



PREDICTION OF ARTILLERY CORRECTION ELEMENTS BY SOFTWARE FOR BALLISTICS MODULE OF FIRE CONTROL SYSTEMS

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Abstract: At present, by software for Ballistics Module –BM of Fire Control Systems -FCS can be calculated GUN elements - elevation in vertical plane and angle of its direction in horizontal plane. There is serious reason, say forced, for recording part of actual trajectory, not observing impact point of projectile fall, and on its values predict point of projectile fall, say only one. This method enables firing two projectiles for acceptable time period, firing second projectile with corrected GUN elements before first projectile is reached impact point - target. Finally, method for prediction artillery correction elements is based on comparing part of numerical data of actual path trajectory and corresponding data of nominal trajectory..

Keywords: Software, Ballistic Module extension, Artillery shooting

1. INTRODUCTION

The complexity of preparing data for opening artillery fire can be seen from the procedure for obtaining data that affects the flight of projectiles. The procedure is intended for complex formations of artillery units, and it is the same for an individual gun - howitzer. What characterizes the procedure is the time and means for collecting the necessary data in battle-field. It is enough to mention the determination of the initial speed of the projectile, as the most important factor affecting the flight of the projectile, and the elements of meteorological data in which the projectiles flight takes place, as the most important speed and direction of the air wind and temperature. Both meteorological elements are random quantities characterized by mean values and their dispersion. In other words, meteorological elements are statistical quantities whose determination requires a certain number of samples, far greater than one in order for the measured quantity to be faithfully presented. Today, the requirement is to hit the target with one projectile, and the statistical method of operation is applicable only during polygon tests in proving ground. In the past, and even today, a simplified method of preparing the elements for opening fire, called fire correction is used. With the initial elements, the projectile is fired, its fall position in relation to the target is recorded, and based on the distance difference in relation to the target and the shooting direction, previous elements for opening fire can be corrected. The time duration projectile flight today makes this fire correction procedure unacceptable. It is applicable only after time projectile ended is flight which is relatively long today and risk to be destroyed place of gun-howitzer on its position is certain. Another, important factor that makes correcting fire difficult is target position - enemy territory, notice its relation to the target and its delivery to the gun-howitzer position. The author see into

a partial way out of these difficulties in the process of predicting where the projectile will fall based on measured data on the position of the projectile at the beginning of its path trajectory. These measurements of the position of the projectile in flight are the basis for extending the author's software of the Ballistic Module – BM. Base calculations of BM are the initial elements for opening fire and performing their renewing. It goes without saying that there is designed original author suitable way of measuring the position of the projectile at the initial part of its trajectory and transferring the measured data to the computer running the authors BM extended software.

2. CALCULATION CORRECTIONS BASED ON FIRING TABLES DATA

Long time ago, Firing Tables –*FT* (collection on projectile trajectory data) exists as the only source of trajectory elements data. Nowadays, its frequency for using is reduced, for it is *FCS* are widely in use in army troops for Artillery preparation fire data. Range and Deflection corrections can be done for meteorological and ballistic conditions different from the *FT* nominal conditions, named meteorological and ballistic *deviations*. Their influence on the range and deflection and sign are given in auxiliary **Table P1** of *FT* (see *NORA FT*). Calculation total Deflection and Range corrections can be done as:

a) For Deflection (mils)

$$\Delta Z_{\Sigma} = Z \times K_{\Gamma} + 0.1 \times \Delta Z_W \times K_{\Gamma} \times W_Z + \Delta Z_{\Gamma\Phi} \quad (1)$$

b) For Range (m) accordingly to Taylors expansion

$$\text{neglecting second terms } f(x_i) = f_0 + \sum \frac{\partial f}{\partial x_i} \Delta x_i :$$

$$\begin{aligned} \Delta X_{\Sigma} = & 0.1 \times \left[(\Delta X_W \times K_{\Gamma} \times W_X) + \right. \\ & \left. + (\Delta X_H + 0.1 \times \Delta X_H \times \Delta H) \times \Delta H + \right. \\ & \left. + (\Delta X_{t_b} \times K_{\Gamma} \times \Delta t_b) + (\Delta X_T \times K_{\Gamma} \times \Delta T) + \right. \\ & \left. + (\Delta X_{V_o} \times K_{\Gamma} \times \Delta V_o) \right] + \Delta X_{\Gamma\Phi} \end{aligned} \quad (2)$$

Where:

GP – Altitude of GUN fire position (m)

$$K_{\Gamma} = 1.0 + GP(m) / 10000$$

Z -Drift (mils) –[12] –(Column 12, Table C1 of **FT**)

ΔZ_W – Correction (mils) for $W_z=10(m/s)$ –[13]

W_Z – Ballistic lateral wind component (m/s)

$\Delta Z_{\Gamma\Phi}$ – Defl. Dev. due to Earth rotation (mils) –[G1]

ΔX_H – Correction for $W_x=10(m/s)$ –[14]

W_X – Ballistic longitudinal wind component (m/s)

ΔX_H – Correction for 10mbar –[16]

ΔH – Ballistic atmospheric Pressure -mbar

ΔX_{t_b} – Corr. for powder temperature 10°C –[18]

Δt_b – Powder temp. difference from Table $t_b=21^{\circ}\text{C}$

ΔX_T – Corr. for air temp. diff. –[15], for $\Delta T=10^{\circ}\text{C}$

ΔT – Ballistic difference air temperature -°C

ΔX_{V_o} – Correction for $\Delta V_o=10(m/s)$ –[17]

ΔV_o – Muzzle velocity difference -(m/s)

$\Delta X_{\Gamma\Phi}$ – Range deviation due to Earth rotation (m) –[G1]

ΔX_m – Corr. for proj. mass difference $\Delta m=1$ mark –[19]

n – Number of projectile mark –mass difference

According to the **FT**, GUN barrel angle – elevation ought to be determined for distance X_T calculated as

$$X_T = X_r + \Delta X_{\Sigma} \quad (3)$$

where X_r is geographic horizontal distance from GUN to the target.

This calculations can be automatized, and they are somewhere realized, but its validity is limited only to the nominal **FT** conditions, ie. for the same height level GUN

and target and other approximation to the used deviations data and its effects on projectile flight. For the *mountain* conditions can be used appropriate **FT**, but other auxiliary **FT** Tables complicate this calculations.

3. PROJECTILE MATHEMATICAL MODEL SOLUTION

By solving so called *inverse* External Ballistic -**EB** problem, the elevation angle θ which should be given the barrel of the artillery GUN-HOWITZER is obtained in order to achieve the required range – distance from GUN to target X_c . The most significant part of the necessary ballistic calculations ought to be performed by the software of the Ballistic Module -**BM** of the Fire Control System -**FCS**.

The software of existing **BM** consists of two parts, first is mathematical model that describes the flight of the projectile in space – gives path trajectory elements. Modeled ballistic characteristics of the projectile and current flight conditions (the most important elements of the state of the atmosphere: pressure, temperature and wind with its strength and direction) serves to solve the *direct* **EB** problem. Another mathematical model is designed for resolving of *inverse* **EB** problem. They together make **BM** of **FCS** software and are coupled, so might not be isolated to work independently, except in case now introduced as extension of existing for correction range in advance. For the solution of the *direct* **EB** problem, a various mathematical models can be used that describes the motion of the projectile –its flight throw atmosphere. As example of complexity chosen projectile mathematical model, it is given Modified Point Mass and five degrees of freedom Model –[10].

$$\begin{aligned} \frac{dx}{dt} &= V_x = V \cos \theta \cos \psi \\ \frac{dy}{dt} &= V_y = V \sin \theta \\ \frac{dz}{dt} &= V_z = V \cos \theta \sin \psi \end{aligned} \quad (4)$$

$$(V_R)_x = V_x - W_x, \quad (V_R)_y = V_y$$

$$(V_R)_z = V_z - W_z$$

$$V_R = \sqrt{(V_R)_x^2 + (V_R)_y^2 + (V_R)_z^2}$$

$$\theta = \arctg \frac{V_y}{V_x}, \quad \psi = \arctg \frac{V_z}{V_x}$$

$$\frac{dV_x}{dt} = a_x$$

$$\frac{dV_y}{dt} = a_y$$

$$\frac{dV_z}{dt} = a_z$$

$$\frac{d\omega}{dt} = k_7 + k_8 \frac{\omega}{V} \quad (5)$$

$$J = C_{et} \cdot K \cdot \xi(y) \cdot M_R^2 \cdot GAK(M_R)$$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} a_{11}a_x + a_{12}a_y + a_{13}a_z \\ a_{21}a_x + a_{22}a_y + a_{23}a_z \\ a_{31}a_x + a_{32}a_y + a_{33}a_z \end{bmatrix}$$

$$a_{11} = 1 - C_1 \left((V_R)_y^2 + (V_R)_z^2 \right)$$

$$a_{12} = C_1 (V_R)_x (V_R)_y + C_3 (V_R)_z$$

$$a_{13} = C_1 (V_R)_x (V_R)_z - C_3 (V_R)_y$$

$$a_{21} = C_1 (V_R)_x (V_R)_y - C_3 (V_R)_z$$

$$a_{22} = 1 - C_1 \left((V_R)_x^2 + (V_R)_z^2 \right)$$

$$a_{23} = C_1 (V_R)_y (V_R)_z + C_3 (V_R)_x$$

$$a_{31} = C_1 (V_R)_x (V_R)_z + C_3 (V_R)_y$$

$$a_{32} = C_1 (V_R)_x (V_R)_y - C_3 (V_R)_z$$

$$a_{33} = 1 - C_1 \left((V_R)_x^2 + (V_R)_y^2 \right)$$

$$b_1 = C_4 (V_R)_x - C_6 (V_R)_z$$

$$b_2 = C_4 (V_R)_y - C_5 (V_R)_z$$

$$b_3 = C_4 (V_R)_z + C_6 (V_R)_x$$

$$C_1 = k_3 \left(\frac{\omega}{V} \right) (\alpha_b - \alpha_a), \quad C_3 = -k_2 (\alpha_b - \alpha_a)$$

$$C_4 = -k_1 - k_9 (\alpha^2 + \beta^2) - k_3 \alpha_b \left(\frac{\omega}{V} \right) g (V_R)_y$$

$$C_5 = g \left(1 - k_3 \left(\frac{\omega}{V} \right) \alpha_b \right), \quad C_6 = k_2 \alpha_b g$$

$$\alpha_b = k_6 \left(\frac{\omega}{V} \right) \frac{1}{k_2 k_4 + k_3 k_6 \left(\frac{\omega}{V} \right)^2}$$

$$\alpha_a = k_2 k_{11} \left(\frac{\omega}{V} \right) \frac{1}{k_2 k_4 + k_3 k_6 \left(\frac{\omega}{V} \right)^2}$$

$$k_i = k_0 \frac{p_0}{p_{0N}} \frac{m_0}{m_{0N}} k_i \xi(y) M^2 GAK(i)$$

$$GAK(1) = C_X$$

$$GAK(2) = C_Y^\sigma$$

$$GAK(3) = C_Z^{\omega, \sigma}$$

$$GAK(4) = m_Z^\sigma$$

$$GAK(5) = m_Z^{\Omega + \dot{\sigma}}$$

$$GAK(6) = m_Y^{\omega, \sigma}$$

$$GAK(7) = m_X^\varepsilon$$

$$GAK(8) = m_X^{\omega_X}$$

$$GAK(9) = C_X^{\sigma^2}$$

$$k'_1 = k'_2 = k'_9 = \frac{d^2}{m} 10^3, \quad k'_3 = \frac{d^3}{m} 10^3$$

$$k'_4 = \frac{d^3}{mi_Y^2} 10^3, \quad k'_5 = k'_6 = \frac{d^4}{mi_Y^2} 10^3$$

$$k'_8 = \frac{d^4}{mi_X^2} 10^3, \quad k'_{10} = \frac{rtg \mu}{i_X^2} 10^3$$

$$k'_{11} = \frac{i_X^2}{i_Y^2} 10^3$$

Given mathematical model as equations of physical projectile with projectile characteristics incorporated as parameters in model, in fact form nonlinear system of differential and ordinary equations. It cannot be solved analytically, but its numerical solution for certain initial values -equation (6), for introduced kinematic parameters determines values projectile center of mass coordinate in space and other projectile characteristic.

Initial values and projectile characteristic are:

$$(V_{x,y,z})_0, (X, Y, Z)_0, (\theta, \psi, \omega)_0, a_{i,j=3}, k_{i=11}$$

$$W_{x,z} = f(Y), GAK(i), i=1, 9 = f(M) \quad (6)$$

M – Mahs number = $\frac{V}{a}$, a – speed of sound.

It is worth to notice that equation (6) determines derivation of high speed rotation projectiles, but step size for numerical integration is sufficiently acceptable great.

Sequences calculated values can serve to represent path trajectories as in Fig. 1, and the value of the distance X_C that is achieved for a given projectile and flight conditions.

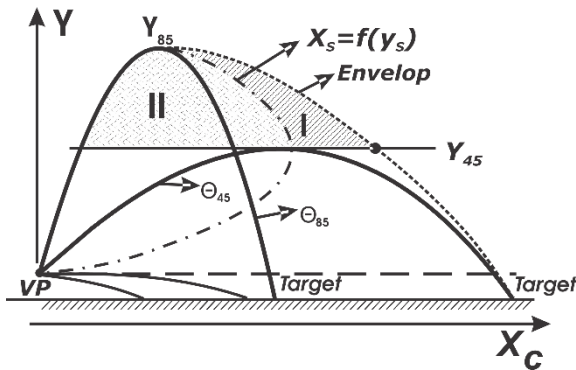


Figure. 1 Projectile path trajectories in plane X-Y

The solutions of the differential equations are obtained for known initial conditions, and one of them is an arbitrary value of the elevation angle θ for which, after calculation, the size of the distance X_C is found, which is definitely not equal to the required distance X_C to impact point - target. Therefore, an algorithm for the solution of the inverse SB problem must be used, but regardless of its type, by intuition it can be concluded that there is no way to solve the set task after only one calculation: for the required distance to the target X_C , determine the required elevation angle θ . In this, as well as in other similar cases, it is necessary to know the initial values θ - predictions that serve as a starting point to achieve at the final solution through correction "if we are going in the right direction". In Fig. 1 are given two path trajectories for extreme angles θ : $-\theta_{45}$ which gives maximum range for lower angle fire –LAF, and $-\theta_{85}$ maximum angle elevation for upper angle fire –UAF. Space occupied with path trajectory branches is at same time space where targets can be positioned.

4. BM DISCRETE MODEL SOLUTION EXTENDING

By solving the direct SB problem, a series of discrete values $\theta_i - X_i$ pairs is obtained, which, when plotted on the graph and connected to each other, have the shape as shown in Fig. 2. The shape of the curve shown in Fig. 2 is in relation on the mutual ratio of the altitudes of the firing position of the weapon Y_{VP} and the target Y_{CH} , and the given corresponds to the case when $Y_{VP} = Y_{CH}$. Having a discrete shape of the curve as in Fig. 2 a suitable prediction of the angle θ_c can be found for any distance to the target X_C in range $0 - X_{45}$, where X_{45} is maximum range

obtained for θ equal to 45 degrees. The value of the found θ prediction should be corrected due to non-standard meteorological - ballistic conditions and due to errors caused by connecting the nodal points with straight lines and others. This correction is done by successive iteration solving direct EB problem.

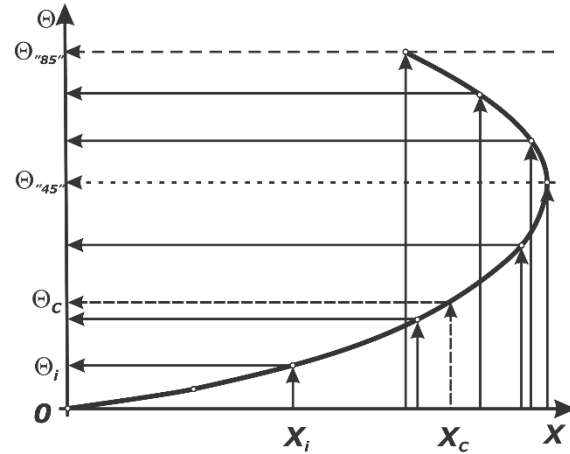


Figure. 2 Discrete form $\theta = f(X)$ and $X = f^{-1}(\theta)$

This is the most common and simplest case, and other forms of approximate polynomials are also possible to connect the nodal points.

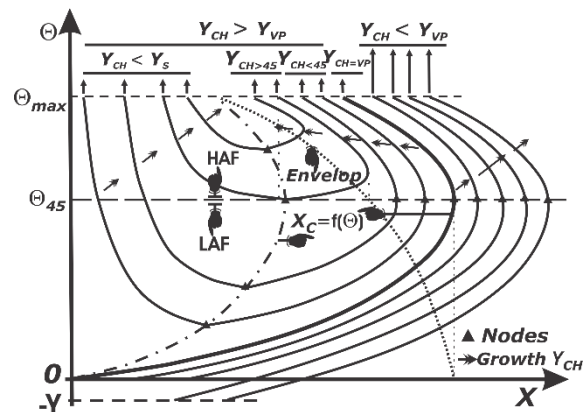


Figure. 3 Graph of discrete points – pairs $\theta - X_C$

In [7], the space in plane $\theta - X$ –Fig. 3 described by the projectile's path trajectories is divided into characteristic areas (domains), and for each of them, the affiliation of $\theta = f(X)$ to the groups of angles – LAF-UAF is indicated, and it is indicated whether an increase in elevation also results in an increase or diminution the range.

As it is shown in Fig. 3, the discrete form $\theta_i = f(X_i)$ or $X = f^{-1}(\theta)$ depends from area to area (domain), and since there are a large number of them, the question arises as to how to form. That practically means, what data should be possessed in order to obtained various discrete forms $\theta_i = f(X_i)$ that characterize certain areas. It was found that the generator (source) of all necessary curved paths lies in special partition trajectory path. Since it is not possible to have analytical form trajectory path that would define it, the starting –ascend and falling –descend trajectory path were discretized with straight segments. The discretization of the trajectory path is effective only if the

discretization points are foreseen in advance. Of the several possibilities, the most economical is the one that, for the selected value of the elevation angle θ_i and the reached height of the vertex (summit) of the path Y_S at point "S", requires knowledge of the distances for the specified values Y_S of the height of the vertex of the path Y_S . The reason is simple, for the selected values θ_i it is necessary to have only the Y_S value and a series of distances obtained at the intersection of the horizontal line at the specified heights. By more trials, it is found [7,8], that best approximation of the starting, ascent trajectory path ought to be at heights: $0 \times Y_S$ (or Y_{GP}); $0.6 \times Y_S$; $0.9 \times Y_S$; Y_S , and on the falling -descent path: Y_S ; $0.9 \times Y_S$; $0.6 \times Y_S$; $0.3 \times Y_S$; $Y_{VP}=Y_{CH}$; $0 \times Y_S$.

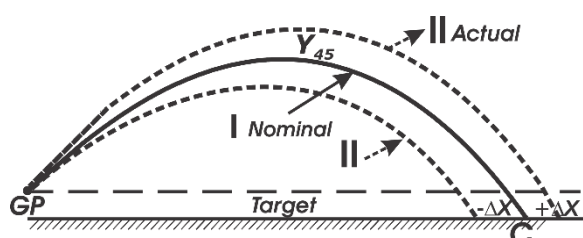


Figure. 4 Actual and nominal trajectory dispositions

After establishing a data generator (source) for construction graphics of type shown in Fig. 2 in the X - Y plane as it is showed in Fig. 1, served to form possible layouts for the observed characteristic areas - Fig. 3 - [7,8,9,11] transformed into the plane θ - X .

Built-in algorithms in the software of the Ballistic Module -**BM** of the Fire Control System -**FCS** based on the interpolation of the nodal points θ_i - X_i proved to be very effective in determining the prediction of the elevation angle θ_i that should be given by the barrel of the artillery GUN to achieve the required distance to the target X_C - [9,11].

There is one step to predict correction $\pm \Delta X$ as it is signed in Fig. 4 for one condition: $\pm \Delta X$ must be calculated earlier then projectile reached impact point "C" at appointed figure. Equation (2) cannot be used for it is not known data for its creation. On the very beginning of creation Ballistic model elements for the ARS prototype (Integral procedure) -[1-6], it is assumed that this data are known. Difficulties for its obtaining are well known for it is user obligated to collect them in battlefield and generally there is not its sources for user disposition.

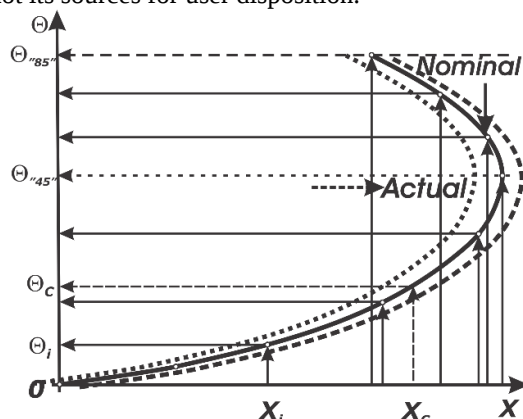


Figure. 5 Nominal and actual form $\theta = f(X)$

There is sufficient software in existing **BM** to perform that job, except data of projectile coordinates from its flight and software for its processing. This software is based on recorded data of actual path trajectory at very beginning -GUN to same ascending trajectory point. It is assumed that recorded data ought to be sent to **BM** of **FCS** for processing with extended and modified present **BM** software on the basis created graph shown in Fig. 5.

5. CONCLUSION

By software for Ballistics module of Fire Control Systems -**FCS** can be calculated GUN elements - elevation -angle in vertical plane and its angle of direction in horizontal plane. Field artillery firing is done indirectly mainly on long distances and with various projectiles (unguided rockets, classical artillery projectiles and mines). Trajectories of this projectiles are various, from flat for direct fire and indirect (lower angle group -**LAF**) to mortar type trajectories (upper angle group -**UAF**). Ballistics module of **FCS** is based on method solution of inverse external ballistics problem for prediction firing elements and its successive corrections. Software for *prediction* artillery correction elements is not developed up to now. This software is based on recording data of actual path trajectory at very beginning GUN -ascending path to same trajectory point. It is assumed that recorded data ought to be sent to Ballistics Module -**BM** of **FCS** for processing with extended and modified present **BM** software. There is serious reason, say forced, for recording part of actual trajectory, not observing impact point of projectile fall, and on its values predict impact point of projectile fall, say only one. Target (impact point of projectile fall) is long away from GUN position and is situated in enemy land! If time of projectile flight is added to previous reason (say, 120 seconds), new situation is come to view - during that time gun will be eliminated from the air with airplane rocket or bomb. This method enables firing two projectiles for acceptable time period, firing second projectile with corrected starting gun elements before first projectile is reached impact point of fall - target. Finally, method for prediction artillery correction elements is based on comparing part of numerical data of actual path trajectory and corresponding data of nominal trajectory.

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