11TH INTERNATIONAL SCIENTIFIC CONFERENCE
ON DEFENSIVE TECHNOLOGIES
OTEH 2024
Serbia, Tara, 9-11 October 2024

OPTIMAL SOURCE LOCALIZATION IN A REAL RADIO CHANNEL BASED ON TDOA APPROACH USING THE HYBRID DIFFERENTIAL EVOLUTION ALGORITHM

MAJA ROSIĆ
Faculty of Mechanical Engineering, University of Belgrade, Belgrade, mrosic@mas.bg.ac.rs

MILOŠ SEDAK
Faculty of Mechanical Engineering, University of Belgrade, Belgrade, msedak@mas.bg.ac.rs

Abstract: *Accurate localization of sources in real radio channels is crucial for various applications ranging from military and civilian domains, notably in security, radar, and sonar systems. This paper presents a novel approach for source localization utilizing the Time Difference of Arrival (TDOA) method in a real radio channel environment. The localization problem is formulated as an optimization task, where the objective is to determine the optimal source location based on TDOA measurements obtained from multiple receivers with known positions, with the objective function derived using the least squares (LS) method. To address the complexity of the optimization problem, a hybrid approach that combines the Differential Evolution (DE) algorithm with conventional Nelder-Mead optimization algorithm has been proposed. The performance of the proposed hybrid algorithm is extensively evaluated and compared with traditional methods using numerical simulations. Results demonstrate the efficacy of the proposed approach in achieving superior localization accuracy in real radio channels, highlighting its potential for practical deployment in diverse applications.*

Keywords: *Localization, Optimization, Least Squares, Time Difference of Arrival, Wireless Sensor Networks.*

1. INTRODUCTION

The problem of determining the unknown location of a source based on Time Difference of Arrival (TDOA) measurements from multiple receivers with known positions is critical in various applications, including military target tracking, environmental monitoring, telecommunications, security systems, and wireless sensor networks. TDOA-based source localization is intensively studied and applied in many fields due to its high ranging accuracy and relatively simple hardware requirements [1]. The core requirement in these applications is to accurately estimate the source location from a set of noisy measurements.

Localization algorithms typically employ various techniques such as Time of Arrival (TOA), TDOA, Received Signal Strength (RSS), or Angle of Arrival (AOA), depending on the available hardware. This paper focuses on source localization using TDOA measurements due to their high ranging accuracy and relatively simple hardware requirements [2].

The source location can be estimated using powerful methods like Least Squares (LS) and Maximum Likelihood (ML), which are effective in practical applications [3]. The TDOA measurement errors necessitate formulating the localization problem as an optimization task, specifically as a least-squares problem. The LS problem involves minimizing the sum of squared errors between the estimated and measured distances. LS estimators are categorized into two types: Linear Least Squares (LLS), which provide closed-form solutions, and Nonlinear Least Squares (NLS). Weighted Least Squares (WLS) is an optimization technique where each data point is assigned a weight based on its variance, minimizing the sum of the weighted squared differences between observed and predicted values to improve estimation accuracy in the presence of heteroscedasticity. In this paper, the NLS estimator is used to accurately determine the source location from noisy TDOA measurements. The complexity of this problem makes the objective function of the NLS estimator highly nonlinear and multimodal [4]. Consequently, conventional optimization algorithms may struggle to find the global optimal solution due to their dependence on the initial solution.

To address this challenge, hybrid approach that combines the Differential Evolution (DE) [5] algorithm with the Nelder-Mead (NM) [6] method has been proposed. The DE algorithm is employed to explore the search space and identify promising regions, while the NM method is used to refine the solutions within these regions, thereby improving the convergence to the global optimal solution. This hybrid approach leverages the strengths of both algorithms: the global search capability of DE and the local optimization efficiency of NM. Hybrid optimization methods have shown superior performance in solving complex global optimization problems, as evidenced by numerous benchmark tests [7].

Corresponding author: M. Rosić;
E-mail: mrosic@mas.bg.ac.rs
DOI: 10.5937/OTEH24065R

The paper is organized as follows: Section 2 presents the source localization problem based on TDOA measurements from multiple receivers with known positions. Section 3 describes the formulation of the localization problem as a least-squares estimation task, using both NLS and WLS approaches. Section 4 introduces the hybrid DE-NM method. Section 5 provides the Cramer-Rao Lower Bound (CRLB) for TDOA measurements, which serves as a benchmark for evaluating localization accuracy. Section 6 presents the simulation results of the proposed hybrid optimization algorithm compared to conventional methods. Finally, Section 7 concludes the paper and suggests directions for future research.

2. PROBLEM FORMULATION

In this section, the two-dimensional (2D) target localization problem using TDOA measurements under the line-of-sight (LOS) environment has been proposed. To determine the unknown position of a target, the considered localization problem requires measurements from at least four receivers with known positions, placed at coordinates $(x_i, y_i), i = 1,2,3,4$, as illustrated in Fig. 1.

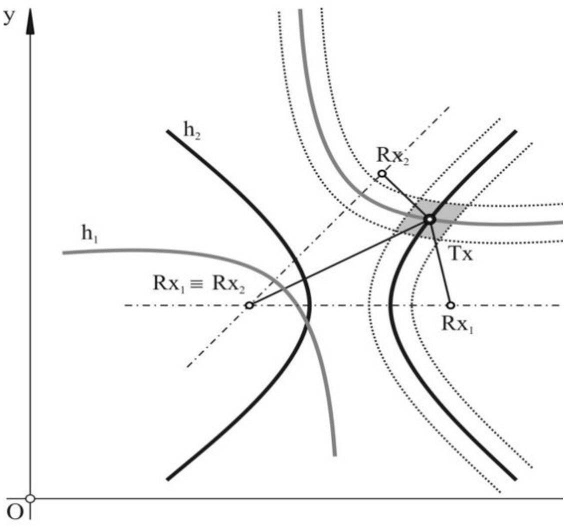

Figure 1. Geometrical model based on TDOA.

Assume that the range difference errors n_i can be modeled as independent Gaussian random variables with zero mean and variance σ^2, i.e., $\Delta d_{ij} \sim \mathrm{N}\,(0, \sigma^2)$. Without loss of generality, the first receiver is set at (x_1, y_1) as the reference receiver.

Using geometrical relationships between the target and the receivers, the target's unknown location (x, y, z) is determined. The unknown distances d_i from the target to the receivers are calculated by multiplying the measured times by the speed of light c. These distances can be calculated using the formula

$$d_i = c \cdot t_i \tag{1}$$

where t_i is the time taken for the signal to travel from the target to the i-th receiver. The distances between the target and the receiver pairs i and j can be expressed as follows

$$d_i - d_j = c \cdot (t_i - t_j) \tag{2}$$

This equation can be rewritten in terms of the coordinates of the target and the receivers

$$\sqrt{(x - x_i)^2 + (y - y_i)^2} - \sqrt{(x - x_j)^2 + (y - y_j)^2} + n_i = \Delta d_{ij} \tag{3}$$

where Δd_{ij} represents the measured range difference between receivers i and j.

Without the noise, the TDOA measurements create hyperbolic equations that relate the target's position to the known receiver positions. For each pair of receivers, the hyperbola h_i (denoted on Figure 1) is defined by the property that the difference in distances from any point on the hyperbola to the two foci (receiver positions) is constant, specifically

$$\sqrt{(x - x_i)^2 + (y - y_i)^2} - \sqrt{(x - x_j)^2 + (y - y_j)^2} = \Delta d_{ij} \,. \tag{4}$$

As shown in Fig. 1, the intersection of multiple 2D hyperbolas provides the geometric model for determining the target's unknown coordinates using TDOA data in the absence of noise.

In practical scenarios, noise is always present, causing the hyperbolas not to intersect at a single point. Therefore, it is necessary to use an appropriate optimization technique to minimize the localization error. The localization problem has been formulated as a NLS problem, aiming to minimize the sum of squared differences between the measured and calculated range differences.

3. LEAST SQUARE METHODS

This section presents the formulation of the LS method for the source localization model using the TDOA measurements described in the previous sections. Generally, the NLS problem formulation precedes the LS problem formulation. The objective function $J_{NLS}(\mathbf{x})$ is defined as the sum of squared residuals between the estimated and measured TDOA values, expressed as

$$J_{NLS}(\mathbf{x}) = \sum_{i=1}^{N} (r_i(\mathbf{x}))^2 \tag{5}$$

where $\mathbf{x}$ denotes the vector of decision variables, and the residual $r_i(\mathbf{x})$ is calculated as

$$r_i(\mathbf{x}) = d_i - \hat{d}_i(\mathbf{x}) \tag{6}$$

Here, d_i represents the measured TDOA values, and $\hat{d}_i(\mathbf{x})$ represents the estimated TDOA values. The minimization problem aims to find the optimal solution $\mathbf{x}^*$ by solving

$$\mathbf{x}^* = \arg\min J_{NLS}(\mathbf{x}) . \tag{7}$$

Since the TDOA problem results in nonlinear equations of hyperbolas, the following steps are necessary to transform these nonlinear equations into a suitable set of linear equations. First, squaring both sides of the hyperbola equations and introduce a new variable, resulting in

$$\begin{aligned}&(x_i - x_1)(x - x_1) + (y_i - y_1)(y - y_1) + r_{i,1}R_1 \\ &= 0.5\left[(x_i - x_1)^2 + (y_i - y_1)^2 - r_{i,1}^2\right] + d_i n_{i,1} + 0.5 n_{i,1}^2, \\ &i \in \{2,3,...,N\},\end{aligned} \tag{8}$$

where

$$R_1 = d_1 = \sqrt{(x - x_1)^2 + (y - y_1)^2}, \tag{9}$$

getting

$$\begin{aligned}&(x_i - x_1)(x - x_1) + (y_i - y_1)(y - y_1) + r_{i,1}R_1 \\ &= 0.5\left[(x_i - x_1)^2 + (y_i - y_1)^2 - r_{i,1}^2\right] + d_i n_{i,1} + 0.5 n_{i,1}^2, \\ &i \in \{2,3,...,N\},\end{aligned} \tag{10}$$

By neglecting second-order terms and linearizing the system of equations, the following is obtained

$$\mathbf{A}\boldsymbol{\theta} = \mathbf{b} + \mathbf{m}, \tag{11}$$

in which

$$\mathbf{A} = \begin{bmatrix} x_2 - x_1 & y_2 - y_1 & r_{2,1} \\ x_3 - x_1 & y_3 - y_1 & r_{3,1} \\ \vdots & \vdots & \vdots \\ x_N - x_1 & y_N - y_1 & r_{N,1} \end{bmatrix}, \tag{12}$$

$$\boldsymbol{\theta} = \begin{bmatrix} x - x_1 & y - y_1 & R_1 \end{bmatrix}^T , \tag{13}$$

$$\mathbf{b} = 0.5\begin{bmatrix} (x_2 - x_1)^2 + (y_2 - y_1)^2 - r_{2,1}^2 \\ (x_3 - x_1)^2 + (y_3 - y_1)^2 - r_{3,1}^2 \\ \vdots \\ (x_N - x_1)^2 + (y_N - y_1)^2 - r_{N,1}^2 \end{bmatrix}, \tag{14}$$

$$\mathbf{m} = \begin{bmatrix} m_{2,1} & m_{3,1} & \cdots & m_{N,1} \end{bmatrix}^T . \tag{15}$$

where $\mathbf{A}$ and $\mathbf{b}$ are derived matrices from the linearized equations. Based on this linear matrix form, the WLS objective function is defined as:

$$J_{WLS}(\boldsymbol{\theta}) = (\mathbf{A}\boldsymbol{\theta} - \mathbf{b})^T \mathbf{W} (\mathbf{A}\boldsymbol{\theta} - \mathbf{b}) \tag{16}$$

where $\mathbf{W}$ is the weighting matrix. The localization problem is then formulated as:

$$\mathbf{x}^* = \arg\min J_{WLS}(\boldsymbol{\theta}) \tag{17}$$

The linear LS problem formulated above provides an algebraic closed-form solution, which for the WLS method is denoted as $\mathbf{x}^*$. This solution minimizes the linearized objective function $J_{WLS}(\boldsymbol{\theta})$. The optimal solution $\mathbf{x}^*$ can be obtained using

$$\mathbf{x}_{WLS}^* = \left(\mathbf{A}^T \mathbf{W} \mathbf{A}\right)^{-1} \mathbf{A}^T \mathbf{W} \mathbf{b} \tag{18}$$

The primary advantages of the WLS method are its simple implementation and computational efficiency. However, it may not achieve sufficient accuracy for highly nonlinear and complex problems.

4. HYBRID DE-NM METHOD

The proposed hybrid DE and NM method is introduced in this section in the context of its application to the problem of locating emitting sources using TDOA measurements. The primary objective of hybridizing different optimization algorithms is to devise the most effective technique for solving the given optimization problem. By hybridizing the algorithms, it is possible to leverage the advantages of each algorithm while mitigating their respective disadvantages.

DE is a robust stochastic global optimization method that explores the search space by generating initial solutions randomly within defined boundary constraints, ultimately finding the global or near-global optimal solution. On the other hand, the NM method is a deterministic local search algorithm that refines the initial solution found by DE to locate the best global optimal solution for the problem at hand.

In this paper, the DE algorithm has been combined with the efficient NM local search method to create a hybrid DE-NM algorithm, enhancing both the efficiency and accuracy of the DE solution. Initially, the DE method explores the search space and identifies promising regions. Subsequently, the NM method is applied to these regions to fine-tune the solutions, thus achieving a more precise localization of the source.

This section provides a detailed overview of the DE and NM methods individually, followed by the description of the hybridization procedure that integrates these two techniques. The hybrid approach not only capitalizes on the global search capabilities of DE but also benefits from the local optimization strength of NM, resulting in a powerful and effective solution for the TDOA-based source localization problem in real radio channels.

4.1 Differential evolution algorithm

The DE algorithm, developed by Storn and Price [6], is a stochastic population-based search method designed for global optimization in continuous search spaces. The DE algorithm is composed of four fundamental steps: initialization, mutation, crossover, and selection, each playing a crucial role in the evolutionary process aimed at finding the global optimal solution of the objective function.

4.1.1 Initialization

The DE algorithm starts with an initial population of N_p individuals within the search space, where each individual represents a potential solution. The i-th individual in the population at generation G is denoted as $\mathbf{x}_{i,G} = [x_{i,1,G}, x_{i,2,G}, ..., x_{i,D,G}]$, where D is the dimensionality of the problem. The j-th component of the i-th individual is initialized randomly within its bounds

$$x_{i,j,0} = x_{j,min} + \text{rand}(0,1)\left(x_{j,max} - x_{j,min}\right) \tag{19}$$

where $rand(0,1)$ is a uniformly distributed random number between 0 and 1.

4.1.2 Mutation

In the mutation step, a new mutant vector $\mathbf{v}_{i,G}$ is generated for each target vector $\mathbf{x}_{i,G}$. This is achieved by adding the weighted difference between two randomly selected vectors to a third vector. Common mutation strategies include
DE/rand/1:

$$\mathbf{v}_{i,G} = \mathbf{x}_{r1,G} + F\left(\mathbf{x}_{r2,G} - \mathbf{x}_{r3,G}\right) \tag{20}$$

DE/rand/2:

$$\mathbf{v}_{i,G} = \mathbf{x}_{r1,G} + F\left(\mathbf{x}_{r2,G} - \mathbf{x}_{r3,G}\right) + F\left(\mathbf{x}_{r4,G} - \mathbf{x}_{r5,G}\right) \tag{21}$$

DE/best/1:

$$\mathbf{v}_{i,G} = \mathbf{x}_{best,G} + F\left(\mathbf{x}_{r1,G} - \mathbf{x}_{r2,G}\right) \tag{22}$$

DE/best/2:

$$\mathbf{v}_{i,G} = \mathbf{x}_{best,G} + F\left(\mathbf{x}_{r1,G} - \mathbf{x}_{r2,G}\right) + F\left(\mathbf{x}_{r3,G} - \mathbf{x}_{r4,G}\right) \tag{23}$$

Here, r_1, r_2, r_3, etc. are distinct integers randomly selected from the set $\{1, 2, ..., N_p\}$ and F is a scaling factor that controls the amplification of the differential variation.

4.1.3 Crossover

The crossover step generates a trial vector $\mathbf{u}_{i,G}$ by mixing the components of the mutant vector $v_{i,G}$ with the target vector $\mathbf{x}_{i,G}$. The binomial crossover method can be expressed as

$$u_{i,j,G} = \begin{cases} v_{i,j,G} & \text{if } \text{rand}(0,1) <= C_r \text{ or } j = j_{rand} \\ x_{i,j,G} & \text{otherwiswe} \end{cases} \tag{24}$$

where C_r is the crossover rate, and j_{rand} is a randomly chosen index to ensure that $\mathbf{u}_{i,G}$ gets at least one component from $\mathbf{v}_{i,G}$.

4.1.4 Selection

Finally, in the selection step, the trial vector $\mathbf{u}_{i,G}$ is compared with the target vector $\mathbf{x}_{i,G}$. For a minimization problem, the vector with the better objective function value is chosen for the next generation:

$$\mathbf{x}_{i,G+1} = \begin{cases} \mathbf{u}_{i,G} & \text{if } f(\mathbf{u}_{i,G}) <= f(\mathbf{x}_{i,G}), \\ \mathbf{x}_{i,G} & \text{otherwise} \end{cases} \tag{25}$$

This process of mutation, crossover, and selection is repeated until a termination criterion, such as the maximum number of generations, is reached.

4.2 Nelder Mead Algorithm

The NM simplex method is a popular local search technique that does not require derivative information of the objective function [7]. It is primarily used for minimizing an objective function in an n-dimensional Euclidean space. The NM method begins with an initial simplex, a polyhedron with $n+1$ vertices. The objective function is evaluated at each vertex, and the vertices are ranked based on their function values, with the best vertex denoted as $\mathbf{x}_{\text{best}}$ and the worst vertex as $\mathbf{x}_{\text{worst}}$.

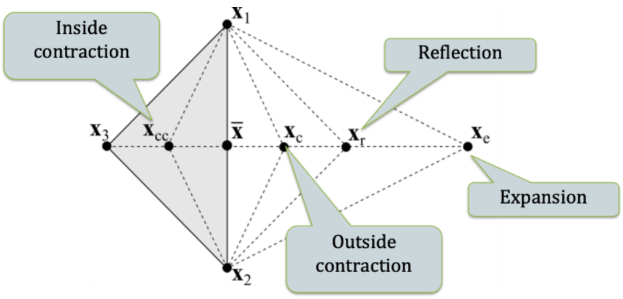

Figure 2. Operations of the NM algorithm

The NM method employs four geometric transformations: reflection, expansion, contraction, and shrinkage. These transformations iteratively improve the simplex's position, moving it closer to the optimal solution.

4.2.1 Steps of the Nelder-Mead Method

Initialization

The initial simplex is generated using the initial vertex $\mathbf{x}_0$ obtained from the DE algorithm. The remaining n vertices $\mathbf{x}_i$ are generated as follows:

$$\mathbf{x}_i = \mathbf{x}_0 + \delta \mathbf{e}_i \quad \text{for } i = 1, 2, \ldots, n \tag{26}$$

where e_i is the unit vector along the i-th coordinate axis, and δ is the initial step size.

Sorting

The vertices of the current simplex are evaluated and ranked based on their objective function values:

$$f(\mathbf{x}_{\text{best}}) \leq f(\mathbf{x}_2) \leq \ldots \leq f(\mathbf{x}_{\text{worst}}) \quad (27)$$

Reflection

A new vertex $\mathbf{x}_r$ is generated by reflecting the worst vertex $\mathbf{x}_{\text{worst}}$ about the centroid $\mathbf{x}_c$ of the best n vertices:

$$\mathbf{x}_r = \mathbf{x}_c + \alpha(\mathbf{x}_c - \mathbf{x}_{\text{worst}}) \quad (28)$$

where α is the reflection coefficient (typically $\alpha = 1$). If $f(\mathbf{x}_r) < f(\mathbf{x}_{\text{best}})$, replace $\mathbf{x}_{\text{worst}}$ with $\mathbf{x}_r$ and terminate the iteration. Otherwise, proceed to expansion.

Expansion

If the reflection improves the solution, the algorithm tries to expand further in the same direction:

$$\mathbf{x}_e = \mathbf{x}_c + \gamma(\mathbf{x}_r - \mathbf{x}_c) \quad (29)$$

where γ is the expansion coefficient (typically $\gamma = 2$). If $f(\mathbf{x}_e) < f(\mathbf{x}_r)$, replace $\mathbf{x}_{\text{worst}}$ with $\mathbf{x}_e$. Otherwise, replace $\mathbf{x}_{\text{worst}}$ with $\mathbf{x}_r$.

Contraction

If the reflection does not yield a better solution, the algorithm contracts the simplex:

Outside Contraction

If $f(\mathbf{x}_r) < f(\mathbf{x}_{\text{worst}})$:

$$\mathbf{x}_c = \mathbf{x}_c + \beta(\mathbf{x}_r - \mathbf{x}_c) \quad (30)$$

where β is the contraction coefficient (typically $\beta = 0.5$). If $f(\mathbf{x}_c) < f(\mathbf{x}_r)$, replace $\mathbf{x}_{\text{worst}}$ with $\mathbf{x}_c$. Otherwise, proceed to shrinkage.

Inside Contraction

If $f(\mathbf{x}_r) \geq f(\mathbf{x}_{\text{worst}})$:

$$\mathbf{x}_c = \mathbf{x}_c + \beta(\mathbf{x}_{\text{worst}} - \mathbf{x}_c) \quad (31)$$

If $f(\mathbf{x}_c) < f(\mathbf{x}_{\text{worst}})$, replace $\mathbf{x}_{\text{worst}}$ with $\mathbf{x}_c$. Otherwise, proceed to shrinkage.

Shrinkage

If contraction fails to improve the solution, the simplex is shrunk towards the best vertex $\mathbf{x}_{\text{best}}$:

$$\mathbf{x}_i = \mathbf{x}_{\text{best}} + \sigma(\mathbf{x}_i - \mathbf{x}_{\text{best}}) \quad (32)$$

where σ is the shrinkage coefficient (typically $\sigma = 0.5$).

These steps are repeated until a termination criterion, such as the maximum number of iterations or a tolerance threshold, is met.

4.3 Proposed hybrid DE-NM method

The proposed hybrid Differential Evolution-Nelder-Mead (DE-NM) method for source localization effectively combines the global optimization strengths of the DE algorithm with the local refinement efficiency of the NM method. This hybrid approach operates in two main phases: the global search phase and the local refinement phase.

Initialization

An initial population of N_P candidate solutions is generated randomly within the search space boundaries. Each candidate solution represents a potential source location.

DE Phase

During the DE phase, iterations are performed to explore the search space by applying Mutation, Crossover and Selection operators. After N_{DE} iterations iterations, the best solution $\mathbf{x}_{DE}$ is selected from the DE population.

NM Phase

The best solution $\mathbf{x}_{DE}$ from the DE phase is used as the initial solution for the NM method. The NM method then refines this solution through geometric transformations (reflection, expansion, contraction, and shrinkage) applied to the simplex formed by the current solution and neighboring points. This phase iteratively improves the solution to achieve higher accuracy.

The refined solution $\mathbf{x}_{NM}$ is compared against a predefined accuracy threshold. If the solution meets the accuracy requirements, the algorithm terminates. If not, a new DE cycle is repeated.

This hybrid approach leverages the ability of DE algorithm to avoid local minima and the efficiency of NM in fine-tuning solutions. By alternating between global and local search phases, the DE-NM method enhances convergence rates and accuracy, making it suitable for precise source localization in noisy environments.

5. CRAMER-RAO LOWER BOUND

The Cramer-Rao Lower Bound (CRLB) is a theoretical lower bound on the variance of unbiased estimators [5]. It is an important concept in the field of statistical estimation theory, providing a benchmark for evaluating the efficiency of an estimator. The CRLB is derived from the inverse of the Fisher Information matrix (FIM), which quantifies the amount of information that an observable random variable carries about an unknown parameter upon which the likelihood depends. The relationship between CRLB and variance is

$$E\left[\left(\hat{\mathbf{x}}-\mathbf{x}\right)\left(\hat{\mathbf{x}}-\mathbf{x}\right)^{T}\right]\geq \mathrm{CRLB}\left(\mathbf{x}\right)=\mathrm{trace}\left\{\mathbf{I}\left(\mathbf{x}\right)^{-1}\right\} \quad (33)$$

where $E[\cdot]$ denoted the expectation operator and $\mathbf{I}(\mathbf{x})$ is FIM given by

$$\mathbf{I}\left(\mathbf{x}\right)=-E\left[\frac{\partial^{2}\ln\left(f\left(\mathbf{r}|\mathbf{x}\right)\right)}{\partial\mathbf{x}\partial\mathbf{x}^{T}}\right], \quad (34)$$

The probability density function $f(\mathbf{r}|\mathbf{x})$ can be defined as

$$f\left(\mathbf{r}|\mathbf{x}\right)=\frac{1}{\left(2\pi\right)^{(N-1)/2}\left|\mathbf{C}\right|^{1/2}} \cdot\exp\left(-\frac{1}{2}\left(\mathbf{r}-\mathbf{d}\left(\mathbf{x}\right)\right)\right)^{T}\mathbf{C}^{-1}\left(-\frac{1}{2}\left(\mathbf{r}-\mathbf{d}\left(\mathbf{x}\right)\right)\right), \quad (35)$$

where $\mathbf{C}$ is covariance matrix is given as

$$\mathbf{C}=\begin{bmatrix}\sigma_1^2+\sigma_2^2 & \sigma_1^2 & \cdots & \sigma_1^2\\ \sigma_1^2 & \sigma_1^2+\sigma_3^2 & & \sigma_1^2\\ \vdots & & \ddots & \vdots\\ \sigma_1^2 & \sigma_1^2 & \cdots & \sigma_1^2+\sigma_N^2\end{bmatrix}. \quad (36)$$

After mathematical operations, the FIM can be obtained as

$$\mathbf{I}\left(\mathbf{x}\right)=\left[\frac{\partial\mathbf{d}\left(\mathbf{x}\right)}{\partial\mathbf{x}}\right]^{T}\cdot\mathbf{C}^{-1}\cdot\left[\frac{\partial\mathbf{d}\left(\mathbf{x}\right)}{\partial\mathbf{x}}\right]. \quad (37)$$

6. SIMULATION RESULTS

This section presents the results of numerical simulations performed to compare the localization performance of the proposed hybrid DE-NM method with the well-known WLS method and the derived CRLB. The simulation considers five receivers positioned at known coordinates: $[150,250]^T$ m, $[1300,200]^T$ m, $[150,1200]^T$ m, $[1410,1400]^T$ m and $[600,800]^T$ m.

For simulation purposes, the true position of the source is assumed to be at $[250,450]^T$. To evaluate and compare the localization performance of the different algorithms, the Root Mean Square Error (RMSE) measure is employed, which is defined as:

$$RMSE=\sqrt{\frac{1}{N}\sum_{n=1}^{N}\left\|\hat{\mathbf{x}}(n)-\mathbf{x}\right\|_2^2}. \quad (38)$$

where $\mathbf{x}$ and $\hat{\mathbf{x}}(n)$ are the true and estimated positions of the source, respectively, and N is the number of Monte Carlo simulation runs.

Firstly, an illustration of the local search process of the proposed novel hybrid ADENM algorithm for the given passive target localization problem is depicted in Fig. 3.

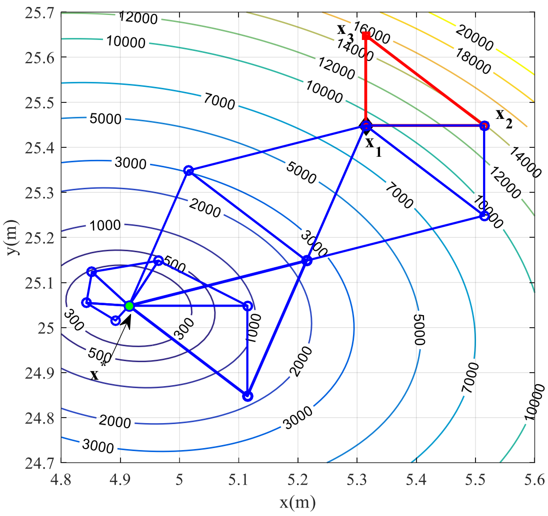

Figure 3. Illustration of the local search process of the proposed novel hybrid DE-NM algorithm

As can be observed from Fig. 3, the initial simplex is marked by a red triangle, in which the initial point $\mathbf{x}_1$ for NM method is obtained as the global best solution found by DE algorithm, and the remaining vertices $\mathbf{x}_2$ and $\mathbf{x}_3$ are then generated. At each NM iteration, the vertices of the current simplex are ordered according to the objective function values. If a new vertex has a smaller objective function value than at least one of the existing vertices, it replaces the worst vertex, and by this way the new simplex is obtained. Thereafter, the new simplex moves through the search space in order to minimize an objective function by applying a series of geometric transformations such as reflection, expansion, contraction and shrinkage. As the evolution process proceeds the size of the simplexes are gradually reduced. This iterative process is repeated until the simplex are close enough to each other. Finally, the coordinates of the global optimum $\mathbf{x}^*$ for a given objective function are found.

Next, the accuracy of the localization algorithms is evaluated based on different levels of TDOA measurement noise. In Figure 4 the RMSE of the DE-NM and WLS methods is plotted as a function of Signal-to-Noise Ratio (SNR), along with the calculated CRLB.

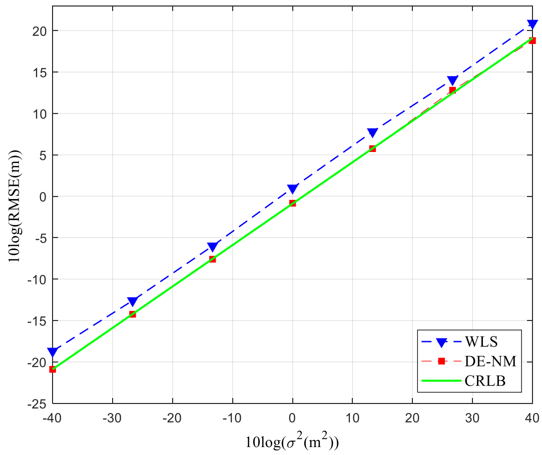

Figure 4. Comparison of RMSE versus SNR levels for different considered algorithms

The provided diagram illustrates a comparative analysis of two methods for source localization the WLS and DE-NM methods. The CRLB, depicted by a green solid line, serves as the benchmark for the best achievable performance. The DE-NM method, consistently aligns closely with the CRLB, indicating superior accuracy. In contrast, the WLS method, deviates more significantly from the CRLB, reflecting lower accuracy. The trends suggest that while both methods follow the general trajectory of the CRLB, the DE-NM method demonstrates a substantial accuracy advantage. This performance is underscored by the DE-NM method's ability to approach the theoretical lower bound more closely than the WLS method. In conclusion, the diagram validates that the proposed hybrid DE-NM method offers a more reliable and precise approach for source localization in noisy environments compared to the traditional WLS method, significantly enhancing localization accuracy.

Next, in with the aim to examine further the localization performance the cumulative distribution functions (CDFs) of the DE-NM and WLS localization methods are compared at SNR=20dB level.

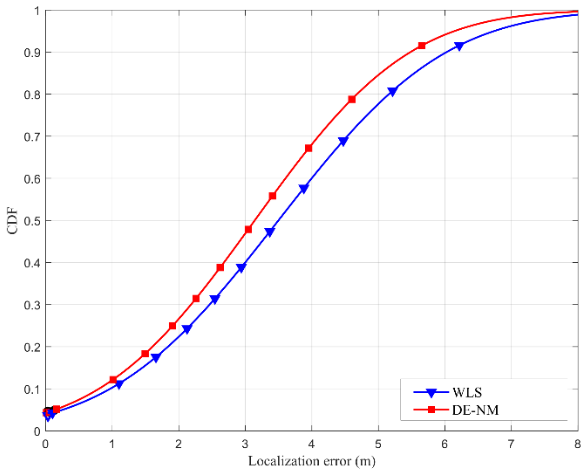

Figure 5. CDFs of the localization error of different localization algorithms for SNR = 20 dB.

The provided CDF diagram compares the localization error performance of the WLS method and the hybrid DE-NM method. The DE-NM method consistently exhibits higher CDF values than the WLS method across the error range, demonstrating that it achieves lower localization errors more frequently. Notably, at a localization error of approximately 2 meters, the DE-NM method achieves a CDF of around 0.4, surpassing the WLS method. This trend continues, with the DE-NM method maintaining a performance lead, especially at lower error thresholds. Overall, the CDF analysis underscores the superior accuracy and reliability of the hybrid DE-NM method in source localization tasks, confirming its robustness and effectiveness compared to the traditional WLS method.

In summary, the numerical simulations validate the effectiveness of the hybrid DE-NM method, showing significant improvements in localization accuracy over traditional methods under various noise conditions.

7. CONCLUSION

In this paper, a novel approach for source localization based on TDOA measurements has been proposed, utilizing a hybrid DE-NM method. The proposed method aims to improve the accuracy and reliability of source localization in real radio channel environments, addressing the inherent challenges posed by noisy measurements and complex optimization landscapes.

The hybrid DE-NM method combines the global search capability of the DE algorithm with the local refinement efficiency of the NM method. The DE algorithm is employed to explore the search space and identify promising regions. This phase ensures that the algorithm avoids local minima and covers a broad search area. Once the DE phase identifies a potentially optimal region, the NM method takes over to fine-tune the solution.

Our simulation results demonstrate the effectiveness of the proposed hybrid DE-NM method compared to traditional localization methods such as WLS. The DE-NM method consistently achieved lower RMSE values across various SNR levels, approaching the theoretical CRLB. This indicates that the DE-NM method not only outperforms the WLS method but also closely approaches the optimal performance achievable for unbiased estimators.

The CDF analysis further supports these findings, showing that the DE-NM method achieves higher cumulative probabilities for lower localization errors. This implies that the DE-NM method is more likely to achieve precise localization, making it a robust and reliable choice for practical applications requiring high accuracy.

In conclusion, the proposed hybrid DE-NM method offers a significant improvement over conventional localization techniques. Its superior accuracy, robustness, and efficiency make it well-suited for real-world applications where precise source localization is crucial. Future work may focus on further enhancing the algorithm's efficiency, exploring adaptive strategies for parameter tuning, and extending the approach to other types of localization problems.

Aknowlegment

The research of M. Rosić was supported by the Serbian Ministry of Education and Science under Grant TR35029. The research of M. Sedak was supported by the Serbian Ministry of Education and Science under Grant No. TR35006.

References

[1] K. Lui, J. Zheng, and H. So, "Particle swarm optimization for time-difference-of-arrival based localization," in Proc. 2007 *15th European Signal Processing Conference*, 2007, pp. 414-417. doi: 10.5281/ZENODO.40289.

[2] B. Xu, G. Sun, R. Yu, and Z. Yang, "High-Accuracy TDOA-Based Localization without Time Synchronization," *IEEE Trans. Parallel Distrib. Syst.*, vol. 24, pp. 1567-1576, 2013. doi: 10.1109/TPDS.2012.248.

[3] Z.-F. Han, C. Leung, H. So, and A. Constantinides, "Augmented Lagrange Programming Neural Network for Localization Using Time-Difference-of-Arrival Measurements," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 29, pp. 3879-3884, 2018. doi: 10.1109/TNNLS.2017.2731325.

[4] B. Huang, L. Xie, and Z. Yang, "TDOA-Based Source Localization With Distance-Dependent Noises," *IEEE Trans. Wireless Commun.*, vol. 14, pp. 468-480, 2015. doi: 10.1109/TWC.2014.2351798.

[5] R. Storn and K. Price, "Differential Evolution – A Simple and Efficient Heuristic for Global Optimization over Continuous Spaces," *J. Glob. Optim.*, vol. 11, no. 4, pp. 341-359, 1997. doi: 10.1023/A:1008202821328.

[6] J. A. Nelder and R. Mead, "A Simplex Method for Function Minimization," *Comput. J.*, vol. 7, no. 4, pp. 308-313, 1965. doi: 10.1093/comjnl/7.4.308.

[7] M. B. Rosić, M. Sedak, M. Simić, and P. Pejovic, "An Improved Chaos Driven Hybrid Differential Evolution and Butterfly Optimization Algorithm for Passive Target Localization Using TDOA Measurements," *Appl. Sci.*, vol. 13, 2023. doi: 10.3390/app13020684.