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UNSTEADY INTERNAL BALLISTIC CALCULATION OF ATYPICAL PROCESSES IN ROCKET MOTOR

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Abstract: *In the case of an gyro-stabilized artillery rocket projectile, several atypical processes occur during the operation of a rocket motor with a free-standing propellant charge. A high rotation rate is required for the gyroscopic stabilization of the projectile, and consequently the elements of the rocket motor are exposed to significant centrifugal acceleration. Two effects arise as a consequence of rotation. The first effect is that the high centrifugal load in some cases leads to breakage of the propellant charge, and also to deviation from the predicted change of the burning area of the propellant charge. The second effect is the influence of centrifugal acceleration on the combustion process, and the increase in the burning rate of the propellant. Last effect which is considered is the nozzle erosion, which leads to a change in pressure in the combustion chamber. A non-stationary calculation of the pressure change in the combustion chamber was applied, which can successfully model the UB processes in the RM, which was adapted to successfully include the mentioned atypical processes.*

Keywords: *rocket motor; solid propellant, thrust, internal-ballistic calculation, burning area.*

1. INTRODUCTION

The internal-ballistic (IB) calculation of a solid propellant rocket motor (RM) is reduced to the calculation of the effects of the combustion of the propellant charge in the combustion chamber [1]. The initial parameters that are taken into account are the characteristics of the propellant, materials and construction of the rocket motor itself [2]. In most cases, rocket motors have an approximately stationary or quasi-stationary mode of operation, operating time from a few tenths of a second to several minutes. Transient processes are only present at the beginning and at the end of motor operation [3]. In these cases, calculations with average values (stationary UB calculation) are traditionally applied. Quasi-stationary calculations [3] are also often used.

The importance of IB calculation is highlighted in the stages of designing a rocket motor, but it is also very important in the procedures of overhaul and other modifications of existing RM [4]. Quality IB calculation is also of great importance when identification of unexpected phenomena in RM, observed in experimental work, is required. The most common requirements for the design of missile systems is to predict the mass-functional characteristics of the RM as accurately as possible, because these values are needed to determine the behaviour of the missile. These characteristics are RM total mass, mass of inert elements RM, thrust, operating pressure, dimensions, ignition time, etc [5].

The input data required for the calculation are the physico-chemical characteristics of the propellant, the geometrical characteristics of the rocket motor, the change of the burning area of the propellant grain *Ab(x)* (burn-back analyses), the function of the combustion rate change (the combustion law, e.g. in the form of Saint-Robert's law) [3]:

$$r = b \cdot p_o^n \tag{1}$$

The simplest form of IB RM calculation implies that the combustion surface and motor geometry are constant in time. The IB calculation [6] is reduced to analytical expressions that calculate the mean value of the operating pressure in the chamber, thrust, mass flow and motor operating time.

$$p_o = \left(\frac{\rho_p \cdot A_b \cdot b \cdot c_*}{A_t} \right)^{\frac{1}{1-n}} \tag{2}$$

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The RM ignition and shutdown processes are considered separately, also mostly in analytical terms [3].

A somewhat more complex procedure is the quasi-stationary calculation in cases where there is a multi-phase (staged) or continuously variable operating mode [3]. The quasi-stationary procedure implies a time-discretized successive calculation, which gives time-dependent results, which accuracy depends on the time resolution. With this procedure, it is possible to predict the change in the operating regime of the RM mostly with satisfactory accuracy.

In certain cases, some atypical processes may occur in the RM, which complicate the IB calculation. Effects that are generally not covered by a typical IB calculation are:

– The function of burning rate is most often represented by Saint-Robert's law, which generally gives satisfactory accuracy. Using experimental methods, it is possible to determine the exact burning rate - pressure function, which can significantly deviate from this law (plateau and mesa effects);
– Spin-stabilized rocket projectiles are not frequently used in rocket technology, however there are a certain number of cases. In those conditions, the effect of centrifugal acceleration on the burning rate occurs, which complicates the IB calculation.
– In some cases, centrifugal or axial high intensity inertial force of can lead to the propellant fracture, changing the burning area, which additionally changes UB processes in RM;
– Erosion of the nozzle occurs due to insufficient thermal resistance, it also leads to the complex IB process, which complicates the calculation;
– The burning rate can be changed by erosive combustion, due to the occurrence of increased mass flux of the propellant products and heat transfer on the on the surface of the propellant charge;
– In the case of short work time or impulse RM, transient process becomes more influential in RM operation, because the duration of those processes and the working period are of the same order of magnitude.

All these processes are much more difficult to describe with the mentioned stationary or quasi-stationary UB calculations, so for their modeling the application of unsteady UB calculations is necessary.

2. UNSTEADY INTERNAL BALLISTIC CALCULATION

The basis of the unsteady IB calculation is the determination of the total pressure change in the combustion chamber, which can be calculated from the equation of continuity (Figure 1) [7].

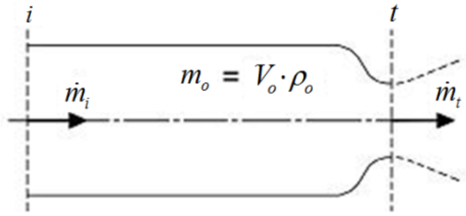

Figure 1. Mass balance of combustion products in the combustion chamber.

In transient flow of the propellant combustion products through the empty space of the rocket motor, the continuity equation has the following form:

$$\dot{m}_i(t) = \frac{d}{dt} m_o(t) + \dot{m}_t(t) \tag{3}$$

The mass flow rate of combustion products at the entrance of the calculation domain is determined by the amount of burned propellant per time:

$$\dot{m}_i(t) = A_b(t) \cdot r(t) \cdot \rho_p \tag{4}$$

The change in the mass of combustion products in the RM chamber free volume is:

$$\frac{d}{dt} m_o(t) = \frac{d}{dt}(V_o(t) \cdot \rho_o(t)) = V_o(t) \cdot \frac{d}{dt} \rho_o(t) + \rho_o(t) \cdot \frac{d}{dt} V_o(t) \tag{5}$$

Using the equation of state of an ideal gas:

$$\rho_o(t) = \frac{p_o(t)}{R \cdot T_o} \tag{6}$$

and the following relations:

$$\frac{d}{dt} V_o(t) = A_b(t) \cdot \frac{dx}{dt} = A_b(t) \cdot r(t)$$
$$V_o(t) = V_o(0) + \int_0^t A_b(t) \cdot r(t) dt \tag{7}$$

expression (5) takes the following form:

$$\dot{m}_o(t) = \frac{A_b(t) \cdot r(t) \cdot p_o(t)}{R \cdot T_o} + \frac{V_o(t)}{R \cdot T_o} \cdot \frac{d}{dt} p_o(t) \tag{8}$$

The mass flow rate of combustion products through the nozzle throat is:

$$\dot{m}_t(t) = \rho_t(t) \cdot A_t \cdot v_t \tag{9}$$

Based on gas dynamics relations for one-dimensional flows, it is possible to express critical values of flow parameters from eq. 8 as a function of total values (which these parameters have in the combustion chamber). These relations are valid for an ideal gas (single-phase and chemically inert), when the flow is isentropic, inviscid and one-dimensional. The mass flow rate through the nozzle throat takes the following form:

$$\dot{m}_t(t) = A_t \cdot \sqrt{\frac{\kappa}{R \cdot T_o} \cdot \left(\frac{2}{\kappa + 1}\right)^{\frac{\kappa+1}{\kappa-1}}} \cdot p_o(t) \tag{10}$$

When expressions 4 – 10 are combined with equation 3, it becomes a first-order differential equation, where combustion chamber total pressure is the dependent and time is the independent variable:

$$\frac{d}{dt}p_o(t)=\frac{R\cdot T_o\cdot A_b(t)\cdot r(t)\cdot \rho_p}{V_o(t)}-A_t\cdot\sqrt{\kappa\cdot R\cdot T_o\cdot\left(\frac{2}{\kappa+1}\right)^{\frac{\kappa+1}{\kappa-1}}}\cdot p_o(t)-\frac{A_b(t)\cdot r(t)\cdot p_o(t)}{V_o(t)} \quad (11)$$

In this equation, the functions $A_b(t)$, $V_o(t)$ and $r(t)$ are not explicitly defined, and they must be determined indirectly. The function $A_b(x)$ is known, which is obtained by burn back analyses. In order to translate it into the time domain, the next substitution is used:

$$A_b\left[x(t)\right]=A_b\left[\int_0^t r(t)\,dt\right] \quad (12)$$

Time change of the free volume of the RM can be calculated based on the change of the burning area:

$$V_o\left[x(t)\right]=V_o(0)+\int_0^x A_b\left[x(t)\right]dt \quad (13)$$

Similarly, the burning rate function $r(p_o)$ is known, which, due to the dependence of pressure on time, also becomes time-dependent:

$$r(t)=r\left[p_o(t)\right] \quad (14)$$

The thrust coefficient can be determined theoretically [1] or experimentally [2]:

$$C_F=\frac{F_{real}}{p_{o\ real}\cdot A_t} \quad (15)$$

And finally the thrust of a rocket motor as a function of time can be determined based on the following equation:

$$F(t)=C_F\cdot A_t\cdot p_o(t) \quad (16)$$

3. GYRO-STABILIZED ROCKET CHARACTERISTICS

The Plamen-D is 128 mm caliber rocket designated as spin stabilized artillery projectile; it is an old design and a mass-produced piece of ordnance. During its production, old technologies were used which resulted in low production costs, average material quality, but high construction reliability. The result of this concept is a cheap, reliable rocket with average performance. The construction of the rocket and the basic components of the rocket motor are given in figure 1.

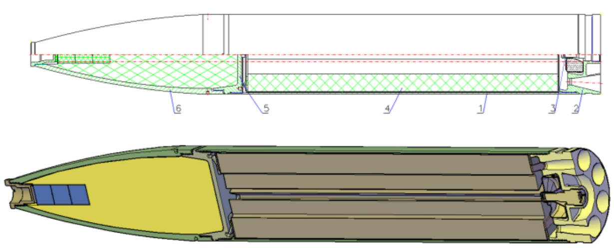

Figure 2. Cross-section of the PLAMEN-D rocket.

1 - Case, 2- Multi-nozzle block, 3 - Igniter,
4 - Free standing grain, 5 - Forward closure, 6 - Warhead.

RM is with a free propellant charge of double base propellant. A star configuration is applied, inhibited on the outside and on both bases, which gives an approximately neutral motor operating regime. To ensure reliable RM assembly manufacturing tolerances resulting in gap between the grain and the chamber of about 0.5 mm.

The nozzle block of RM is made of steel and is realized in the form of a closure with eight nozzles. The nozzles are turned in the tangential direction in relation to the rocket axis, ensuring the circumferential component of the thrust force, i.e. the rotational moment. Thanks to this feature, the rocket achieves a high spin level, around 27.000 min^{-1}, i.e. gyroscopic stabilization.

The mentioned characteristics of RM cause the following atypical processes, which modeling is the subject of this work:

- erosion of the nozzle and change in the operation regime of the RM;
- fracture of the propellant grain due to centrifugal force;
- increase in burning rate due to the effect of centrifugal acceleration.

4. NOZZLE EROSION INFLUENCE ON ROCKET MOTOR OPERATION

The application of a steel nozzle block with multiple beveled nozzles is a practical and economical solution. On the other hand, the performance of this type of nozzle is relatively low, and erosion of the critical section also occurs, because the steel does not have sufficient thermal resistance to the RM combustion products. The operating time of the RM is relatively short, about one second, so a massive nozzle block can successfully perform the function, at the cost of moderate erosion.

RM nozzle erosion is not necessarily a negative phenomenon. Increase of the nozzle throat resulting the pressure decreasing, but thrust can remain constant or even increase. If the erosion is moderate, the nozzle can be designed so that the specific impulse and the thrust coefficient remain approximately constant. However, in most cases, this phenomenon is not reproducible, so there is a greater dispersion of the parameters of the RM operation, which can affect the accuracy of the rocket projectile. This is especially important with projectiles with a nozzle block. If there is a larger difference in the erosion of individual nozzles, an increase in the non-axiality of thrust will occur.

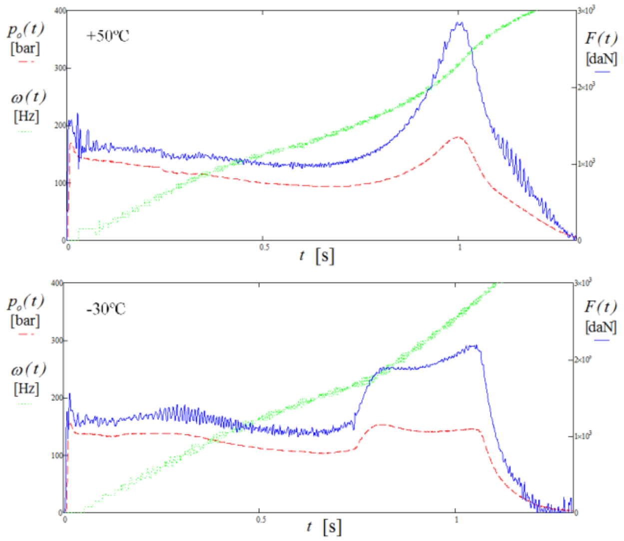

Figure 3. Pressure, thrust and spin - time curves at extreme temperatures of +50ºC and -30ºC.

The process of erosion is very complex, but it can be simulated in a relatively simply by empirical function of time. Typical results of pressure, thrust and spin - time curves measured at extreme temperatures -30ºC and +50ºC are shown in Figure 3.

The initial value of the diameter of the nozzle throat d_t^0 is known before the RM firing, and also the initial value of the critical section area A_t^0.

If the motor operating regime is approximately neutral and there is no nozzle erosion, the thrust coefficient will be approximately constant. The thrust coefficient can be determined at the beginning of the motor operation when throat erosion is low.

$$C_F = \frac{F(0)}{p_o(0)\cdot A_t^0} \qquad (17)$$

Based on this value, the critical section - time function, i.e. the average value for all nozzles of the throat diameter - time function can be calculated:

$$A_t(t) = \frac{F(t)}{p_o(t)\cdot C_F} \qquad d_t(t) = \sqrt{\frac{A_t(t)}{8}\frac{4}{\pi}} \qquad (18)$$

The obtained time dependences can be approximated by linear functions. The following piecewise-linear functions are adopted for rocket motor operation at extreme temperatures:

$$d_t^i(t) = D_i + (D_{i+1} + D_i)\frac{t - T_i}{T_{i+1} - T_i} \qquad (19)$$

Coefficients D and T for this function are given in table 1.

Table 1. Throat erosion equation coefficients

Temperature -30ºC				**Temperature +50ºC**			
T_0	0	D_0	0.01	T_0	0	D_0	0.01
T_1	0.5	D_1	0.011	T_1	0.5	D_1	0.011
T_2	1	D_2	0.012	T_2	1	D_2	0.013
T_3	5	D_3	0.012	T_3	5	D_3	0.013

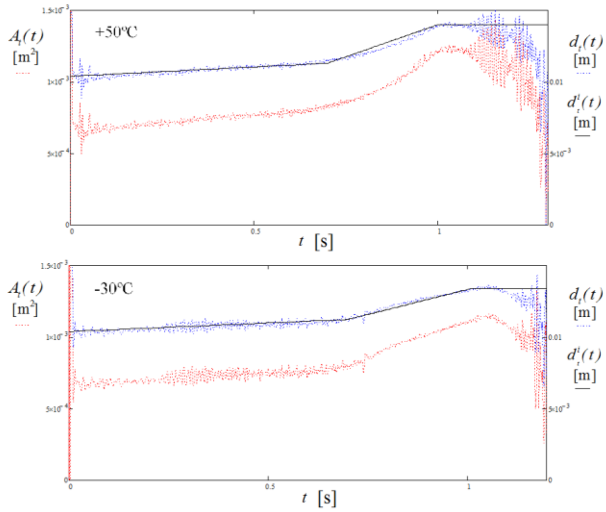

Figure 4. The average diameter and area of the nozzle throat, compared with linear approximation at extreme temperatures +50ºC and -30ºC.

After the RM test, the measured average values of the eroded critical sections approximately correspond to the final values of the calculated functions, 0.012 and 0.013 mm, respectively.

5. PROPELLANT GRAIN BREAKING DUE TO CENTRIFUGAL FORCE

The geometry of the propellant grain is shown in Figure 6. The configuration of the grain is a classic "star" profile. The burnback analysis of this type of grain is well known and can be performed using analytical methods [8].

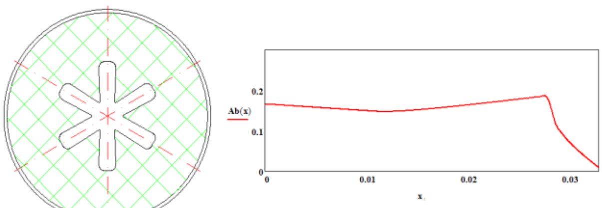

Figure 5. Cross-section of the propellant grain and the burning area curve.

The geometry, working conditions and characteristics of the propellant at -30ºC are such, that in most cases the propellant cracks during the operation of the RM. This phenomenon was confirmed experimentally in a special RM. The test was carried out in static conditions on a special test stand, which enables free rotation simulating the rocket spin in the flight. In all experiments at low temperatures, a stepped pressure jump was observed at the end of RM operation (Figure 3).

In order to confirm the assumption that it is a fracture of the grain, an experiment was performed with special grains. They are pripared in such a way that their shape approximately corresponds to the grain geometry at the moment when the breaking occurs (Figure 6). When are grains was subjected to the appropriate rotation, it was determined that all propellant charges had breaking at the minimum web locations.

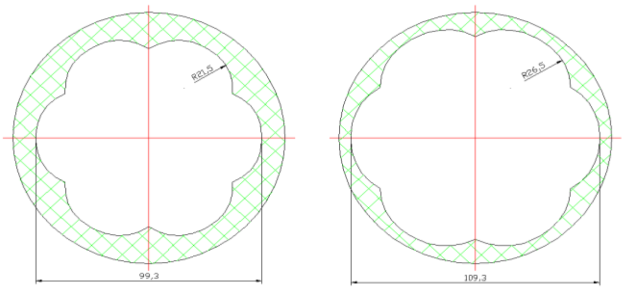

Figure 6. Cross-section of the propellant grain at two kracking moments.

The burn-back analysis including the grain breaking no longer fits the analytical model. In order to perform the calculation, a special numerical program for burn-back analysis SVOD [7] was applied. The calculation of the Burning area change was performed, taking into account the cracking of the PP at different web thickness. The calculation results are given in Figure 7.

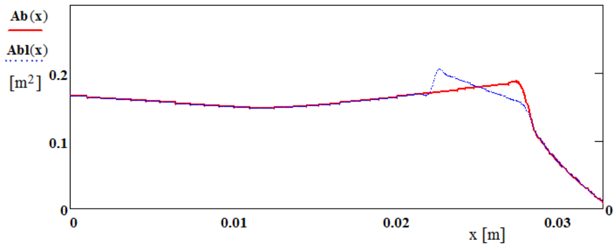

Figure 7. Burning area change without *Ab(x)*, and with grain breaking *Ab1(x)*.

6. THE SPIN INFLUENCE ON THE BURNING RATE

The change in burning rate due to acceleration is a phenomenon in internal ballistics that has been known for a long time. The phenomenon is complex, it depends on a number of factors, such as: propellant type, geometry of the propellant grain, direction of the acceleration vector [9], etc. The applied gain in the periods near the end of the operation has surfaces that are almost perpendicular to the radial direction of the RM (Figure 6), so this effect is very pronounced.

The physicality of this process is reflected in the following phenomena:

1. Rotation reduces the mass flow rate at the nozzle throat, due to the redistribution of the flow field during rotation. This increases chamber pressure and consequently higher burning rate may result;

2. Viscous and boundary layer flow patterns are set up in the motor, increasing heat transfer to the propellant surface through greater mass transfer;

3. The radial acceleration forces can cause greater retention of the solid phase combustion products near the propellant surface.

All those phenomena are mostly complex and require thorough theoretical-experimental work [10]. Only the first phenomenon can be described by known relations from gas dynamics. Semi-empirical and experimental methods [10] are mainly used for modeling these processes.

The burning rate law of the applied propellant was experimentally determined. At extreme temperatures, the burning rate was measured in a certain range of pressure. Due to the application of ballistic modifiers in propellant, the burning rate laws have a plateau effect and irregular shapes, shown in Figure 8.

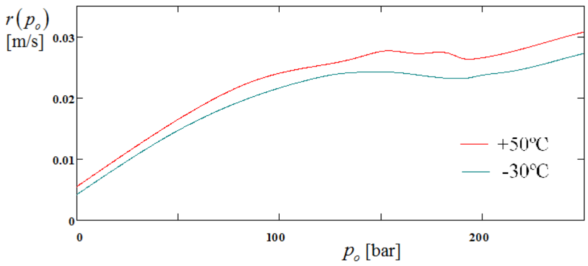

Figure 8. The burning rate curves at extreme temperatures of +50ºC and -30ºC.

The effects of rotation can be modeled using a semi-empirical methods [10]. The increase in the burning rate depends exponentially on the centrifugal acceleration, i.e. on the product of the square of the angular velocity and the radial distance of the burning surface from the axis of rotation:

$$r(p_o, x) = r(p_o)\left[1 + A \cdot e^{B \cdot r_s(x) \cdot \omega(x)^2}\right] \qquad (20)$$

The burning surface radial distance can be approximated by a linear function. The influence of centrifugal acceleration comes to the fore in the second half of the RM operation, when the burning surface is approximately circular in shape (Figure 6):

$$r_s(x) = \begin{Bmatrix} r_s(0) + x, & x \le x_k \\ r_s(x_k), & x \ge x_k \end{Bmatrix} \qquad (21)$$

where x_k is the grain web thickness.

The angular velocity of the projectile is proportional to the thrust, and inversely proportional to the moment of inertia. Measurements show that the angular velocity of the projectile can be approximated by a linear function (Figure 3).

$$\omega(x) = \begin{Bmatrix} \omega_k \cdot \dfrac{x}{x_k}, & x \le x_k \\ \omega_k, & x \ge x_k \end{Bmatrix} \qquad (22)$$

The coefficients A and B in equation (22) can be determined based on the experimental results of the motor static test, until agreement with the calculated pressure curve is achieved.

7. SIMULATION OF RM OPERATION AND EXPERIMENTS COMPARISON

In addition to the described functions of the throat erosion, burn-back analysis and burning rate law, the input data for the unsteady IB calculation are the characteristics of the propellant, combustion products and RM (given in Table 2). Combustion products are obtained by thermo-chemical calculation [11].

Table 2. The RM characteristics

Characteristic	Value
Density [kg/m^3]	1617
Specific heat ratio	1.261
Temperature of combustion [K]	2381
Molar mass [kg/Kmol]	24.152
Nozzle throath diameter [m]	$10.4 \cdot 10^{-3}$
Start rocket motor free space [m^3]	$7.7 \cdot 10^{-4}$

After the calculation, the results were obtained in the form of a pressure - time curve. A comparison with the experimental results shows an extremely good agreement (Figure 9).

Applying the constant value of the thrust coefficient (17) in equation (16), the thrust-time curves are obtained (Figure 10).

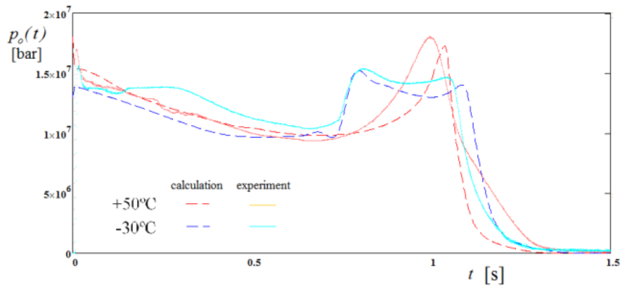

Figure 9. Comparison of calculated and experimental pressure-time curves, at extreme temperatures.

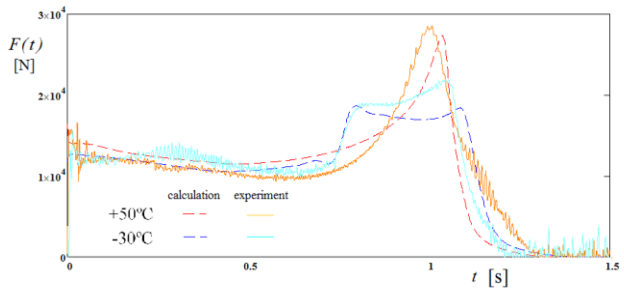

Figure 10. Comparison of calculated and experimental thrust-time curves, at extreme temperatures.

8. CONCLUSION

The described non-stationary internal-ballistic calculation can be successfully used to simulate the operation of a rocket motor with several atypical processes. An example of a spin-stabilized rocket is given, which design results with complex internal ballistic processes.

Nozzle erosion is not a rare phenomenon, but its modeling complicates the calculation. The method of identifying the function of the critical section change in time, based on measurements in a static experiment of a rocket motor is presented.

It was experimentally identified the occurrence of the fracture of the propellant charge due to the centrifugal load. Numerical burning surface analysis of the propellant charge was performed, which at a certain moment creates a multiple grain, when the burning area increase rapidly. Based on this analysis, a function of the burning area was formed depending on the burned web.

The interpolation method was applied to create the burning rate law, as dependence of the burning rate from the pressure in the motor chamber. This function is valid when the motor is running in static, without rotation. This function is extended to add dependency from centrifugal acceleration, using spin correction function. This dependency is based on an approximate linear function of the rocket spin and the burned web, and it is adjusted by two empirical coefficients until is achieve best agreement between calculation and experimental curves.

Nomenclature

A_b burning area
At throat area
b burn rate coefficient
C_F thrust coefficient
c^* caracteristic velocity
d_t nozzle throath diameter
F thrust of a rocket motor
F_{real} experimental measured thrust
$\dot{m}_i$ mass flow rate at inlet
m_o mass of products in the RM chamber
$\dot{m}_t$ mass flow rate through the nozzle throat
n pressure exponent
p_o total pressure at chamber
R gas constant of combustion products
r burning rate
T_o total temperature at chamber
V_o free volume of the RM chamber
v_t velocity at nozzle throat
ρ_o density of combustion products in the chamber
ρ_p density of propellant grain
ρ_t density of combustion products at the nozzle throat
γ specific heat ratio of combustion products

Acknowledgements

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