



IMPROVEMENT OF THE SIGNAL-TO-NOISE RATIO BY USING A STATISTICAL FILTER

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Abstract: *The need to increase the signal-to-noise ratio is a significant task in almost all sensor applications that are exposed to a significant influence of ambient noise. The most common methods used to reduce ambient and/or measurement noise are based on the use of analog and digital filters or wavelet signal decomposition. The paper describes a new approach to signal filtering based on the application of the so-called a statistical filter that was realized by applying statistics of higher order, i.e. the fourth order cumulants. The statistical filter is successfully tested in analysis various types of sensor signals such as: acoustic, hydro acoustic, seismic, and other.*

Keywords: statistical filter, analog filter, digital filter, wavelet, signal analysis.

1. INTRODUCTION

Signals given by different sensors carry information of interest, almost always, they are compromised by noise, the nature of which can be different. It will almost never be possible to avoid the existence of the so-called ambient noise, which is a basic feature of the medium where the useful signal appears. Ambient noise is not the only component of signal interference. Namely, the noise of measuring system itself should be added to the ambient noise. In addition, to the unwanted components of the signal, one should add artificial noise that humans intentionally create in order to make the reception of the useful signal more difficult. This is especially emphasized in military applications.

Higher-order statistics (spectra) have begun to find wide applicability in many diverse fields, e.g.:

- sonar,
- radar,
- plasma physics,
- biomedicine
- seismic data processing,
- image reconstruction,
- harmonic analysis,
- harmonic retrieval,
- time-delay-estimation,
- adaptive filtering,
- array processing,
- blind equalization, and etc.

High-order statistics is one of the most effective tool in statistical pattern recognition approaches. The key basis

for such an attitude is in fact that cumulants can remove the influence of Gauss noise completely. In other words, higher-order statistics are applicable when we are dealing with non-Gaussian, or, in some cases possibly nonlinear processes, see [1,2].

The number of higher order statistics applications gave us an impulse to put into practice something that can be called statistical signal filtering by applying higher – order techniques. In the past few years, we have published several papers on this topic, see literature [3-5].

Until now, the usual procedure for removing the noise of signals, and improving Signal to Noise Ratio (SNR), involved the use of electronic filters. Electronic filters allow some parts of signal to pass, but stop others. Actually, electronic filters allow some signal frequencies applied at their input terminals to pass through their output terminals with little, or no reduction in signal level. In general, it can be said that there are two classes of electronic filters, analog and digital. Analog electronic filters are present in just about in every part of electronic equipment. Analog signal filtering can be based on the use of passive and/or active filters. Design of digital filters include infinite impulse response (IIR), or finite impulse response (FIR). FIR filter design is based on using digital signal processors, and IIR filters are based on analog filter design, see [6,7].

The function of digital FIR filters is realized by passing a digitized signal through a series of discrete delay elements and then multiplying the output of each delay element by a number, or coefficient, see [7].

Based on the above, a signal can be described in the time domain, or in the frequency domain. The time domain is

where an event, such as change of amplitude, is measured over time. The frequency domain is where the amplitude of signal is measured relative to the frequency. In many cases Fourier analysis of the signal is extremely useful because in many practical cases frequency content of the signal is important. By transforming signal into frequency domain the information about time is lost. That is not problem when we are dealing with stationary signals. But in non-stationary case that approach is not so good. In non-stationary case is used another approach which is based on so called short time Fourier analysis to correct these deiciency of classical Fourier approach.

Wavelet analysis makes next step in signal analysis which is based on windowing techniques with variable sized regions. The use of the time intervals of different size enable us to get more precise frequency informations of interest, shorten time intervals for high frequencies, and longer time intervals for low frequencies. In other words, a wavelet transform is able to measure the time-frequency variations of spectral components, but at the same time it has a different time-frequency resolution, see [8].

In this paper is introduced new domain for signal analysis – statistical domain. This domain enable to analyze how statistical properties of the signal change in time, that is, to what extent the statistical distribution of the signal deviates from the Gaussian distribution. This approach is justified by the fact that the events we want to analyze are usually caused by the release of some form of energy in a short time interval.

In this statistical approach, the fourth-order cumulants are used. If a random process is symmetrically distributed, then its third-order cumulant equals zero; hence, for such a process we have to use the fourth-order cumulants. The process, which is distributed with Laplace, Uniform, Gaussian, and Bernoulli – Gaussian distributions are symmetric, as it is known.

2. DEFINITIONS OF THE CUMULANTS

The starting point in defining of the cumulants are the moments of order n , where n is natural number. In a case of a stationary discrete time random process, $X(k)$, where k denotes discrete time, the moments of order n are given by Eq.(1), see [2].

$$m_n^x(\tau_1, \tau_2, \dots, \tau_{n-1}) = E\{X(k)X(k + \tau_1) \dots X(k + \tau_{n-1})\}, \quad (1)$$

where $E\{\cdot\}$ denotes expectation operator, and $(\tau_1, \tau_2, \dots, \tau_{n-1})$ $(n-1)$ cumulant lags. The cumulants of n -th order, c_n^x , up to 4-th order ($n=1,2,3,4$), are given by following equations.

$$c_1^x = m_1^x = E\{X(k)\} \quad (2)$$

$$c_2^x(\tau_1) = m_2^x(\tau_1) - (m_1^x)^2 \quad (3)$$

$$c_3^x(\tau_1, \tau_2) = m_3^x(\tau_1, \tau_2) - (m_1^x)[m_2^x(\tau_1) + m_2^x(\tau_2) + m_2^x(\tau_2 - \tau_1)] + 2(m_1^x)^3$$

$$c_4^x(\tau_1, \tau_2, \tau_3) = m_4^x(\tau_1, \tau_2, \tau_3) - m_2^x(\tau_1)m_2^x(\tau_1)m_2^x(\tau_1 - \tau_2) - m_2^x(\tau_2)m_2^x(\tau_3 - \tau_1) - m_2^x(\tau_3)m_2^x(\tau_2 - \tau_1) - m_1^x[m_3^x(\tau_2 - \tau_1, \tau_3 - \tau_1) + m_3^x(\tau_2, \tau_3) + m_3^x(\tau_2, \tau_4) + m_3^x(\tau_1, \tau_2)] + (m_1^x)^2[m_2^x(\tau_1) + m_2^x(\tau_2) + m_2^x(\tau_3) + m_2^x(\tau_3 - \tau_1) + m_2^x(\tau_3 - \tau_2) + m_1^x(\tau_2 - \tau_1)] - 6(m_1^x)^4 \quad (4)$$

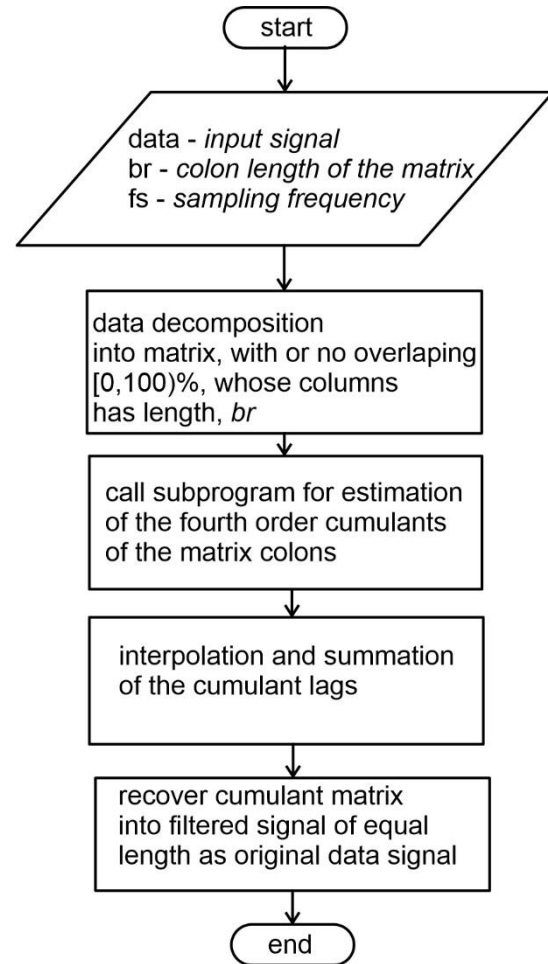


Figure 1. The flow chart of the algorithm of statistical filter, which is, based on the fourth order cumulants.

The analysis of the signal in so-called statistical domain is enabled by using of short-time the fourth-order cumulant estimation of the signal. The flow chart of the algorithm is shown in Fig.1. The approach is very similar to the short-time Fourier signal analysis in time-frequency domain where we track how the frequency content of the signal is changing in time, which is of particular interest in the case of non-stationar signals. The algorithm allows signal segmentation to be performed using rectangular windows of arbitrary duration, with or without overlapping. It should be noted that the application of this technique in signal analysis is a computationally very demanding process. Namely, it is necessary to calculate the fourth-order cumulant, see Eq. 5, of each time interval. Depending on the specific need, the number of time intervals can be relatively large, especially if the technique of overlapping time intervals is used.

The correctness of this concept in signal analysis has been demonstrated in many cases. Using this methodology, signals generated in various fields such as acoustics, hydroacoustics, seismics and others were analyzed. What's more, there are numerous situations when the application of this method is the only possible and effective one. With the introduction of the algorithm whose flow chart is shown in the Fig. 1, a practically new domain for signal analysis has been opened, the statistical domain, which is not behind the time and frequency domains.

3. EXAMLES OF STATISTICAL FILTER APPLICATION

The application of the method of statistical filtering of the signal using the filter described in previous section, which is based on the application of the fourth-order cumulants, has proven to be effective in removing ambient and measurement noise in many cases. If we assume that ambient and measurement noise have the Gaussian distribution, then by applying this method, these noises are completely removed from the signal. The Signal-to-Noise-Ratio (SNR) has become maximal. Such a result of maximizing the SNR is almost impossible to achieve by classic filtering of the signal in the time and frequency domain.

3.1. Examples in acoustics

In the examples that follows, statistical filtering of a very noisy acoustic signals, originated firing from a 122 mm cannon at the test range, are shown, see Figs. 2-4.

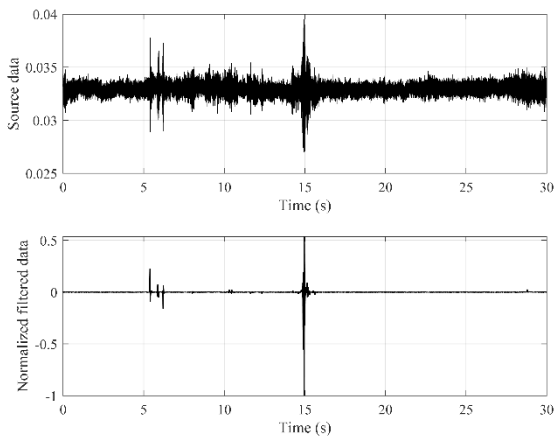


Figure 2. Source signal is acoustic, which originates from firing of a 122 mm cannon at a distance of about 3 km from measuring microphone.

By looking at the diagrams in the Fig.2, it can be concluded that ambient and measurement noise are completely suppressed. The same acoustic signal was also analyzed using the wavelet transform. After detailed checks, it was shown that the Debauchies coefficient of the seventh order, D_7 , most closely describes the recorded acoustic signal, see Fig. 3.

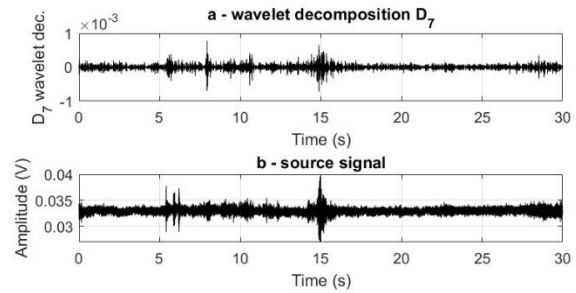


Figure 3. The Debauchies coefficient, D_7 , of the same source signal from Fig. 2.

Based on the analysis of the results shown in Figs. 2 and 3, it can be concluded that the wavelet transformation did not significantly contribute to the noise reduction of initial acoustic signal.

In order to additionally illustrate the potential of the statistical filter in suppressing noise in acoustic signals, the results of noise suppression of a similar acoustic signal, but significantly more noisy than in the previous example is presented in Fig. 4.

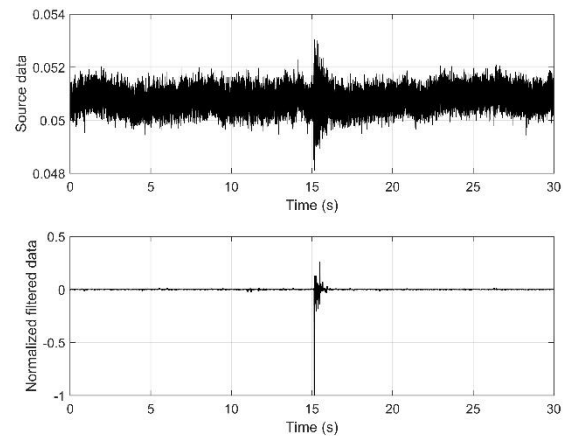


Figure 4. Very noisy acoustic signal produced by firing of a 122 mm cannon at the test range, diagram up, and statistically filtered signal, diagram down.

The diagrams in Fig. 4 perhaps best illustrate the power and potential of this method, bearing in mind that ambient and measurement noise are completely removed.

Finally, in Fig. 5, the source acoustic signal is shown, with good SNR even without additional filtering. But it is undoubtedly that the additional filtering of the signal made the SNR excellent – maximum.

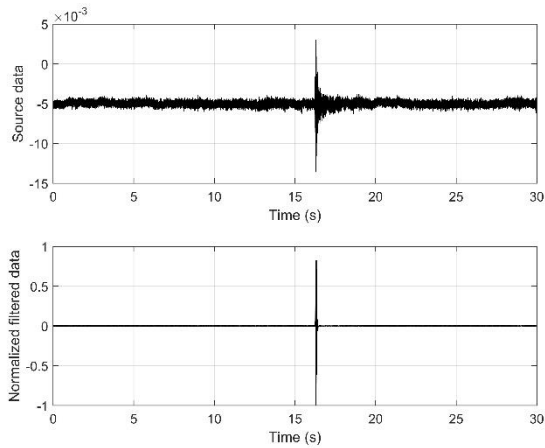


Figure 5. Relatively good acoustic source signal after filtration became excellent with maximum SNR.

The application of the statistical filtering method is confirmed in this case as well. Relatively good SNR of the source signal itself, became excellent, bearing in mind that all noise was completely suppressed, see Fig 5.

All acoustic signals were recorded using the microphones that were placed on the test range at the relatively large distances of several hundred meters from each other. The cannon, source of the acoustic signal, was located several kilometers from the measuring microphones.

3.2. Hydroacoustic data

Hydroacoustic data were downloaded from the internet [9], see Fig. 6. The launch of the torpedo was recorded using a hydrophone. The hydroacoustic signal that was recorded on that occasion was analyzed in two ways, the first, using the described statistical filtering method, and the second, method based on wavelet transformation.

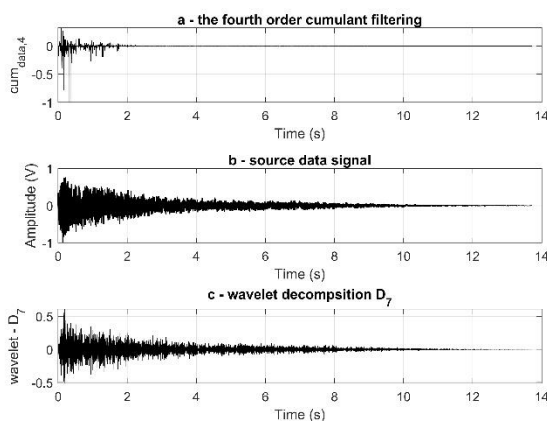


Figure 6. The noise of the torpedo during firing. a – filtered data, b – source signal, and c – wavelet decomposition, D_7 .

By looking at the obtained result, see diagram a, it can be concluded that by applying the method of statistical filtering from the generated hydroacoustic noise, it is possible to significantly reduce the influence of reverberation, which is not possible with the application of wavelet transformation.

Results of the analysis, using a statistical filter, of the radiated hydroacoustic noise of the large ship, see Fig. 7.

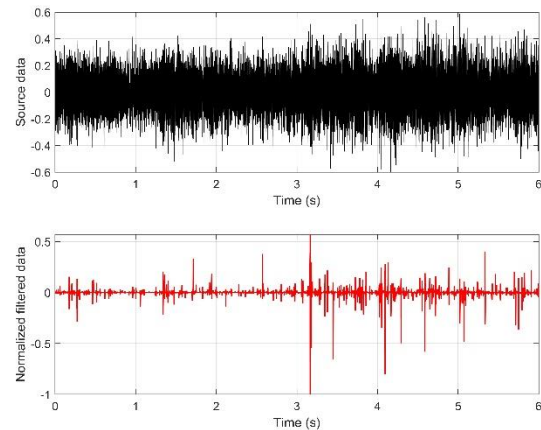


Figure 7. Radiated hydroacoustic noise of the large ship, speed 10 kn, and hydrphone, B&K 8104, was placed at 2 m under sea surface, and filtered data – diagram down.

Based on the conducted analysis, it can be concluded that the ship transfers a significant part of the propulsion energy into the water environment, creating significant underwater noise.

The next example, see the Fig. 8, refers to the movement of much quieter ship, with the same movement parameters: speed, trajectory and position of the measuring hydrophone.

Based on these two examples, it can be concluded that non-linear processes occur in the water environment as the result of the release of propulsion energy, and based on the amplitudes of the filter data, hypotheses can be derived about their size, propulsion system, and/or their class.

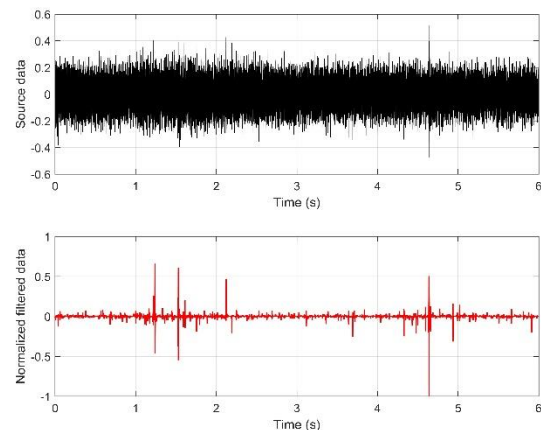


Figure 8. Radiated hydroacoustic noise of the small ship (diagram up), and filtered data (diagram down).

It should be pointed out that the noise recordings were not conducted in open sea conditions and far from human activities, but near the coast where human activities are very pronounced. This can explain the appearance of several very pronounced local maxima, especially in the case of a smaller and quieter ship, see Fig. 8.

3.3. Seismic data

Seismic data, with a lot of measurement noise, were taken by courtesy of the German company Sonicon GbR, Tübingen, in order to test the possibilities of the statistical filter.

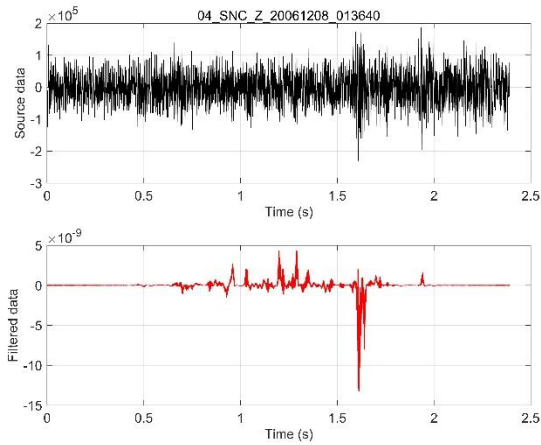


Figure 9. Statistical filtering of the very noisy seismic data, obtained by courtesy of Sonicon GbR, Tübingen. Source data diagram up, and filtered data diagram down.

The measurement noise is completely suppressed, which was confirmed by the representative of this German company.

4. CONCLUSION

The paper gives a detailed description of the working principle of the statistical filter, which is based on the application of the fourth-order cumulants. Its application has enabled the maximization of the SNR in numerous, or in all cases of the analyzed data. Characteristic signals were analyzed in numerous areas, starting with acoustics, hydroacoustics and seismic.

The proposed concept of a statistical filtering, allows introducing the statistical domain as an equal one in addition to the time and frequency domain in signal analysis, taking into account the obtained results.

In addition to the fact that the practical application of the statistical filter is computationally demanding, it can be said that its application is future, bearing in mind the

intensive development of computer technology.

Future work should provide an answer to the question of whether there is, and if so, what is the relationship between frequency and statistical domain.

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