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OPTIMIZATION OF SINGLE STAGE PLANETARY GEARBOX PARAMETERS USING GENETIC ALGORITHM

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Abstract: *Planetary gearboxes are a mechanical devices consisting of multiple gears arranged in a circular configuration around a central sun gear. This layout enables an efficient gearbox with high torque in a compact design, thus making it well-suited for a wide range of industrial and military applications such as industrial motors, rotorcraft, vehicles, wind turbines, and more. However, when it comes to aircraft applications, weight and strength are essential considerations in the design process. Genetic algorithms are used to optimize the parameters of a single-stage planetary gearbox in order to achieve the necessary balance between weight, strength, and performance in aircraft applications. The study focuses on formulating the optimization problem in an appropriate way while also developing constraints that guarantee the effective functioning of the planetary gearbox. In order to effectively address this complex and multimodal constrained optimization problem, this paper suggests utilizing an enhanced genetic algorithm (NSGA-II), which is widely recognized as the most commonly employed evolutionary optimization technique. In comparison with conventional GA algorithm, the numerical simulation results demonstrate that the suggested method exhibits enhanced optimization performance in relation to the quality of the achieved solutions.*

Keywords: *Planetary gearboxes, optimization, efficiency, genetic algorithm.*

1. INTRODUCTION

Planetary gearboxes, also known as epicyclic gear trains, are critical components in many mechanical systems due to their high power density, compact size, and versatility in providing a wide range of speed ratios. These gearboxes consist of a central sun gear, multiple planet gears rotating around the sun gear, and an outer ring gear that meshes with the planet gears. This configuration allows for efficient power transmission and torque distribution, making planetary gearboxes essential in various applications, including automotive transmissions, aerospace mechanisms, wind turbines, and industrial machinery [1], [2].

The optimization of planetary gearbox parameters, such as gear ratios, gear module, tooth number, and material properties, presents a significant engineering challenge. This complexity stems from the numerous interdependent variables that influence the overall performance of the gearbox, including efficiency, load distribution, noise levels, and reliability. Achieving an optimal balance among these factors requires sophisticated optimization techniques that can navigate the intricate trade-offs involved [1]. In this regard, the optimization of gearbox and gear transmission systems has traditionally focused on single-objective optimization. Single-objective optimization aims to improve a specific performance criterion such as minimizing weight or maximizing efficiency. However, these approaches often fail to meet the complex requirements of modern engineering applications.

Single-objective optimization methods, while effective for specific criteria, do not suffice for the increasingly stringent and complex demands of modern engineering applications. Contemporary requirements often involve balancing multiple conflicting objectives, such as minimizing weight while maximizing efficiency and durability. Multi-objective (MO) optimization provides a more comprehensive approach by considering multiple performance criteria simultaneously. One exemplary study [3] explores a multi-objective optimization of a two-stage spur gearbox. This research incorporated constraints such as scuffing and wear, with objectives to reduce weight and power losses using a discrete version of the Non-Dominated Sorting Genetic Algorithm II (NSGA-II). The study demonstrated significant improvements in power loss reduction compared to single-objective optimization.

In [4] authors applied multi-objective optimization to vehicle drivetrains, aiming to improve fuel consumption, acceleration, and emissions. Using the Interactive Adaptive-Weight Genetic Algorithm, the study optimized gear ratios and other variables under various driving cycles, achieving a balanced enhancement in acceleration, fuel savings, and emission reductions. Additionally, in [5] researchers implemented a multi-objective optimization for gear unit design, targeting power loss and vibrational excitation minimization through a multi-scale approach.

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The NSGA-II was used to optimize both macro and micro-geometry characteristics, resulting in reduced power loss and improved performance metrics compared to single-objective approaches.

To tackle the complexity of multi-objective optimization, evolutionary optimization algorithms (EAs) have been extensively employed. These algorithms are inspired by the principles of natural evolution and use mechanisms such as selection, crossover, and mutation to evolve solutions over successive generations. EAs are particularly effective in exploring large and complex solution spaces, making them suitable for solving multi-objective problems [6]. Several EAs have been developed for multi-objective optimization, including Differential Evolution (DE), Particle Swarm Optimization (PSO), and Ant Colony Optimization (ACO), each with its unique strengths and applications [6].

Among these, Genetic Algorithms (GAs) have proven to be particularly robust and flexible for solving complex optimization problems [7]. GAs encode potential solutions as chromosomes, which are then evolved through genetic operations to explore and exploit the solution space efficiently, enabling the finding of optimal or near-optimal global solutions. Since it is designed for single objective optimization, the application to MO problems requires specific modifications. The earliest advancements in the application of genetic algorithms to multi-objective optimization involved the weighted sum method. In this approach, each objective function is assigned a specific weight, and the weighted objectives are summed to transform the problem into a single-objective optimization task. Despite its simplicity, this method has significant limitations, particularly in the context of non-convex Pareto sets, where it often fails to identify global optimal solutions [8].

Genetic Algorithms for MO optimization, such as the NSGA-II [9] and the Strength Pareto Evolutionary Algorithm (SPEA2), have been specifically designed to handle multiple objectives. These algorithms maintain a diverse population of solutions and employ selection mechanisms that favor non-dominated solutions, ensuring a comprehensive exploration of the trade-offs between objectives [9].

In this study, the aim is to optimize the parameters of a single-stage planetary gearbox using a multi-objective genetic algorithm. By leveraging the strengths of GAs in handling complex optimization problems, the aim is to enhance the performance and efficiency of planetary gear systems through a rigorous and systematic approach. Our research focuses on the application of NSGA-II to identify optimal trade-offs among conflicting objectives, ultimately contributing to the advancement of gearbox design and performance. Furthermore, the performance of NSGA-II algorithm is compared to the more conventional GA using weighted sum method.

The structure of this paper is as follows: Section 2 formulates the optimization problem and introduces the penalty method used for handling constraints. Section 3 details the procedure of the multi-objective genetic algorithm method. Section 4 presents the corresponding simulation results. Finally, conclusions are discussed in Section 5.

2. FORMULATION OF THE CONSIDERED OPTIMIZATION PROBLEM

The planetary gearbox considered in this study is a single-stage planetary gearbox. This gearbox includes three planet gears and two central gears. The sun gear, which is one of the central gears with external gearing, is directly connected to the input shaft. The other central gear, which has internal gearing, remains stationary and secures the assembly. Power is distributed from the input shaft and sun gear to the planet gears and subsequently transmitted to the carrier connected to the output shaft. The schematic representation of the gearbox under consideration is shown in Fig. 1.

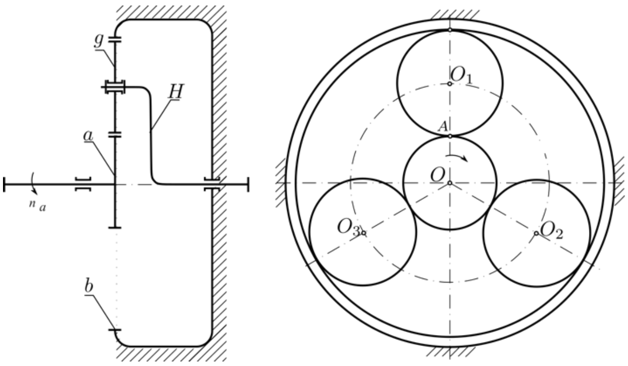


Figure 1. Schematic representation of a single-stage planetary gearbox considered in the optimization model

The primary goal of this study is to develop a reliable and lightweight planetary gearbox that maintains high efficiency. This necessitates creating a MO optimization model that integrates design variables, objectives, and constraints to optimize conflicting objectives, such as weight reduction, decreased power loss, and improved component reliability.
The general form of the multi-objective optimization problem can be expressed as:

$$\min\ f_m(\mathbf{x}), \quad m = 1,\ldots,M \text{ and } i = 1,\ldots,N_P \tag{1}$$

subject to:

$$g_k(\mathbf{x}) \geq 0, \quad k = 1,\ldots,K \tag{2}$$

$$h_l(\mathbf{x}) = 0, \quad l = 1,\ldots,L \tag{3}$$

$$x_{i,j} \in \left[x_j^{\text{Lower}}, x_j^{\text{Upper}} \right], \quad j = 1,\ldots,n \tag{4}$$

where M represents the number of objectives (≥ 2), $g_k(x_i)$ represents the k-th inequality constraint out of a total of K inequality constraints, $h_l(x_i)$ represents the l-th equality constraint out of a total of L equality constraints, $\mathbf{x}$ is the vector of decision variables whose components are $x_{i,j}$ bounded by x_j^{Lower} and x_j^{Upper}, and N_P is the number of solutions in the population.

2.1 Objectives of the considered problem

The considered planetary gearbox optimization model takes into consideration a number of objective outlined in the following subsections the objectives are outlined.

Volume of the Gears

One of the crucial requirements, especially in aerospace industry, is the requirement of mass minimization. Under the assumption of constant density, the problem can be reduce to the problem of minimizing the volume of the gears. The volume of the gears in the considered gearbox can be calculated as

$$W(\mathbf{x}) = \rho\frac{\pi}{4}b\left[d_{(a)}{}^2 + n_w\left(d_{a(b)}^2 - D^2\right) + \left(d_{(g)}^2 - d_s^2\right)\right] \quad (5)$$

where $V(x)$ denotes the volume of the gears, b is the gear face width, where $d_{(a)}$ is the pitch circle of the sun gear, $d_{a(b)}$ is tip diameter of the ring gear, d_s is outside bearing diameter, $d_{(g)}$ is the pitch circle of the planet gear, $n_w = 3$ is the number of planet gears, D denotes outside diameter of a ring gear.

Contact Ratio

The contact ratio is introduced as an objective to ensure smooth and continuous gear operation by maintaining multiple teeth in contact, which distributes load evenly and reduces vibrations and noise, as follows

$$\varepsilon_\alpha = \frac{0.5\left(\sqrt{d_{2a(a)} - d_{2a(b)}} + \sqrt{d_{2a(g)} - d_{2a(g)}}\right) - a\sin(\alpha_{wt})}{\pi m_t \cos(\alpha_t)}$$

(6)

where ε_α denotes the contact ratio, a is the center distance, α_{wt} and α_t are appropriate pressure angles and m_t is the module of the gear.

Safety Against Bending for Central Gears

Maximizing the safety against bending for the sun gear and other gears:

$$S_{F(a)(x)} = \frac{[\sigma_F]_{M(a)}}{\sigma_{F(a)}}, \qquad S_{F(b)(x)} = \frac{[\sigma_F]_{M(b)}}{\sigma_{F(b)}} \quad (7)$$

where $S_{F(a)(x)}$ and $S_{F(b)(x)}$ are the safety factors against bending for gears (a) and (b), $[\sigma_F]_{M(a)}$ and $[\sigma_F]_{M(b)}$ are the allowable bending stresses, and $\sigma_{F(a)}$ and $\sigma_{F(b)}$ are the actual bending stresses for gears (a) and (b), respectively.

Safety Factor for Contact Stress

Minimizing the contact stress:

$$S_{H(a)(x)} = \frac{[\sigma_H]_{M(a)}}{\sigma_{H(a)}} \quad (8)$$

where $S_{H(a)(x)}$ is the safety factor for contact stress, $[\sigma_H]_{M(a)}$ is the allowable contact stress, $\sigma_{H(a)}$ is the contact stress.

Gearbox Efficiency

The second objective of the optimization issue pertains to the efficiency of the planetary gear box and can be mathematically expressed by the given statement

$$\eta_{aH}^b = \frac{1 - \eta_{ag}^H \eta_{gb}^H u_{ab}^H}{1 - u_{ab}^H} \quad (9)$$

where η_{ag}^H and η_{gb}^H denote relative efficiency of the sun-planet gears and relative efficiency of planet-ring gears, respectively. These efficiencies can be determined according to the numerical procedure given in [10]. Furthermore, u_{ab}^H denotes the relative gear ratio.

2.2 Constraints

Bending Constraint

Ensuring the gear tooth bending strength:

$$g_{1,2,3} = \frac{[\sigma_F]_{M(a,g,b)}}{\sigma_{F(a,g,b)}} - S_F > 0 \quad (10)$$

where $[\sigma_F]_{M(a,g,b)}$ is the allowable bending stress for gears a, g, and b, $\sigma_{F(a,g,b)}$ is the actual bending stress for gears a, g, and b, and S_F is the safety factor.

Pitting Constraints

Ensuring resistance to surface fatigue:

$$g_4 = \frac{[\sigma_H]_{M(a,g)}}{\sigma_H} - S_H > 0, \qquad g_5 = \frac{[\sigma_H]_{M(g,b)}}{\sigma_H} - S_H > 0 \quad (11)$$

where $[\sigma_H]_{M(a,g)}$ and $[\sigma_H]_{M(g,b)}$ are the allowable contact stresses for gear pairs (a,g) and (g,b), σ_H is the actual contact stress, S_H is the safety factor for pitting.

Space Requirement

Ensuring appropriate clearance for gear assembly:

$$g_7 = 2a\sin\left(\frac{\pi}{n_w}\right) - f_z - d_{a-g} \geq 0 \tag{12}$$

where f_z is radial clearance and d_{a-g} is the addendum circle diameter of the sun gear.

Assembly Condition

Preventing teeth interference during meshing:

$$h_1 = \frac{z_a z_b}{n_w D(z_g, z_b)} - i = 0 \tag{13}$$

where z_a and z_b are the numbers of teeth on gears a and b, D is the pitch diameter, z_g is the number of teeth on the planet gear, i is the gear ratio.

3. MULTI OBJECTIVE GENETIC ALGORITHM

Genetic algorithm is a metaheuristic optimization algorithm, inspired by the principles of natural genetics and natural selection. This algorithm is widely utilized to find optimal solutions to multimodal optimization problems at a global level. The process of utilizing a GA to identify the global optimum solution begins with the generation of a population of initial solutions. In traditional GAs, also known as single-objective GAs, each solution is evaluated based on a single objective function, and the algorithm aims to find the best solution according to that objective [8].

3.1 Regular Genetic Algorithm

A GA operates through three primary evolutionary operators: selection, crossover, and mutation.

Selection

During the selection phase, the GA uses the objective function value of each individual to choose chromosomes from the population that have favorable mating qualities. Chromosomes with lower objective function values (for minimization problems) are more likely to be chosen. The selection probability of the i th individual is calculated as:

$$P_i = \frac{f_i}{\sum_{j=1}^{N} f_j}, \tag{14}$$

where f_i denotes the objective function value of the chromosome, and N is the population size. The cumulative probability C_i of the i th individual is determined by:

$$C_i = \sum_{j=1}^{i} P_j \,. \tag{15}$$

Individuals are then selected based on a random number r generated between 0 and 1, where:

$$C_{i-1} \leq r < C_i \,. \tag{16}$$

Crossover

The crossover operator combines the genetic information of two selected individuals to produce offspring that share important features from both parents. A common method is the two-point crossover, where two random points along the chromosome length are selected, and the segments between these points are exchanged between the two parents.

Mutation

Mutation introduces new genetic structures into the population by randomly altering some genes of selected chromosomes. This operator helps maintain genetic diversity within the population and prevents premature convergence to local optima. The mutation rate is typically kept low to avoid disrupting valuable genetic information. These operators are applied iteratively, evolving the population towards the global optimal solution until a termination condition is met, which is usually defined by a maximum number of iterations or a convergence threshold.

3.2 Multi-objective Genetic Algorithm (NSGA-II)

In multi-objective optimization problems, multiple conflicting objectives must be optimized simultaneously. Unlike single-objective optimization, there is no single optimal solution but rather a set of solutions known as the Pareto front, where each solution is non-dominated with respect to the others. A solution is considered non-dominated if no other solution is better in all objectives simultaneously. To extend GAs for multi-objective optimization, non-dominated sorting and crowding distance mechanisms are introduced. One of the most widely used algorithms in this domain is the NSGA-II.

The NSGA-II [9] is an advanced version of the GA algorithm, designed to enhance the process of multi-objective optimization by focusing on non-dominated sorting. This method evaluates the quality of each solution $\mathbf{x}$ based on a fitness function that considers not only the solutions dominating $\mathbf{x}$ but also those that $\mathbf{x}$ itself dominates. An essential aspect of NSGA-II is the crowding distance metric, which calculates the Euclidean distance of a solution from its neighboring solutions in the criterion space, thereby preserving solution diversity within the population.

Non-dominated Sorting

The NSGA-II algorithm classifies all chromosomes in the current population into different fronts based on their level of non-domination. Each solution $\mathbf{x}_i$ is assessed for the number of solutions n_i that dominate it and the set S_i of solutions it dominates. Solutions not dominated by any member of the population form the first non-dominated front (PF1). Subsequent fronts are formed by decrementing the dominance count n_j for each solution in S_i until all solutions are classified. This iterative sorting process ensures that all solutions are grouped into their respective non-dominated fronts, as illustrated in Fig.2, which depicts the maximization of two conflicting criterion functions.

After the non-dominated sorting, a new population for the next iteration is constructed by sequentially adding solutions from the highest-ranked fronts until the population size limit is reached. If adding an entire front would exceed the population limit, solutions from that front are sorted based on their crowding distance, and the top solutions are selected to fill the remaining slots. This method ensures that the new population maintains a balance between convergence and diversity.

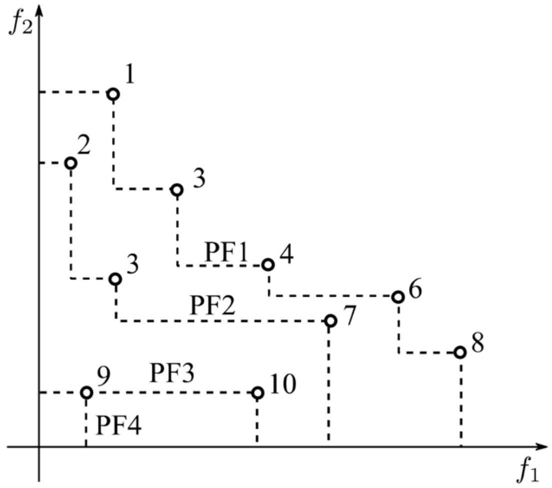


Figure 2. Illustration of the domination principle of NSGA-II algorithm

Crowding Distance

The crowding distance metric in NSGA-II is calculated for each solution $\mathbf{x}_i$ as the average Euclidean distance between the solution and its nearest neighbors along each criterion dimension. For a population of N_P chromosomes and a solution $\mathbf{x}_a$ with a vector function value $f(\mathbf{x}_a)=[f_1(\mathbf{x}_a), f_2(\mathbf{x}_a),\ldots, f_m(\mathbf{x}_a)]^T$, the maximum and minimum values of the nearest solutions in the population are determined. The crowding distance for solution $\mathbf{x}_a$ is then computed using the formula:

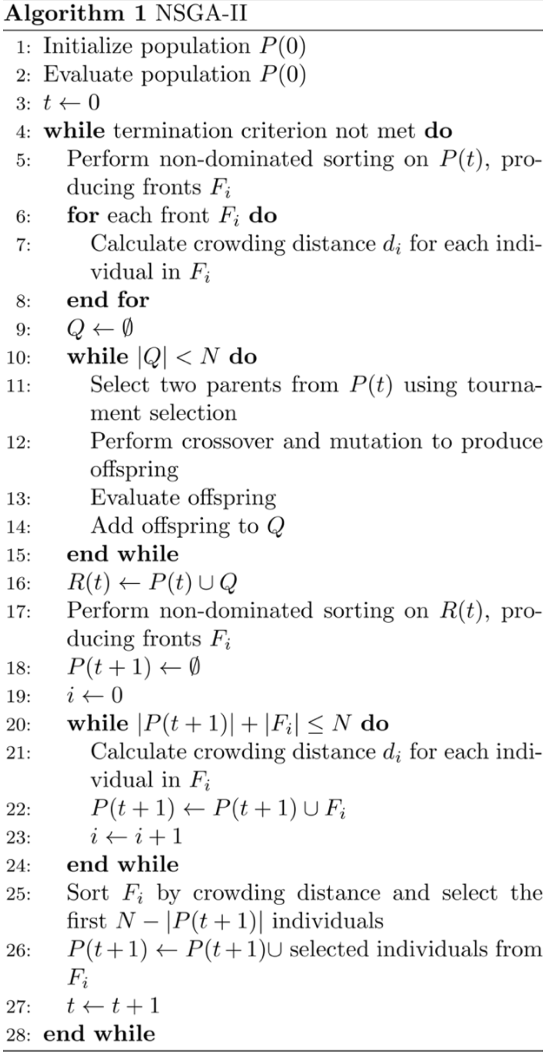
$$d(\mathbf{x}_a)=\sum_{i=1}^{m}\left(f_{\max,i}(\mathbf{x}_a)-f_{\min,i}(\mathbf{x}_a)\right)$$

Solutions located in densely populated regions of the criterion space have lower crowding distance values, while those at extreme values tend towards infinity. The crowding distance ensures a uniform distribution of solutions along the Pareto front, preventing premature convergence and maintaining diversity.

The NSGA-II algorithm effectively balances convergence and diversity by utilizing non-dominated sorting and crowding distance metrics. This approach results in a robust and efficient method for solving multi-objective optimization problems, and the pseudocode of this algorithm is shown in the Fig. 3

5. SIMULATION RESULTS

This section presents results of optimizing the parameters of planetary gearbox using the NSGA-II method compared to the conventional single objective GA method using the weighted sum method to apply to MO problems [8]. Table 1 shows the essential design characteristics of the planetary gearbox under consideration.
In the context of optimizing planetary gearboxes, the Pareto frontier serves as a critical tool for balancing multiple objectives. Therefore, firstly the combination of the contact ratio and gearbox efficiency as objectives have been analyzed, as shown in Fig. 4.



```
Algorithm 1 NSGA-II
Initialize population $P(0)$
Evaluate population $P(0)$
$t \leftarrow 0$
while termination criterion not met do
    Perform non-dominated sorting on $P(t)$, producing fronts $F_i$
    for each front $F_i$ do
        Calculate crowding distance $d_i$ for each individual in $F_i$
    end for
    $Q \leftarrow \emptyset$
    while $|Q| < N$ do
        Select two parents from $P(t)$ using tournament selection
        Perform crossover and mutation to produce offspring
        Evaluate offspring
        Add offspring to $Q$
    end while
    $R(t) \leftarrow P(t) \cup Q$
    Perform non-dominated sorting on $R(t)$, producing fronts $F_i$
    $P(t+1) \leftarrow \emptyset$
    $i \leftarrow 0$
    while $|P(t+1)| + |F_i| \leq N$ do
        Calculate crowding distance $d_i$ for each individual in $F_i$
        $P(t+1) \leftarrow P(t+1) \cup F_i$
        $i \leftarrow i+1$
    end while
    Sort $F_i$ by crowding distance and select the first $N - |P(t+1)|$ individuals
    $P(t+1) \leftarrow P(t+1) \cup$ selected individuals from $F_i$
    $t \leftarrow t+1$
end while
```

Figure 3. Pseudocode of NSGA-II algorithm

Table 1. Parameters of the planetary gear set used in this paper

Parameters	Symbol	Value
Input Power [kW]	P_a	180
Input speed [rpm]	n_a	2750

Parameters	Symbol	Value
Pressure angle [degree]	α_n	20
Gear material	18CrNi8	
Gear surface Roughness [μm]	R_a	0.8
Factor of safety against bending	$S_{F\min}$	1.5
Factor of safety against pitting	$S_{H\min}$	1.25
Number of planet gears	n_w	3

From the Fig. 4 it can be seen that there exists a trade-off between contact ratio and gearbox efficiency as the objectives, where the increase in one, leads to the simultaneous decrease in the other.

Furthermore, regarding the optimization algorithms for a planetary gearbox optimization problem Fig. 4 showcases the effectiveness of NSGA-II in producing a more detailed Pareto frontier compared to a conventional Genetic Algorithm based on weighted sum method.
Next, the contact ratio and the contact stress are taken as the objectives, as shown in Fig. 5.

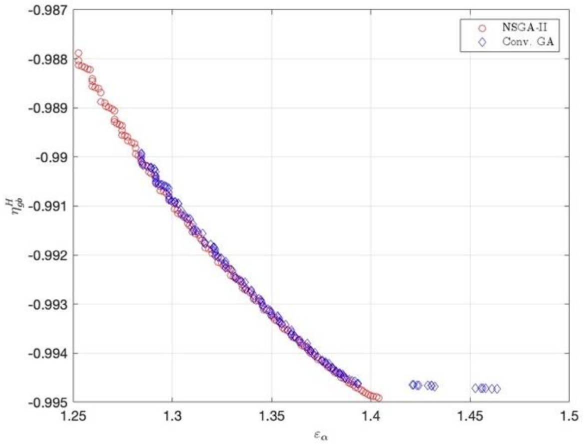


Figure 4. Pareto frontier for planetary gearbox for contact ratio and gearbox efficiency as objectives.

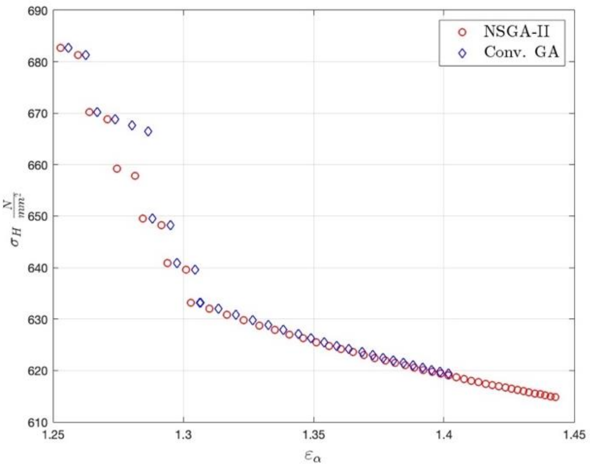


Figure 5. Pareto frontier for planetary gearbox for contact ratio and contact stress.

From the results depicted in Fig. 5 it can be observed that the considered objective functions are conflicting. Again the NSGA-II algorithm showed comparably better performance.

When compared to the industrial gearbox reference [11], it results in a 12% decrease in gearbox weight and a 0.3% increase in efficiency. The numerical simulations demonstrate that the gear optimization technique, which utilizes the NSGA-II algorithm, can yield superior design solutions in comparison to standard algorithms.

6. CONCLUSION

This study explores the development of a non-linear optimization model to find the best parameters for a planetary gearbox. The model is solved using a metaheuristic optimization technique. In order to determine suitable parameters for a planetary gearbox, the complex multimodal objective functions that involve multiple constraints have been minimized. In the paper, a comparative examination of the application of the widely recognized NSGA-II version of the GA and a more typical GA algorithm implemented using the weighted sum approach have been analyzed. In the given multi-objective optimization issue, the relevant objectives are formulated, including minimizing the weight of the gearbox and maximizing the efficiency of the gearbox. The NSGA-II optimization method enhances the characteristics of the planetary gearbox, resulting in superior performance.

Future research should extend the optimization framework to multi-stage planetary gearboxes and incorporate additional real-world constraints such as manufacturing tolerances and material properties variations. Additionally, combining NSGA-II with other optimization techniques and validating the results through experimental testing will enhance the robustness and practical applicability of the proposed method.

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