and agents of education achieve superior performance, using a clear descriptor from business economics. Therefore, the role, position, and education of (mathematics) teachers, as well as the consequentiality of acting, are so demanding that we are in need of a complex transdisciplinary curriculum that enables professional development in the conditions of relevant dialogue and exchange of practice.

Branislav Petronijević notices that „the existence of thinking, when it is once there, cannot be abolished“ (Петронијевић, 1991, p. 102), and it characterizes thinking as the “content of the thinking function of a conscious spirit, consciousness”, that is, thinking is “only that which is given in consciousness”. Exactly so – psychological fact of thinking is contained within the identity of a mathematics teacher, and by respecting this dimension of teaching, he becomes recognizable in the pedagogical discourse of a particular educational institution. Stimulating the development of mathematical thinking by teaching design and institutional learning presupposes cognitively directed and informed teachers; philosophical, psychological, mathematical settings and the context of thinking, knowledge of mutability of this construct, and manageability of its development generate a set of teaching competencies, directing his professional practice and determining his identity. Socio-cultural complex is an agent of the development of mathematical thinking if it is a cognitive process intentionally designed by the school curriculum as a whole of pedagogical action.

Dispersion of insights – the knowledge of philosophical, socio-cultural, and psychological circumstances of education – points to an eclectic gathering of knowledge in a cognitive process that is related to teaching mathematics. This collection of knowledge is then projected by the teacher into a unique dynamic system; it is, in fact, synthesized into a flexible and changeable didactic resource that generates support for the development of (mathematical) thinking.

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