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THE LEVEL CROSSING RATE OF THE NAKAGAMI-M RANDOM PROCESS

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Abstract: *In this paper, the level crossing rate of the Nakagami-m random process will be determined. To calculate the level crossing rate we will determine the mean value of the first derivative of the Nakagami-m random variable. This mean value is determined using the joint probability density of the Nakagami-m random variable and its first derivative. The level crossing rate of the Nakagami-m random process is one of the statistical characteristics of another type of telecommunication system performance analysis.*

Keywords*: Nakagami-m random variable, first derivative of Nakagami-m random variable, the level crossing rate of the nakagami-m random process*

1. INTRODUCTION

The level crossing rate of the Nakagami-m random process is used in telecommunication system performance analysis, by which we determine the mean failure duration at the output of the wireless system and the joint probability density of the signal envelope at the output of the wireless system and its first derivative. The Nakagami-m distribution is a general distribution, and Rayleigh distributions and one-sided Gaussian distributions can be derived from this distribution as special cases. The Nakagami-m distribution can be used to describe the rapidly changing signal envelope in a linear medium without a dominant multi-cluster component [4,5].

The Nakagami-m random variable has two parameters. The parameter r represents the mean power Nakagami-m random variable. The parameter m is the sharpness of the Nakagami-m fading and has a large change. For small values of the parameter m, the fading is deeper and sharper and the performance of the system is worse. For smaller values of the parameter, the probability of failure and the probability of system error increases. For higher values of the parameter m, the fading depth decreases [8]. In this case, the failure probability decreases, the error probability increases and the channel capacity increases. When the parameter m tends to infinity, the fading disappears. The Rayleigh distribution and the one-sided Gaussian distribution can be obtained from the Nakagami-m distribution. For the parameter m =1 the Nakagami-m distribution changes to the Rayleigh distribution, and for m=0.5 the Nakagami-m distribution changes to the one-sided Gaussian distribution. A squared Nakagami-m random variable can be represented as a sum of τm squared Gaussian random variables. These Gaussian random variables have a mean of zero and a variance of τ^2.

In this paper, the level crossing rate of the Nakagami-m random process will be determined. This mean number is determined as the mean of the first derivative of the Nakagami-m random variable. This mean value is determined using the joint probability density of the Nakagami-m random variable and its first derivative. The first derivative of the Nakagami-m random variable is the Gaussian random variable. Nakagami-m random variable and its first derivative are mutually independent. Based on this, the joint probability density function probability density of the Nakagami-m random variable and the first derivative of the Nakagami-m random variable is equal to the product of the probability density of the Nakagami-m random variable and the probability density of the first derivative of the Nakagami-m random variable. The expression for the level crossing rate Nakagami-m random variables turns into the expression for the level crossing rate Rayleigh random variables for values of the parameter m=1. Using the expression for the probability density of the Nakagami-m random variable, expressions for calculating the moments of the Nakagami-m random variable can be obtained.Nakagami-m random variable can be obtained.

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2. SQUARED NAKAGAMI-M RANDOM VARIABLE

The squared Nakagami-m random variable y can be written as the sum of m squared 2m Gaussian random variables:

$$y^2 = y_1^2 + y_2^2 + \ldots + y_{2m}^2 \qquad (1)$$

Where y_i are Gaussian random variables:

$$P_{y_i}(y_i)\frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{y_i^2}{2\sigma^2}}, \quad i = 1,2\ldots 2m \qquad (2)$$

The first derivative of the squared Nakagami-m random variable y is:

$$\dot{y} = \frac{1}{y}(y_1\dot{y}_1 + y_2\dot{y}_2) \qquad (3)$$

The random variables $\dot{y}_1, \dot{y}_2, \ldots \dot{y}_{2m}$ are Gaussian random variables. Based on this is the linear transformation of Gaussian random variables. The probability density of $\dot{y}$ is Gaussian.

The mean value of the first derivative of the squared Nakagami-m random variable $\dot{y}$ is:

$$\bar{\dot{y}} = \frac{1}{y}(y_1\overline{\dot{y}_1} + y_2\overline{\dot{y}_2} + \cdots + y_{2m}\overline{\dot{y}_{2m}}) = 0 \qquad (4)$$

Since:

$$\overline{\dot{y}_1} = \overline{\dot{y}_2} = \cdots = \overline{\dot{y}_{2m}} = 0 \qquad (5)$$

The variance from the first derivative of the squared Nakagami-m random variable $\dot{y}$ is:

$$\sigma_{\dot{y}}^2 = \frac{1}{y^2}\left(y_1{}^2\sigma\dot{y}_1{}^2 + y_2{}^2\sigma\dot{y}_2{}^2 + \cdots + y_{2m}{}^2\sigma\dot{y}_{2m}{}^2\right) \qquad (6)$$

Where in:

$$\sigma\dot{y}_1{}^2 = \sigma\dot{y}_2{}^2 = \cdots\sigma\dot{y}_{2m}{}^2 = \pi^2 f_m^2\frac{\Omega}{m} = \beta^2 \qquad (7)$$

After substituting (7) into (6) is getting:

$$\sigma_{\dot{y}}^2 = \frac{1}{y^2}\beta^2\left(y_1{}^2 + y_2{}^2 + \cdots + y_{2m}{}^2\right) = \beta^2 \qquad (8)$$

3. JOINT PROBABILITY DENSITY OF THE SQUARED NAKAGAMI-M RANDOM VARIABLE AND THE FIRST DERIVATIVE OF THE SQUARED NAKAGAMI-M RANDOM VARIABLE

The joint probability density of the squared Nakagami-m random variable and the first derivative of the squared Nakagami-m random variable y and $\dot{y}$ is:

$$P_{y\dot{y}}(y\dot{y}) = P_y(y)\,P_{\dot{y}}(\dot{y}) = \frac{2}{\Gamma(m)}\left(\frac{m}{\Omega}\right)^{y^2}\frac{1}{\sqrt{2\pi\beta}}e^{-\frac{y^2}{2\beta^2}} \qquad (9)$$

4. LEVEL CROSSING RATE OF THE SQUARED NAKAGAMI-M RANDOM VARIABLES

The level crossing rate of the squared Nakagami-m random variables y is:

$$N_y = \int_0^{\infty} d_{\dot{y}}\,\dot{y}\,P_{y\dot{y}}(y\dot{y}) = \frac{2}{\Gamma(m)}\left(\frac{m}{\Omega}\right)^m y^{2m-1}\frac{1}{\sqrt{2\pi}}e^{-\frac{m}{\Omega}y^2}\pi f_m\left(\frac{\Omega}{m}\right)^{1/2} \qquad (10)$$

The normalized value of the level crossing rate squared Nakagami-m random variable y is:

$$\frac{N_y}{\sqrt{2\pi f_m}} = \left(\frac{m}{\Omega}\right)^{m-1/2} y^{2m-1} e^{-\frac{m}{\Omega}y^2} \qquad (11)$$

5. NUMERICAL RESULTS

Figure (1) shows the level crossing rate of the Nakagami-m random variable depending on the envelope y for various values of the fading sharpness m and the mean power of the Nakagami-m random variable r. Depending on envelope y, the level crossing rate increases for smaller values of envelope, then the curve reaches a maximum and then the curve decreases. For higher values of the parameters m and r, the maximum of the curve overflows into the area of large values of the parameter y. Mean values of envelope y have the greatest influence on the level crossing rate of the Nakagami-m random process. The shape of the level crossing rate curve does not depend much on the fading sharpness m and the mean power r. The influence of parameters m and r on level crossing rate decreases for higher values of fading sharpness m and mean power r. With an increase in level crossing rate, the quality of transmission decreases, that is, the probability of failure and the probability of error increase, while the channel capacity decreases.

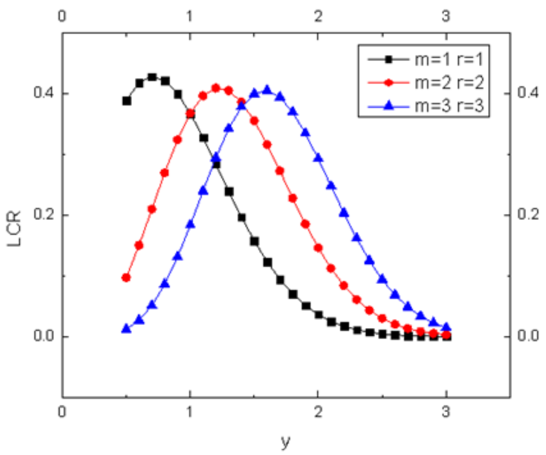

Figure 1. Level crossing rate of Nakagami-m random variable depending on envelope y for various values of fading sharpness m and mean power of Nakagami-m random variable r.

Figure (2) shows the level crossing rate of the Nakagami-m random variable depending on the fading sharpness m for various values of envelope y and mean power r. For parameter values y=1 and r=1, the curve of the level crossing rate increases depending on the parameter m. The slope of this curve is not significant. For smaller values of the parameter m, the influence of the parameter m on the level crossing rate is somewhat greater. For the parameter y=1 and r=1, the level crossing rate curve decreases. The slope of this curve is slightly higher than in the previous case. The level crossing rate curve for y=1 and r=2 decreases with the increase in the sharpness of fading m. The level crossing rate curves for y=1, r=2 and y=2, r=1 intersect. The slope of the curve for y=1 and r=2 is not significant. The curve tends to zero.

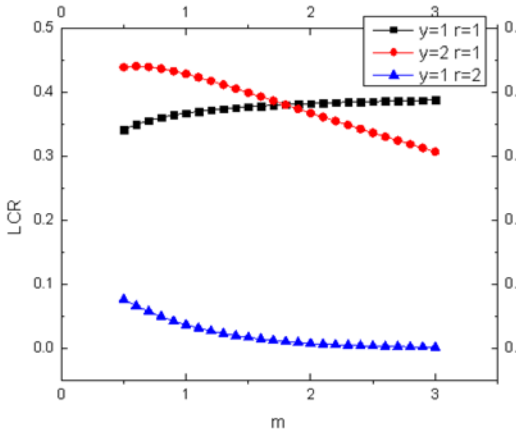

Figure 2. Level crossing rate of Nakagami-m random variable depending on the sharpness of fading m for various values of envelope y and mean power r

Figure (3) shows the curve of the level crossing rate Nakagami-m random variable depending on the mean power for various values of envelope y and sharpness fading m. For parameter values y=1 and m=1, the level crossing rate curve has a slight maximum. The curve rises sharply, reaches a maximum, and then declines slightly. For y=1.5 and m=0.75, the level crossing rate curve increases with the increase in mean power y. The steepest slope of this curve occurs for the mean values of mean power r. The level crossing rate curve y=1.5 and m=1.5 does not differ much from the previous curve.

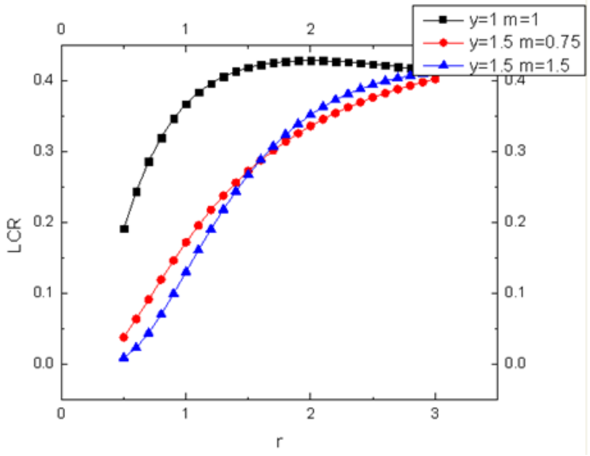

Figure 3. Level crossing rate of the Nakagami-m random variable depending on the average power for various values of envelope y and fading sharpness m

6. CONCLUSION

Fast and slow fading have an impact on system performance and limit channel capacity. It is very important how fading degrades the performance of the system and therefore it is necessary to determine the second-order performance of the telecommunication system such level crossing rate of the envelope signal, the mean duration of the system failure and the joint probability density of the envelope signal and the first derivative of the envelope signal. In this paper, we determined the level crossing rate of the Nakagami-m random process signal, which is applied in the performance analysis of the telecommunications system, by means of which we determine the mean failure duration at the output of the wireless system of the system and the joint probability density of the envelope of the signal at the output of the wireless system and its of the first extract. The results obtained in this way can be used to calculate the average failure time of telecommunication systems in the presence of Nakagami-m fast fading. The mean time of failure of the system can be determined as the quotient of the probability of failure and the level crossing rate. The joint probability density of the signal envelope and the first derivative of the signal envelope is determined by the transformation of the random variables. Using analytical expressions, which are presented in the paper, the system can be simulated, which allows designers to create rational systematic solutions for the desired performance of the system.

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