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Several examples of the applications of definite integral calculus in the mathematics teaching at undergraduate studies in economics and business

Nevena M. Vasović [1, a, *], Nevena Z. Petrović [2, b]
[1]University of Kragujevac, Faculty of Hotel Management and Tourism, Vrnjačka Banja, Serbia
[2]University of Kragujevac, Faculty of Mechanical and Civil Engineering in Kraljevo, Serbia
[a]nevena.vasovic@kg.ac.rs, [b]nevena.z.petrovic@pmf.kg.ac.rs

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Abstract

One of the main problems in teaching mathematics subjects in undergraduate level studies of economics and similar profiles is how to achieve good communication with students who did not acquire sufficient knowledge in mathematics in their previous education. In this paper, we emphasize that it is necessary to show students how to apply a certain mathematical concept as a tool for solving real problems in the field of their education, which directs them to practical skills. We want to mention that the subject of mathematics is mostly taught in the first semester of bachelor studies, so time is also a limiting factor. Here, we will present several examples of the application of definite integral calculus in economics and business, which refer to determining the functions of the total accumulated cost, income, and profit using the corresponding marginal functions, calculating consumer and producer savings, and comparing social status levels. For visualizations, we will use the software Mathematica (Wolfram, 2003). The results of student teaching evaluation surveys for the subject Business Mathematics and Statistics, which is implemented within the Hotel Management and Tourism study program at the Faculty of Hotel Management and Tourism in Vrnjačka Banja, University of Kragujevac, show that students have a positive attitude towards the content of the course and the way the content is presented. At the end of the paper, we will formulate recommendations related to innovative approaches in teaching mathematics on the observed study profiles.

Keywords: definite integral calculus, economic functions and indicators, mathematics teaching methodology.

MSC2020: 97M10, 97M40

1. Introduction and rationale

This section gives a short literature review of similar problems, modern research, and conclusions on this topic. There has been increased attention given to students' understanding and use of some mathematical concepts in a different type of high-level studies in the last decades (e.g. Bajracharya & Thompson, 2014; Jones, 2013; Kouropatov & Dreyfus, 2013; Rasslan & Tall, 2002; Sealey, 2014; Thompson & Silverman, 2008; Wemyss et al., 2011). For example, integral calculus is an important foundational concept for a program of pure mathematics study (see Thomas et al., 2009). However, integral calculus goes much deeper and serves as the basis for many real-world applications in science, engineering, business, economics, etc. (Barnett et al., 2008, Paul et al.,

* Corresponding author
E-mail address: nevena.vasovic@kg.ac.rs

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2013). Unfortunately, there is a significant gap between teaching some mathematical concepts at the mathematical faculties and non-mathematical ones. It is not easy to apply some mathematical concepts in other fields because, for instance, variables in mathematical expressions and equations give a new level of meaning (Dray & Manogue, 2005). Also, students do not always use their mathematical knowledge in their field of education in how they study it in their math classes (Gainsburg, 2006). One of the main aims of the mathematical educator at the non-mathematical faculties should be to cultivate mathematical knowledge that facilitates the application of some mathematical concepts in other domains. In this paper, we concentrate on teaching the concept of definite integral calculus in business and economics studies. There are studies that deal with how different conceptualizations of a certain integral calculus can save resources for students for easier application in a certain field of education. Jones's papers (Jones, 2013; Jones, 2015) showed which conceptualizations of the definite integral are the most useful for making sense of integrals in the other nonmathematical science disciplines. It was concluded that the multiplicatively based summation conception is highly useful and beneficial for understanding that should be promoted in integral calculus curriculum and instruction. In the paper of Milojević et. al (2024), some application of definite integral calculus in the rent calculation was shown. The mentioned studies and investigations are very useful because they could help us to provide students the opportunity to construct knowledge about the integral that will enable them to satisfactorily apply their knowledge in the field of their interest and give us mathematics teachers guidance on how to design our curricula.

In this paper, we gave several examples that present how certain conceptualizations of the definite integral calculus could develop an understanding of the problems in the fields of economic and business studies. Namely, we will show some conceptualizations of the definite integral that are related to three common interpretations of the definite integral: (a) as the area under a curve, (b) as the values of an anti-derivative, and (c) as the limit of Riemann sums (see Salas et al., 2006; Stewart, 2012; Thomas et al., 2009). The reason for concentrating only on these three conceptualizations is that calculus textbooks often focus on area, anti-derivatives, and the Riemann integral during the exposition of and treatment of integrals (see Thomas et al., 2009). Also, a limiting factor is the number of teaching hours, which refers to a certain integral in the high school from where the students come. In some schools, only a few lessons are devoted to this concept, and there is a lack of applications, or students do not learn integral calculus (Official Gazette Republic of Serbia - Educational Gazette, 2007 and 2010). That is why it is very important to consider the way and depth of introducing the concept of integral account. Because of that, many students struggle to understand the integral notion and are, at best, content with formal approaches to problem-solving. This assertion is supported by our teaching experience and the professional literature (e.g. Orton, 1983; Sealey, 2006).

In the next section, we introduce the readers to the real problems that we have in teaching the concept of definite integral calculus, and we introduce our study area. In Section 3, first, we mentioned the necessary skills that students need to develop through the course before they meet with definite integral calculus; then, we introduced the concept of definite integral calculus through the surface calculation under the curve, and finally, we gave several examples of definite integral applications in economics and business. In Section 4, we give some recommendations related to improving mathematics teaching on similar profiles of studies and give the curricula of our units related to definite integral calculus. Also, we incorporated in this section the attitudes of the students who participated in the teaching experiment. In section 5, we gave our conclusions.

For the research and presentation of results in this work, we used standard methods of applied mathematical analysis, descriptive statistics, as well as Mathematica software for visual interpretation of the presented examples (Wolfram, 2003).

2. Study area

The compulsory mathematics subjects at the economic profile faculties of the Universities in the Republic of Serbia usually have 45 hours of classes and 30 hours of practical exercises. They are mostly one-semester courses in the first year of basic academic studies, according to the organization of classes requiring more individual student work, as well as other forms of additional classes to master all studied units successfully. Because of time constraints and insufficient knowledge from secondary school for certain teaching units, it is necessary to develop teaching methods that encourage students to work independently and to become interested in a subject that is largely abstract to them. In the present article, we are concentrating on the following question: How can we help the students of a study program that is not mainly focused on mathematics develop mathematical competencies? Accordingly, we will consider one of the teaching units on the first-semester subject of Business Mathematics and Statistics at the Faculty of Hotel Management and Tourism in Vrnjačka Banja (FHT), University of Kragujevac (major Hotel Management and Tourism). Starting from the data in Table 1, we have that a large percentage of first-year students at the Faculty of Hotel Management and Tourism in Vrnjačka Banja comes from secondary schools where, for example, the integral calculus is not part of the fourth-grade plan of Mathematics in secondary school or the number of lessons for this teaching unit is less. This important unit is very abstract for our students, but there are many problems with bringing this concept closer to them and getting them interested in teaching. Therefore, in this short paper, among other things, we also gave some examples of how to apply definite integral calculus in diverse areas of economics, with problems that are accessible by a first-year student, such as index of income concentration, consumer's and producer savings and determining the functions of the total accumulated cost, income, and profit (see Barnett et al, 2006; Paul et al, 2013).

Table 1. The number of students by type of schools at FHT, University of Kragujevac (2019-2024)

Type of the secondary school						
Academic year	Economic school	Hospitality and tourism school	Medical school	Technical school	Grammer school	Others
2019/20	33%	19%	1%	11%	19%	17%
2020/21	41%	14%	9%	9%	17%	10%
2021/22	20.34%	44.92%	2.54%	6.78%	15.25%	10.17%
2022/23	26.06%	23.94%	2.82%	12.68%	19.01%	15.49%
2023/24	33.03%	26.79%	4.46%	7.14%	12.5%	16.08%

3. Applications of Definite Integral Calculus in Business and Economic

First, we will mention the basic terms from integral calculus, which students must be introduced to before meeting with definite integral.

Through the part of the course of Business Mathematics and Statistics devoted to integral calculus, it is necessary to introduce students to the reverse operation of the derivative, i.e. with an operation that could be explained as the function reconstruction if we start from the given derivative f. We can say that the function F is that wanted reconstruction of the function f if holds $F' = f$. It is necessary to familiarize students with this problem from a geometric and algebraic point of view. From the algebraic side, it is necessary for students to learn the basic formulas and properties of the indefinite integral. Now we will give the basic terms from the definite integral.

Definite Integrals. Suppose that the function $f(x)$ is defined on an interval $[a, b]$ and that the interval $[a, b]$ is divided into n subintervals $[x_{i-1}, x_i]$ of equal width $\Delta x = \frac{b-a}{n}$ (see Fig. 1). The definite integral of f from a to b is

$$\int_a^b f(x)\mathrm{d}x = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x,$$

if limits exist.

If $f(x) \geq 0$ for all x in $[a, b]$, then $\int_a^b f(x)\mathrm{d}x$ represents the area under the curve $y = f(x)$ between $x = a$ and $x = b$. The integrals make sense even when $f(x)$ is not positive, but in this paper, we will consider only the cases when $f(x) \geq 0$. If the function f is the continuous on $[a, b]$, and F is some function with the properties $F' = f$, then $\int_a^b f(x)\mathrm{d}x = F(b) - F(a)$.

In the sequel, we will show several examples of the application of the concept of integral calculus that we work on with first-year students. We choose bellow mentioned examples because through them, students meet with definite integrals as a surface between two curves; they are met with the problem of geometrical representation of the product $f(x_i)\Delta x$ and total change of some economic function at a certain closed interval. Also, through all these examples, students are encouraged to use software. More about the examples we will interpret here can be found in (Lorenz, 1905; Charls, 1986; Camm, 2022).

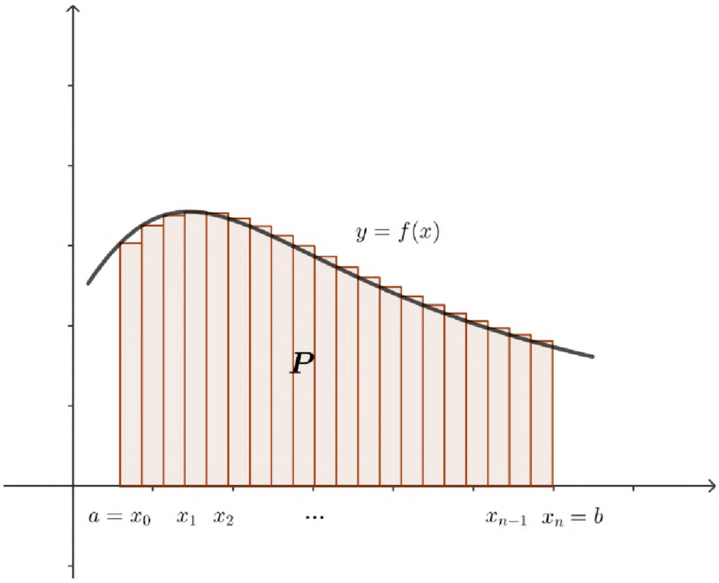

Fig. 1. Definite integral $P = \int_a^b f(x)\mathrm{d}x$

Lorenz curve and income concentration index. The Lorenz curve is usually interpreted in economics as a graphical representation of income or wealth distribution among a population (see Fig. 2). The horizontal axis on a graphic of Lorenz curve typically shows the portion or percentage of total population, and the vertical axis shows the portion of total income or wealth (see Fig. 2). For example, the point with coordinates $(0.4, 0.1)$ on the Lorenz curve in Fig. 2 represents that the first 40% of the population (ranked by income in increasing order) earned 10% of total income.

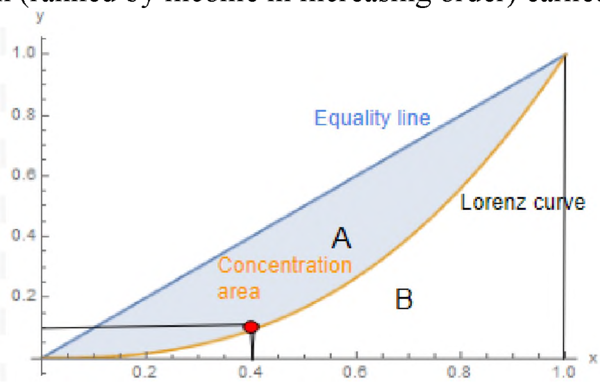

Fig. 2. Lorenz curve and Income concentration

The Lorenz Curve is a convex curve, and greater convexity implies more inequality in income distribution. The ratio of the area bounded by the graph of the function $y = x$ and the Lorenz curve

$y = f(x)$ and the area of the triangle under the line $y = x$ in the limits from $x = 0$ to $x = 1$ is called the income concentration index (Gini Index), and it is given by

$$G = \frac{A}{A+B} = 2\int_0^1 (x - f(x))\mathrm{d}x.$$

Income concentration index 0 indicates absolute equality - all households have the same income, while the income concentration index 1 indicates absolute inequality - all income belongs to one household. Now, we give two examples which could be useful in working with students.

Example 1. The data in Table 2 show the distribution of the national wealth of one country.

Table 2. Data

Distribution of the national wealth of the one country						
x	0	0.20	0.40	0.60	0.80	1
y	0	0.12	0.31	0.54	0.78	1

A) Using software Mathematica, determine the cubic regression equation and determine the equation of the Lorenz curve.

B) Using the obtained regression equation, determine the approximate value of the income concentration index.

Solution. A) The following lines of code in software Mathematica could be used for determining the cubic regression equation (the equation of the Lorenz curve).

```
data = {{0, 0}, {0.20, 0.12}, {0.40, 0.31}, {0.60, 0.54},
{0.80,0.78}, {1, 1}};
Fit[data, {1, x, x^2, x^3}, x]
5.94213*10^-16 + 0.375 x + 1.25 x^2 - 0.625 x^3
```

B) Using the obtained equation of Lorenz curve from the part A), we have that the approximate value of the income concentration index is

$$G = 2\int_0^1 [x - (5.94213 \cdot 10^{-16} + 0.375x + 1.25x^2 - 0.625x^3)]\mathrm{d}x = 0.10.$$

Since the value of indices G is relatively small, we conclude that the given income distribution is uniform (see Fig. 3).

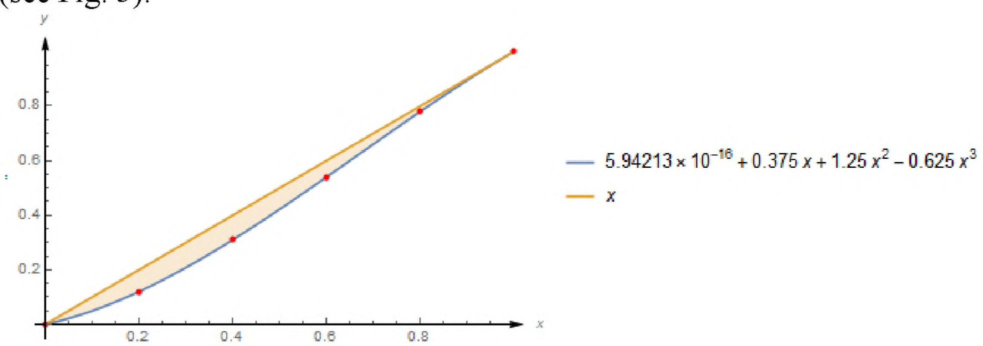

Fig. 3. Lorenz curve (blue line), equality line (orange line) and data set (red dots).

Example 2. The Lorenz curve of income distribution in 2020. for the one country is given with $f(x) = x^{2.3}$. Due to certain changes in the tax system, it is expected that the Lorenz curve for 2025. of the same country will be $f(x) = x^{2.8}$. Determine the income concentration indices for both years and interpret the obtained result.

Solution. The income concentration index for 2020 (see Fig. 4) is given with

$$G_1 = 2\int_0^1 (x - x^{2.3})\mathrm{d}x \approx 0.39,$$

and for 2025 (see Fig. 4) is assumed that it will be

$$G_2 = 2\int_0^1 (x - x^{2.8})\mathrm{d}x \approx 0.47.$$

The income concentration index will increase in 2025, so incomes will be more unevenly distributed than in 2020.

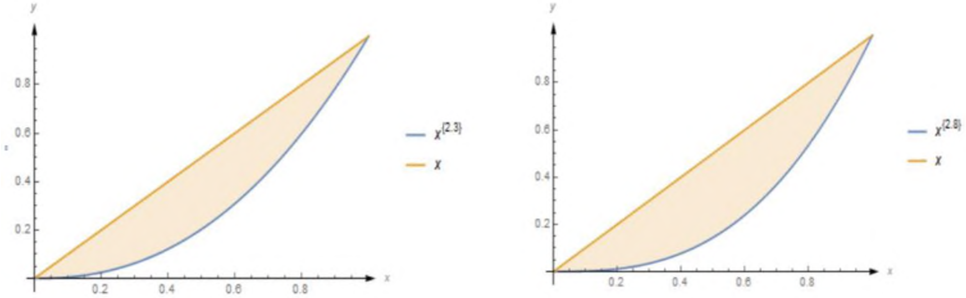

Fig. 4. The Lorenz curve for 2020 (left) and the projection of the Lorenz curve for 2025 (right).

Remark 1. The Gini index and the Lorenz curve can be used for researching the seasonality of tourist destinations and for the presentation of relevant data (see Veljović & Vasović, 2024).

Remark 2. In Example 1 we introduce students to the concepts of interpolation, interpolation curve and programming in software Mathematica.

The consumer and producer's savings. Let $p = D(x)$ be the equation of dependence of product price on demand where x is the number of units of the product that consumers are willing to buy at price p per unit. Let $\bar{p}$ be the current price and let $\bar{x}$ be the number of units that can be sold at that price. Consumers who are willing to pay a price higher than $\bar{p}$ but still buy the product at price $\bar{p}$ have saved money. It is necessary to determine the total amount of savings of all consumers who are willing to pay an amount greater than $\bar{p}$ for the product.

First we consider interval $[c_k, c_k + \Delta x]$, where $c_k + \Delta x < \bar{x}$, and if the price on that interval is constant then the savings per unit of product is $D(c_k) - \bar{p}$, i.e., the saving is the difference between the prices that consumers are willing to pay and the price they actually paid. The total saving on the considered interval will be $(D(c_k) - \bar{p})\Delta x$ (see Fig. 5).

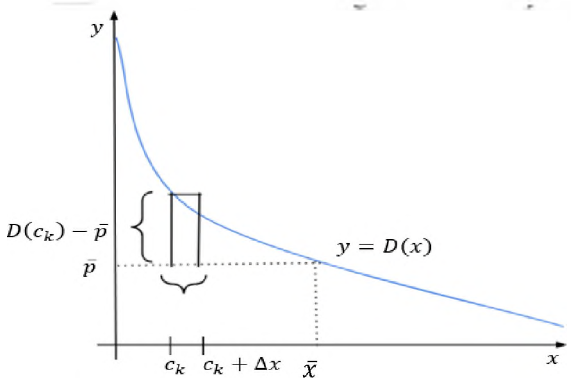

Fig. 5. Consumer savings on interval $[c_k, c_k + \Delta x]$

If we divide interval $[0, \bar{x}]$ on n equal subintervals, the total savings of consumers will be $\sum_{k=1}^{n}[D(c_k) - \bar{p}]\Delta x$ and if we let $n \to \infty$ the previous sum becomes $PP = \int_0^{\bar{x}}[D(x) - \bar{p}]\mathrm{d}x$, which represent the savings of consumers. On the similar way we could consider the savings of producer, and it could be interpreted as $PD = \int_0^{\bar{x}}[\bar{p} - S(x)]\mathrm{d}x$, where $p = S(x)$ is the equation of dependence of product price on supply (see Fig. 6).

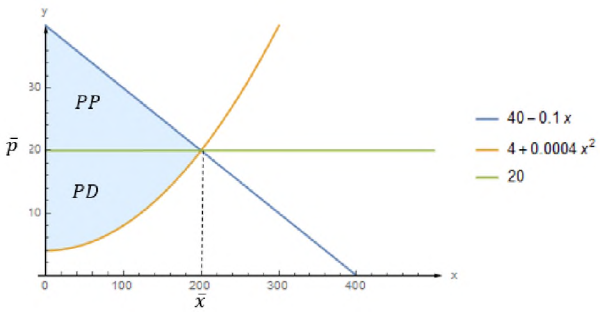

Fig. 6. Consumer savings (PP) and producer savings (PD)

If $p = D(x)$ and $p = S(x)$ are equations of dependence of product price on demand and supply, respectively, and if $(\bar{x}, \bar{p})$ is their intersection point, then we say that $\bar{p}$ is the equilibrium price and $\bar{x}$ equilibrium supply (see Fig. 6).

Example 3. If the dependence equations of product price on demand and supply are given with $D(x) = 50 - 0.002x^2$ and $S(x) = 10 + 0.2x$, respectively, determine the equilibrium price in dollars and calculate the savings of consumers and producers.

Solution. If we equalize $D(x)$ and $S(x)$ and solve equation $D(x) = S(x)$ we get $x_1 = -200$, $x_1 = 100$. Since, x could not be negative, we have that the unique solution of the equation $D(x) = S(x)$ is $\bar{x} = 100$. The equilibrium price could be calculated using $D(x)$ or $S(x)$ and we get $\bar{p} = 30$ (see Fig. 7). The consumer savings is

$$PP = \int_0^{\bar{x}} (D(x) - \bar{p})\, \mathrm{d}x = \int_0^{100} (50 - 0.002x^2 - 30)\, \mathrm{d}x = \$1933.33.$$

The producer savings is

$$PR = \int_0^{\bar{x}} \big(\bar{p} - S(x)\big)\, \mathrm{d}x = \int_0^{100} \big(30 - (10 + 0.2x)\big)\mathrm{d}x = \$1000$$

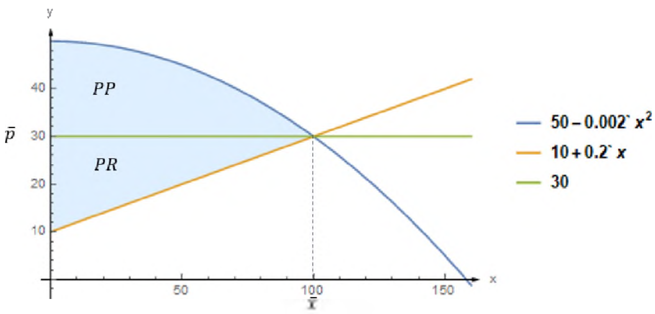

Fig. 7. Consumer savings (PP) and producer savings (PR)

Total accumulated cost, revenue, and profit. Suppose that $C'(x)$ is marginal cost function at x production units, then $\int_a^b C'(x)\mathrm{d}x$ is a total change of cost because of production increasing (or decreasing) from $x = a$ production units to $x = b$ production units (see Fig. 8). The total change of cost could be calculated as the value of $C(b) - C(a)$, where $C(x)$ is the cost function.

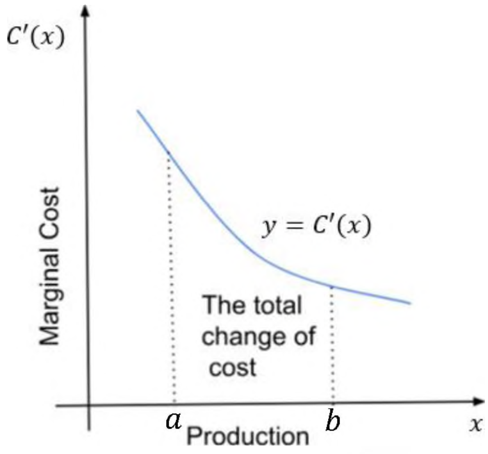

Fig. 8. The total change of cost $\int_a^b C'(x)\mathrm{d}x$

Notice that the marginal cost function $C'(x)$ is the approximation of the cost function of producing one product more with a production volume of x products i.e., $C'(x) \approx C(x+1) - C(x)$ (see Fig. 9). The same interpretation could be formulated for the functions of total revenue $R(x)$ and profit $P(x)$ and the corresponding marginal functions.

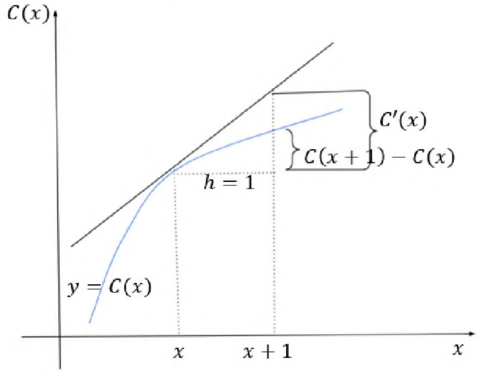

Fig. 9. Interpretation of the marginal cost function $C'(x) \approx C(x+1) - C(x)$

Example 4. The R&D sector of one factory for electric scooter production has presented the marginal cost function $C'(x) = 1000 - \frac{x}{3}, 0 \leq x \leq 1800,$ where x is the number of monthly produced electric scooters and $C'(x)$ is marginal cost function given in dollars ($). Calculate the cost change when production increases from 600 to 1800 electric scooters per month.

Solution. The total change of cost from 600 to 1800 for electric scooters per month is $C(1800) - C(600)$, so we need to calculate the integral $\int_{600}^{1800} C'(x)\mathrm{d}x$. We obtained that the total change of cost is $\int_{600}^{1800}(1000 - \frac{x}{3})\mathrm{d}x = \$72000$.

4. Discussion

Integrals are essential to comprehending economic ideas and provide a methodical way to analyze significant topics. Therefore, by using integral calculus, economists can accurately assess how many variables affect economic results, leading to more precise projections and strategic decision-making. This essential link between integrals and economics gives experts a more comprehensive analytical arsenal and gives students a foundation for studying economic theories and practices.

To analyze the success of our concept of teaching mathematics at FHT, University of Kragujevac, we have used students' comments in the anonymous evaluation surveys for the

Business Mathematics and Statistics subject. These surveys have been conducted yearly at the end of the semester since FHT, University of Kragujevac, was established in 2011. Through surveys, students could express their opinions on the Likert scale (1-5) about the organization of the teaching process, themes on the subject, and teacher skills in presenting teaching material. Also, they could comment on some teaching units and express their own opinions via comments. The results from surveys confirmed that the students have a positive attitude towards the subject and the curriculum of the subject. Student grades in the period 2012-2024. range from very good to excellent, and the analysis of students' comments leads to the conclusion that the lectures are clear and well thought out and that the material covered motivates students to participate in classes actively. Some comments from the students related to the material presented for the teaching units of definite integral calculus are mentioned bellow:

Student 1:” Until now, we have not had the opportunity to familiarize ourselves with the integral account. The course presented an abstract topic with a big application in the field I love. Thanks to the application and software usage for solving problems in this, a complex topic is closer to me.”

Student 2:” We studied integrals in high school but not in this way where we can clearly see the purpose of working with it. I especially like the tasks in the written part of the exam, where interesting applications in business and economics are given.”

Student 3: “I like the integral calculus unit presented through the course because I can see how this course will help me in my future academic development. In my previous secondary education, we worked with integrals but only on the calculation level using the table of integrals. “

Student 4: “Teaching processes are very interesting with a lot of examples and applications, but for the students who were not familiar with this calculus, the short time devoted to this unit is a limiting factor. “

Also based on the data that with which dispose at FHT, University of Kragujevac, it was found that the number of students who passed the course of Business Mathematics and Statistics in the observed period of academic years 2019/20-2023/24 is around 60% of the first time enrolled students in that year and that the average mark was around 7 (on the scale 5-10), which is quite a good percentage if we look again at the data in Table 1 about secondary school profile of students and we need to take into consideration the COVID-19 period and online teaching (De Coito & Estaiteyeh, 2022). In this way, we have one of the indicators of the success of presenting the teaching unit in a way that is oriented towards practice and problems of the real world. The final exam is divided into two parts: oral and written part. In the oral part of the exam, the students are asked to demonstrate through simple examples that they understand the concept of the interpretation of definite integrals as the surface under the curve and between curves, geometrically interpret the product of the form $\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$ and find the graphics of the anti-derivative of some function given with her graphic. In the written part of the exam related to the definite integrals, students are asked to apply the acquired knowledge to examples and real problems from practice. In addition to the examples presented in this paper, the course also covers the Geometric-numerical introduction to the definite integral; then the definite integral as a limit of the sum; the basic theorem of differential and integral calculus; as well as the application of the definite integral to the calculation of the mean value of a function over an closed interval; areas between curves; the total change of some function on an interval, and the use of software.

In accordance with our knowledge of teaching performance at faculties of a similar profile, we have no information that this concept of the teaching is applied on faculties of the similar profiles in Republic of Serbia (see Boričić & Ilić, 2015).

According to our visions and work on teaching mathematics to non-mathematical students, we emphasize several remarks that could be valuable for improving reaching the faculties of the similar profile of the studies. To improve the teaching process in the future and find some gaps in this process, individual in-depth interviews with students can be conducted, and the answers could be analyzed in software for quantitative data processing. Also, it is necessary for deeper analysis of students' enrolment and to design the teaching and subject curriculum in such a way as to achieve homogenization with the material from high school. Practical examples with visual interpretation in software must be included in the lessons because, in that way, the students are encouraged to work independently, to be research oriented, and prepared for other levels of study. This paper explains how a complex mathematical concept - integral calculus, can be brought closer to students who may likely be encountering it for the first time, how we work with them, and how we can get them interested in the mathematical subjects. This problem of interpreting some complex mathematical topics to non-mathematical students is very complex and can be discussed seriously at thematic conferences. we also think there is a need for work and exchange of experiences in this area.

5. Conclusions

In summary, we found that presenting definite integrals through the applications in the specific problems from economics and business fields is highly productive for making sense of applied integral expressions and equations. We found that the students are satisfied with the teaching and the presented applications and that, in this way, they have a clear attitude as to why a certain teaching unit is being studied. They see their further progress in the sphere of interest by using the knowledge acquired in the course. This, in turn, allowed them to demonstrate how the integral applies to economics, highlighting its practical value and emphasizing the importance of incorporating this understanding into calculus curriculum and instruction. We hope that the research in this paper will further deepen this topic and encourage researchers with a similar problem in the teaching process to stronger cooperation.

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