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NUMERICAL INVESTIGATION OF THICKNES, ASPECT AND TAPER RATIO INFLUENCE ON DIVERGENCE VELOCITIES OF COMPOSITE PLATES AT SUBSONIC MACH NUMBERS

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Abstract: *Within the scope of this research, the influence of different geometrical parameters on static aeroelastic instability of thin composite plates is analyzed. Design of the composite plate's geometry is aligned with design of modern subsonic missiles fins. Orthotropic material model is based on the classical laminate theory, mechanical properties of unidirectional laminae are calculated with composite micromechanics theories and laminates design is based on Tsai and Han quasi-isotropic composite lay-ups. Divergence velocities, the crucial static aeroelastic effect, are calculated numerically using commercial finite element method software. Calculation method is based on coupling subsonic aerodynamics models with structural models, and divergence effect on static stability loss is analyzed.*

Keywords: *Divergence, Composites, QI Laminates, Static stability loss.*

1. INTRODUCTION

Since the early days of flying, the aircraft loss of structural stability has been imposed as primary design variable for achieving a safe flight. Starting with the initial design, aircraft's structure is tailored to optimally meet all necessary strucutral requirements within the flight anvelope. Therefore, designers must enough accuretly predict a wide scope of effects that arise from flexibile airframe structure and aerodynamics load's coupling, at the early stages of design.

In this paper, the dependency of divergence velocity of thin composite plates of low speed flight vehicles on plates geometrical parameters in investigated. Torsional divergence is the most representative static aeroelastic effects that has crushing effect on airframe static structural stability. Airframe components are light and flexibile, and therefore they are keen to both static and dynamic instability. During the flight, lifting surfaces deform in such way that leads them to entirely new, deformed shape which has completely new aerodynamic load. Lifting surfaces are being twisted around it's elastic axes, and therefore they “see“ higher angle of attack that produces larger twisting moment. If the aerodynamic loads generated on the deformed surfaces are higher than airframe torsional stifness, lifting surfaces are twissted of the flight vehicles, which ussualy leads to structural failure. Therefore, with the aim of achieving a safe and designed flight trajectory, divergence velocity must be sufficiently higher than the aircraft operational speeds, at all time and under all flight conditions, within the flight anvelope.

Investigated plates are designed accordingly to modern subsonic ASM and AGM missile control surfaces [1]. Divergence velocities and plates inertial characteristics are obtained with usage of analytical and numerical methods. Plates are designed from composite material *E-GLASS / Epoxy 3501 – 6*, in accordance with modern trend and overwhelmed application of composites in aerospace industry [2]. Material and lay-up designs are conducted within CLT theory and with respect to the well-known aerospace composite guidelines [3].

2. ANALYSIS MODEL

2.1. Theoretical model

Analytical model for evaluation of the torsional divergence of lifting surfaces is derived from the commonly used method – *Typical section* method [4], in the early design stages. For the typical section shown in Figure 1, steady state, aerodynamic loads are expressed as:

$$R_z = \frac{1}{2} q \frac{dc_z}{d\alpha} \alpha_a l \qquad M_{ac} = \frac{1}{2} q C_{mac} c l \tag{1}$$

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Typical section is twisted around its elastic axis, which is shown in Figure 2, and twisting moment is generated due to the aerodynamic loads:

$$M_t = M_{ac} + R_z \cdot (el) \quad (2)$$

Following the Saint-Venant's theory of pure twisting of beams, the twisting moment of typical section can be, in the domain of small and linear deformations as a function of its torsional stiffness:

$$M_t = C_{\theta\theta}\theta \quad (3)$$

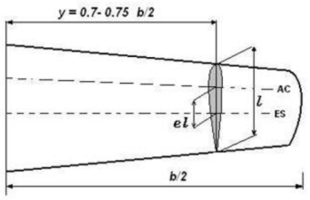

Figure 1. Lifting surface typical section.

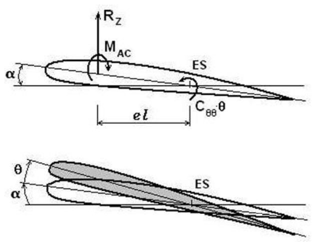

Figure 2. Deformation of typical section.

Since the lifting surface is flexibile and it deformes upwards for the value of angle of twist θ, the characteristic section is then exposed to new angle of attack $(\alpha_a+\theta)$ and new aerodynamic loads. Therefore, based on this theoretical approach, and since the airframe torsional stifness is indepedent of flight conditions, aiframe will become neutrally stable at critical, divergence speed:

$$V_D = \sqrt{\frac{2C_{\theta\theta}}{\rho \frac{dC_z}{d\alpha} S el}} \quad (4)$$

In the above equation, it can be seen that divergence speed has propotrional dependency to the torsional stifness (airframe inertical characteristics) and disproportionately to the aerodynamic loads. Within this paper, lifting surfaces are thin, flat plates, for which the lift gradient is known as $dC_z/d\alpha = 2\pi/rad$. In order to evaluate divergence speed, structure's inertial characteristics are examined.

As mentioned in the above text, for the lineary elastic structure, torsional stifness can expressed analyticaly in terms of it's inertial characteristics which is described in :

$$C_{\theta\theta} = GJ \quad (5)$$

In the above equation, G represents influence of material design and J represents influence of structure's geometry design on the overall structure torsional stifness. Within this research, in-plane shear modulus of QI laminates with long fibers is calculated following the conducted researches [6], [7]:

$$G_{ekv} = \frac{1}{2}G_{12} + \frac{1}{8}\frac{E_1(E_1+E_2-2\nu_{12}E_2)}{E_1-2\nu_{12}^2E_2} \quad (6)$$

Individual laminae mechanical characteristics E_1, E_2, G_{12} and ν_{12} are evaluated with composite micromechanics theory [8], for known mechanical characteristics of individual phases in aerospace composite *E-GLASS / Epoxy 3501 – 6* and fiber volume fraction $V_f = 0.55$. Afterwards, with the usage of identical analogy, material strength characteristics are calculated and shown in Table 1.

Table 1. Designed lamina mechanical properties.

E-Glass / Epoxy 3501 – 6; V_f = 0.55	
E_1 [GPa]	42.1
E_2 [GPa]	14.2
G_{12} [GPa]	5.37
ν_{12} [/]	0.284
ρ [g/cm^3]	2.0015
F_{1t} [MPa]	1100
F_{2t} [MPa]	50
F_{1c} [MPa]	520
F_{2c} [MPa]	140
F_6 [MPa]	100

Finally, inertial characteristic of typical section J is derived from the closed form Prantl's torsional equation and it is simplified for the case of thin plate ($a >> b$) under pure shear loads [9], its valid approximations is as follows:

$$J \approx a^3 b\left(0.333 - 0.21\frac{b}{a}\left(1 - \frac{1}{12}\left(\frac{b}{a}\right)^4\right)\right) \quad (7)$$

2.2. Geometry and lay-up model

Design variables, thickness (t), aspect ratio (AR) and taper ratio (TR) are varied in such way, that the plate geometry could always represent subsonic missile fins – presented in Figure 3, while the planform plate surface was maintained constat at $S = 0.15$ m^2. Plate geometry is obtained with following equations:

$$C_r = \sqrt{\frac{4S}{AR(1+TR)^2}}$$

$$C_t = TR \cdot C_r \quad (8)$$

$$H = AR\frac{c_r}{2}(1 + TR)$$

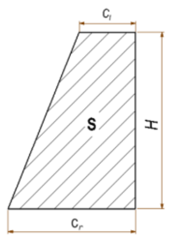

Figure 3. Fin planform geometry.

For the lay-up model, one of the most popular QI lay-up's in aerospace was used: *[0˚, 45˚, -45˚, 90˚]$_s$*. Designed geometries for this research are presented in the Table 2.

Table 2. Designed fin's planform geometries.

No.	*t* [mm]	*AR [/]*	*TR [/]*	C_r [mm]	C_t [mm]	*H [mm]*
1	0.5	0.4	2	391	156	548
2	1	0.4	2	391	156	548
3	1.5	0.4	2	391	156	548
4	2	0.4	2	391	156	548
5	1	0.2	2	456	91	548
6	1	0.6	2	342	205	548
7	1	0.8	2	304	243	548
8	1	1	2	274	274	548
9	1	0.4	0.5	782	313	274
10	1	0.4	1	553	221	387
11	1	0.4	1.5	452	181	474
12	1	0.4	2.5	350	140	612
13	1	0.4	3	128	128	671

2.3. Numerical model

The modern approach to structural stability loss predictions of airframe is to couple numerics and finite element methods. Withint this research, comerical software *Nastran* was used to evaluate the divergence speed and verify theoretical methods. Based on the numerical simulations, two different methods are used for calculation. For both cases, plates were clamped at the root, which simulates fin-body connection. This approach, beside divergence velocity, also renders informations about structure stress-strain field.

First numerical model was based on experimental test and guidelines based on suggested literature for static aeroelastic experimental testing [10] and it is illustrative presented in Figure 4. Based on the equation (3) and in the field of small and linear deformations, twisting moment is applied at the wing tip and deformations (twisting angle) are measured. For the known deformations and applied twisting load, lifting surface torsional stifness is recalculated, and afterwards, divergence velocity is calculated on the basis of equation (9). Small and linear deformations are calculated as a function of leading and trailing edge displacements:

$$\theta = \frac{\Delta_{LE} + \Delta_{TE}}{l_{section}} \qquad (9)$$

Second numerical model represents more realistic simulation of in-flight conditions with generation of three-dimensional aerodynamic loads, which is shown in Figures 5 and 6. Structural model is coupled with aerodynamic model with the usage of infinite plate splines, that transfer displacements and rotation between two models [11]. Steady aerodynamic model is based on VLM method for subsonic speeds [12] and it consists of large number of small aeropanels, that simulates potential flow for the given flight conditions. Divergence speed is adopted as the flight speed that generates such steady aerodynamic loads that leads to static failure of the structure.

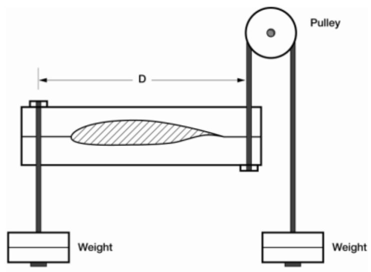

Figure 4. Simulated experimental test [10].

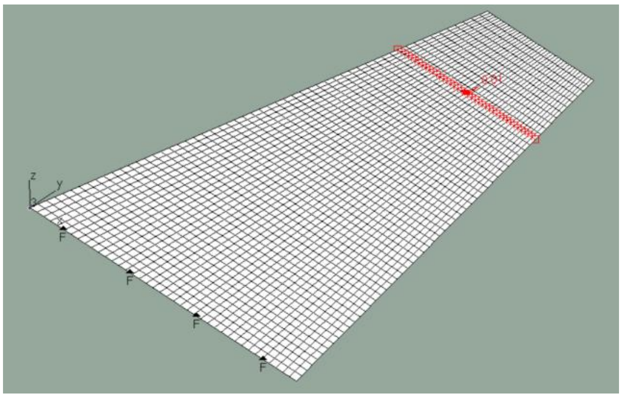

Figure 5. First numerical model setup.

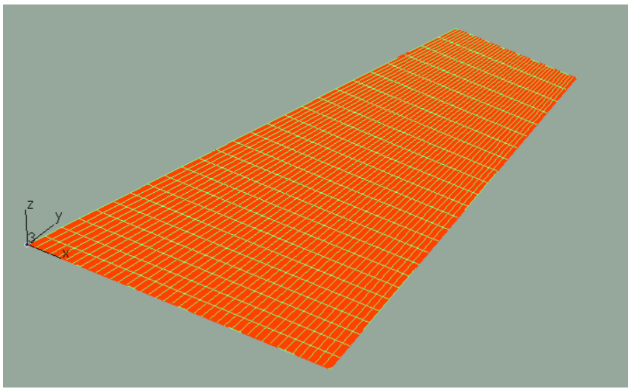

Figure 6. Aerodynamic model.

3. RESULTS AND DISSCUSSION

Analytical approach is verified with numerical models, altough the analytical approach overestimates the loss of static stability, because it doesn't take into consideration the strucutre stress – strain field and doesn't sufficiently enough capture the AR and TR influence on static instability. Therefore, it is appropriate to use the analytical approach at the early design stages in order to obtain an understanding of the order of magnitude of the static instability. The typical deformed shape of missile fin due to twisting is given in Figure 7.

The design variable of paramount importance for strucutre static stability is it's thickness, because it directly and mostly influence the structures inertial characteristics. The influences of AR and TR are drastically lower, and the theoretical method isn't suitable for its prediction. Coupled influence of two or more design variables could be analyzed in the future work. Figure 8 shows the formed dependencies of the divergence speed on the plate thickness, obtained with theoretical and numerical approach.

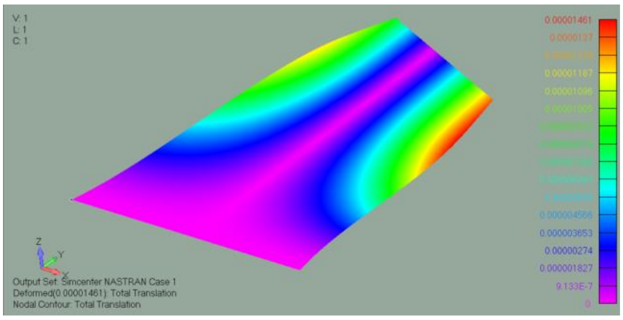

Figure 7. Deformed model, experimental testing simulated with numerics.

Table 3. Divergence velocities obtained theoreticaly.

Plate No. [/]	Divergence velocity [m/s]
1	4.05
2	11.44
3	21.00
4	32.30
5	11.42
6	11.44
7	11.45
8	11.45
9	11.45
10	11.44
11	11.44
12	11.44
13	11.43

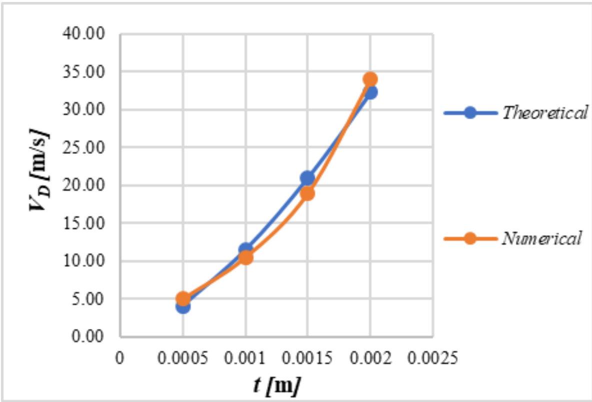

Figure 8. Dependence of divergence speed on plate thickness.

4. CONCLUSION AND FURTHER WORK

Within the content of this research, the influence of geoemtrical parameters on low speed missile fins static instability is analyzed. Results are obtained with theoretical method and afterwards verified with two different numerical methods. It was found out that designers could use either theoretical or experimental method simulated with FEM, in the early design stages, because they are fast, not expensive, and they render enough accurate preliminary results. Difference betwen theoretical and first numerical method is within 20% which is suitable in the preliminary design stage. In the detailed design phase, it is strongly suggested to use such numerical models that couple structural and aerodynamic effects. In the further work, aerodynamic modeling (panel AR) on the static instability will be investigated.

Table 4. Theoretical model verification.

Plate No. [/]	Model	Divergence velocity [m/s]
3	Analytical	21.0
	Experimental setup	19.2
	SOL 144	10.3
4	Analytical	32.3
	Experimental setup	29.2
	SOL 144	14.5

In the following research, complex and detailed theoretical methods will be used in order to predict the static and dynamic plates instability and futher database of parameters that influence aeroelastic effects will be built that will be used to predict the aeroelastic instability with Machine Learning.

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