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Theoretical thermodynamic analysis of the organic Rankine cycle

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Abstract: The present work is a theoretical investigation of the organic Rankine cycle (ORC). A simplified cycle is studied aiming to determine the ORC thermodynamic efficiency in an analytical and simplistic way. Reasonable assumptions have been made and the methodology is based on an analysis of the cycle depiction in the temperature – specific entropy diagram. An analytical solution for the ORC efficiency was developed using the low cycle temperature, the high cycle temperature, the superheating degree, the liquid and vapor-specific heat capacities and the fluid latent heat at the high-temperature level. The reported mean deviation of the suggested analytical model compared to the detailed thermodynamic one was found at 5.03% which is an acceptable value. Moreover, analytical approximations for the efficiency with regression models were created for three different working fluids names n-pentane, toluene and R600. The present approach can be extended to extra working fluids and operating conditions and the present work consists of the first step for the establishment of this methodology.

Keywords: Theoretical cycle, Thermodynamic cycle, Organic Rankine Cycle, Thermal efficiency, Power cycle

1. Introduction

The organic Rankine cycle (ORC) is an established technology for the efficient exploitation of low-grade energy sources like solar [1], geothermal [2] biomass and waste heat [3]. They can also be used for complex systems such as thermal Carnot batteries [4] and trigeneration units [5]. The modeling of ORC is based on the development of analytical thermodynamic models based on energy and mass balances that use thermodynamic properties from libraries and tools [6] [7], something that is reliable, but it creates difficulties when the ORC has to be studied further. Specifically, in dynamic simulation and optimization procedures, there is a need to develop simpler models for the ORC analysis that present simplicity, and they lead to reduced complexity in the simulation and generally reduced computational time.

In the literature, there are some studies that have examined the ORC in detail aiming to develop analytical approaches for its performance. Li [8] and Li et al. [9] performed two studies about the trapezoidal approach for estimating the performance of the ORC. This idea was further expanded by Wang et al. [10] who separated the ORC into three smaller cycles; trilateral, Carnot and Brayton cycles. The same research team performed another work with the trapezoidal design [11] and both studies ([10] [11]) emphasized the working fluid investigation. The separation of the ORC into three cycles was also studied by Scangolatto et al. [12] and they gave emphasis on the detailed modeling of the subcomponents. Moreover, another literature study worked with the entropy generation principle for analyzing the ORC [13].

In this direction, the investigation suggests a new idea regarding ORC modeling by using the proper assumptions. This idea has been based on the previous literature review, and mainly in Ref. [10], but it has some different points aiming to increase the accuracy and to be presented as a simpler one. Therefore, this work has added value to the existing literature. Moreover, the present work shows how the suggested efficiency equation can be used for the ORC analysis and some regression equations are developed to simplify further the ORC performance.

2. Material and methods

2.1 The suggested theoretical cycle modeling

2.1.1 Cycle description

The organic Rankine cycle is modeled in a simplistic way as depicted in Figure 1. The heat input is separated into three parts (1→2), (2→3) and (3→4). The heat input of (1→2) is the preheating, which is an approximately isobaric process, as well as the (3→4) which is the superheating. The evaporation (2→3) is performed under the constant high-temperature level (TH). The heat rejection is conducted in the process of (5→6), (6→7) and (7→1). The temperature is not constant in the process (5→6), while in the others is constant. The work production is the area instant the space 1-2-3-4-5-6-7-1.

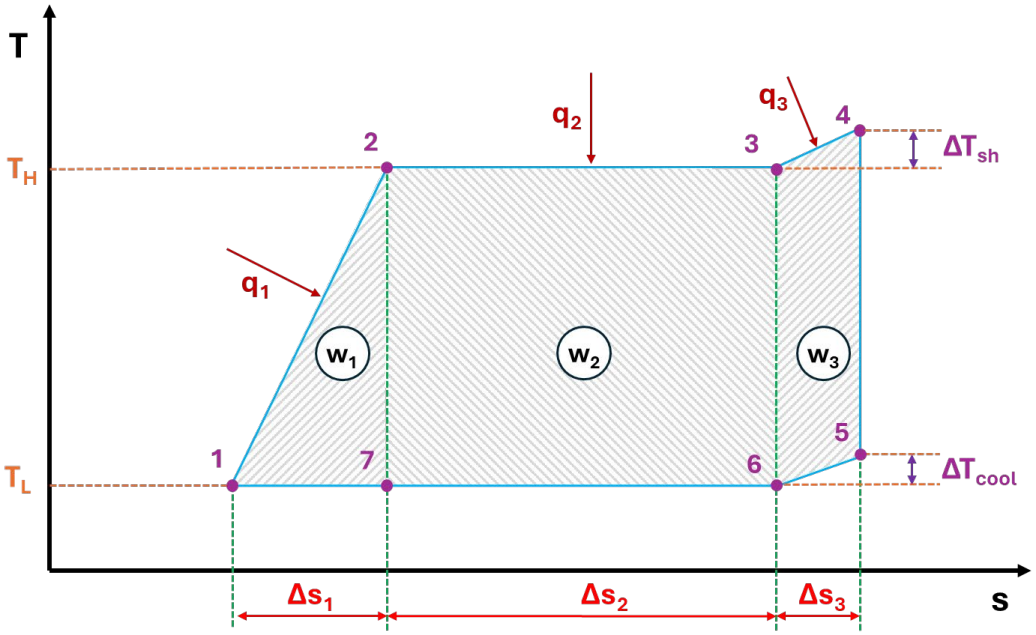

Figure 1. Thermodynamic cycle depiction in temperature (T) – specific entropy (s) diagram

2.1.2 Suggested cycle modeling

The following equations present the mathematical modeling for simulating the ORC cycle efficiency in an analytical way.

The specific work production (w) is calculated as below:

$$w = w_1 + w_2 + w_3 \tag{1}$$

The specific work (w_1) of the space (1-2-7-1) is calculated as:

$$w_1 = \frac{1}{2} \cdot \Delta s_1 \cdot (T_H - T_L) \tag{2}$$

The specific work (w_2) of the space (2-3-6-7-2) is calculated as:

$$w_2 = \Delta s_2 \cdot (T_H - T_L) \tag{3}$$

The specific work (w_3) of the space (3-4-5-6-3) is calculated as:

$$w_3 = \Delta s_3 \cdot (T_H - T_L) + \frac{1}{2} \cdot \Delta s_3 \cdot \Delta T_{sh} - \frac{1}{2} \cdot \Delta s_3 \cdot \Delta T_{cool} \tag{4}$$

The specific heat input (q) is calculated as below:

$$q = q_1 + q_2 + q_3 \tag{5}$$

The specific heat input in the process (1→2) is calculated as:

$$q_1 = \int_{s_1}^{s_2} T(s) \cdot ds = \int_{s_1}^{s_2} \left[T_L + \frac{s - s_1}{s_2 - s_1} \cdot (T_H - T_L) \right] \cdot ds = \frac{1}{2} \cdot \Delta s_1 \cdot (T_L + T_H) \tag{6}$$

The specific heat input in the process (2→3) can be calculated as:

$$q_2 = \Delta s_2 \cdot T_H \tag{7}$$

The specific heat input in the process (3→4) can be calculated as:

$$q_3 = \int_{s_3}^{s_4} T(s) \cdot ds = \int_{s_3}^{s_4} \left[T_H + \frac{s - s_3}{s_4 - s_3} \cdot \Delta T_{sh}\right] \cdot ds = \frac{1}{2} \cdot \Delta s_3 \cdot \left(T_H + \frac{\Delta T_{sh}}{2}\right) \tag{8}$$

The specific entropy generation in the process (1→2) is calculated as below:

$$\Delta s_1 = c_{p,l} \cdot \ln\left(\frac{T_H}{T_L}\right) \tag{9}$$

Practically, this process is approximately the sum of an isentropic compression and of an isobaric liquid heat input. Thus, the previous formula is a very good approximation.

The specific entropy generation in the process (2→3) is calculated as below:

$$\Delta s_1 = \frac{r_H}{T_H} \tag{10}$$

The latent heat of the high-temperature $[r_H = r(T_H)]$ is used in the previous formula.

The specific entropy generation in the process (3→4) is calculated as below:

$$\Delta s_3 = c_{p,v} \cdot \ln\left(\frac{T_H + \Delta T_{sh}}{T_H}\right) = c_{p,v} \cdot \ln\left(1 + \frac{\Delta T_{sh}}{T_H}\right) \approx c_{p,v} \cdot \frac{\Delta T_{sh}}{T_H} \tag{11}$$

Practically, this process is approximately an isobaric gas heat input. Thus, the previous formula is a very good approximation. Moreover, the ratio $\left(\frac{\Delta T_{sh}}{T_H}\right)$ is relatively low, thus the previous approximation can be done (with Taylor series utilization).

The mean specific heat capacity of the liquid $[c_{p,l} = c_{p,l}(T_m)]$ and mean specific heat capacity of the vapor $[c_{p,v} = c_{p,v}(T_m)]$ can be estimated in the mean temperature:

$$T_m = \frac{T_L + T_H}{2} \tag{12}$$

Regarding the temperature difference between the state points (5) and (6), this has to be estimated by using the superheating degree, the low and the high-temperatures. The specific entropy variation in the process (3→4) and (5→6) are the same as absolute value. Assuming that these processes are approximately isobaric, it can be said:

$$\Delta s_3 = c_{p,v} \cdot \ln\left(1 + \frac{\Delta T_{sh}}{T_H}\right) = c_{p,v} \cdot \ln\left(1 + \frac{\Delta T_{cool}}{T_L}\right) \tag{13}$$

Therefore, by assuming the same specific heat capacities, it is concluded:

$$\Delta T_{cool} = \frac{T_L}{T_H} \cdot \Delta T_{sh} \tag{14}$$

The net specific work production can be written as below, after combining the reported equations:

$$w = \left[\frac{1}{2} \cdot c_{p,l} \cdot T_H \cdot \ln\left[\frac{T_H}{T_L}\right] + r_H + c_{p,v} \cdot \Delta T_{sh} + \frac{1}{2} \cdot c_{p,v} \cdot T_H \cdot \left(\frac{\Delta T_{sh}}{T_H}\right)^2\right] \cdot \left[1 - \frac{T_L}{TH}\right] \tag{15}$$

The specific heat input can be written as below, after combining the reported equations:

$$q = c_{p,l} \cdot \left(\frac{T_L + T_H}{2}\right) \cdot \ln\left[\frac{T_H}{T_L}\right] + r_H + c_{p,v} \cdot \Delta T_{sh} + \frac{1}{2} \cdot c_{p,v} \cdot T_H \cdot \left(\frac{\Delta T_{sh}}{T_H}\right)^2 \tag{16}$$

The theoretical ORC efficiency ($\eta_{model,ORC}$) can be written as:

$$\eta_{th,ORC} = \frac{\frac{1}{2} \cdot c_{p,l} \cdot T_H \cdot \ln\left[\frac{T_H}{T_L}\right] + r_H + c_{p,v} \cdot \Delta T_{sh} + \frac{1}{2} \cdot c_{p,v} \cdot T_H \cdot \left(\frac{\Delta T_{sh}}{T_H}\right)^2}{c_{p,l} \cdot \left(\frac{T_L + T_H}{2}\right) \cdot \ln\left[\frac{T_H}{T_L}\right] + r_H + c_{p,v} \cdot \Delta T_{sh} + \frac{1}{2} \cdot c_{p,v} \cdot T_H \cdot \left(\frac{\Delta T_{sh}}{T_H}\right)^2} \cdot \left[1 - \frac{T_L}{TH}\right] \tag{17}$$

The thermal efficiency of the "similar" Carnot cycle is given as below, assuming as high temperature the (T_H) and avoiding the use of superheating which is generally low.

$$\eta_{carnot} = 1 - \frac{T_L}{T_H} \tag{18}$$

Therefore, it can be written:

$$\eta_{model,ORC} = \frac{\frac{1}{2}\cdot c_{p,l}\cdot T_H \cdot \ln\left[\frac{T_H}{T_L}\right] + r_H + c_{p,v}\cdot \Delta T_{sh} + \frac{1}{2}\cdot c_{p,v}\cdot T_H \cdot \left(\frac{\Delta T_{sh}}{T_H}\right)^2}{c_{p,l}\cdot\left(\frac{T_L+T_H}{2}\right)\cdot \ln\left[\frac{T_H}{T_L}\right] + r_H + c_{p,v}\cdot \Delta T_{sh} + \frac{1}{2}\cdot c_{p,v}\cdot T_H \cdot \left(\frac{\Delta T_{sh}}{T_H}\right)^2} \cdot \eta_{carnot} \tag{19}$$

or

$$\eta_{th,ORC} = f \cdot \eta_{carnot} \tag{20}$$

The factor (f) shows how the theoretical ORC is close to the respective Carnot cycle, and it is defined as:

$$f = \frac{\frac{1}{2}\cdot c_{p,l}\cdot T_H \cdot \ln\left[\frac{T_H}{T_L}\right] + r_H + c_{p,v}\cdot \Delta T_{sh} + \frac{1}{2}\cdot c_{p,v}\cdot T_H \cdot \left(\frac{\Delta T_{sh}}{T_H}\right)^2}{c_{p,l}\cdot\left(\frac{T_L+T_H}{2}\right)\cdot \ln\left[\frac{T_H}{T_L}\right] + r_H + c_{p,v}\cdot \Delta T_{sh} + \frac{1}{2}\cdot c_{p,v}\cdot T_H \cdot \left(\frac{\Delta T_{sh}}{T_H}\right)^2} \tag{21}$$

2.1.3 Classical thermodynamic modeling

This subsection includes the main expressions for the typical thermodynamic modeling for the ORC. This modeling is used in the present work to compare the suggested modeling of section 2.1.2 with the typical one. The ORC depiction is given in Figure 2, and it is the simplest possible illustration of this power cycle.

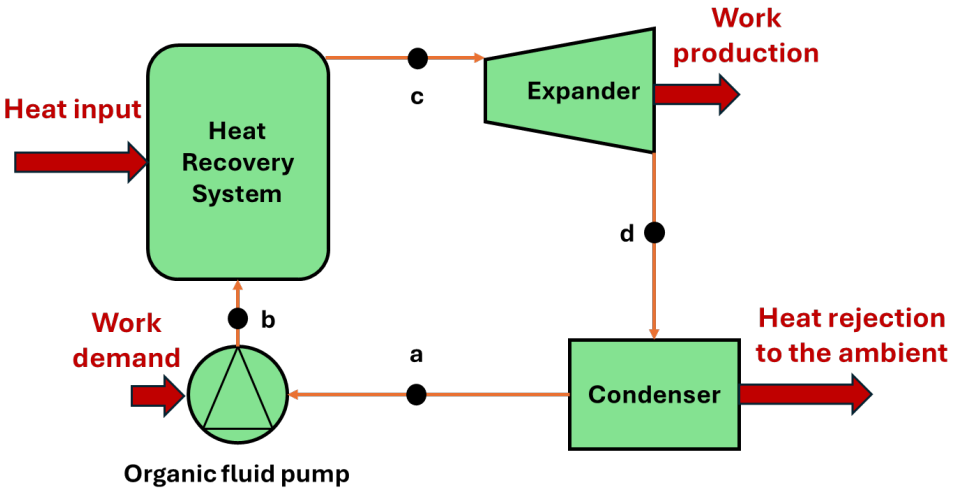

Figure 2. Depiction of the thermodynamic ORC configuration

The specific heat input in the unit is calculated as:

$$q_{in} = h_c - h_b \tag{22}$$

The heat rejection to the environment is given as:

$$q_{out} = h_d - h_a \tag{23}$$

The work production in the expander devise is given as:

$$w_{exp} = h_c - h_d \tag{24}$$

The work demand in the organic fluid pump is given as:

$$w_{pump} = h_b - h_a \tag{25}$$

The net work production of the cycle is given as:

$$w_{net} = w_{exp} - w_{pump} \tag{26}$$

The ORC thermal efficiency is given as:

$$\eta_{real,ORC} = \frac{w_{net}}{q_{in}} \quad (27)$$

The previous formula regards the ORC efficiency that is calculated with the traditional thermodynamic models.

2.2 Followed methodology

In this work, the analytical approach of section 2.1.2 is compared with the analytical thermodynamic approach of section 2.1.3. The main analysis is conducted with n-pentane as a working fluid, while extra analysis is conducted for toluene and R600. Different combinations of low temperature, high temperature and superheating degree are investigated. All the examined scenarios concern subcritical operation. The thermodynamic properties are retrieved by the libraries of Engineering Equation Solver [14]; a tool that is used for the thermodynamic analysis with the modeling of section 2.1.3.

3. Results and discussion

Figure 3 depicts the thermodynamic efficiency of the ORC with the analytical model and with the thermodynamic analysis which is called real efficiency with n-pentane as working fluid. Also, the respective Carnot cycle efficiency is depicted in this figure. The deviation between the model and the real value is generally low with a mean value around 5.03%. Therefore, it is clear that the suggested analytical model is accurate enough. Also, the deviation is generally constant for the studied high temperatures the fact which shows good behavior of the model under different operating conditions.

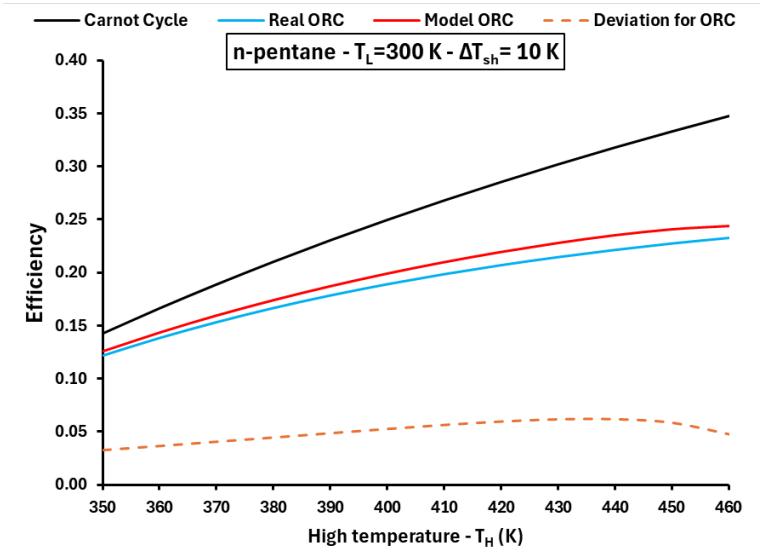

Figure 3. Efficiency cycle for n-pentane with low cycle temperature at 300 K, 10 K superheating degree and variable high cycle temperatures

The next stage is the presentation of an example for the application of the present methodology. Figures 4 and 5 show the variation of the (f) factor for different operating conditions with n-pentane as the working medium. This factor is the ratio of the ORC efficiency to the respective Carnot cycle efficiency. In Figure 4, the (f) is given for low temperature at 300 K, while in Figure 5 for superheating at 10 K. It can be concluded from these figures that the rise of the high temperature decreases the value of (f). Moreover, the increase of the superheating degree in the expander inlet leads to higher values for the ratio (f). The rise of the lower temperature increases the values of (f) in low high temperatures, but after T_H=445 K, the reverse result is obtained. Generally, in low high temperatures, the (f) is close to 90%, while in higher high temperatures is closer to 70%. This interesting result indicates that the ORC is the most appropriate cycle in applications with relatively low-temperature heat sources.

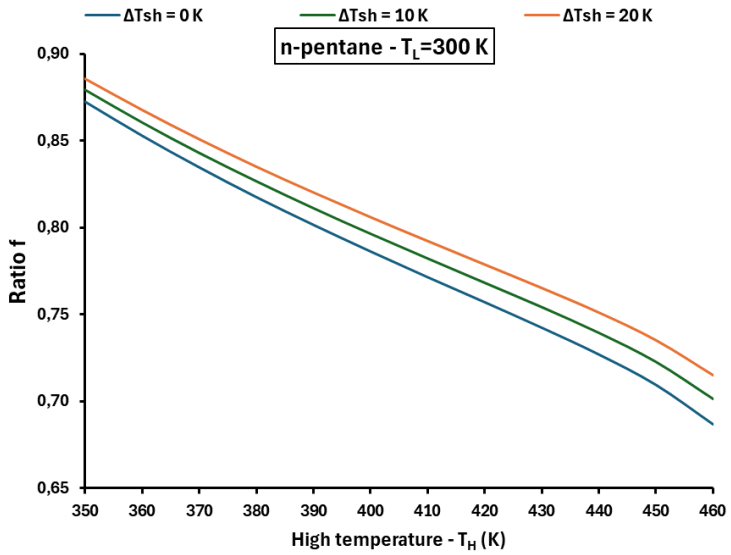

Figure 4. Ratio (f) for n-pentane with low cycle temperature at 300 K, 0-10-20 K superheating degrees and variable high cycle temperatures

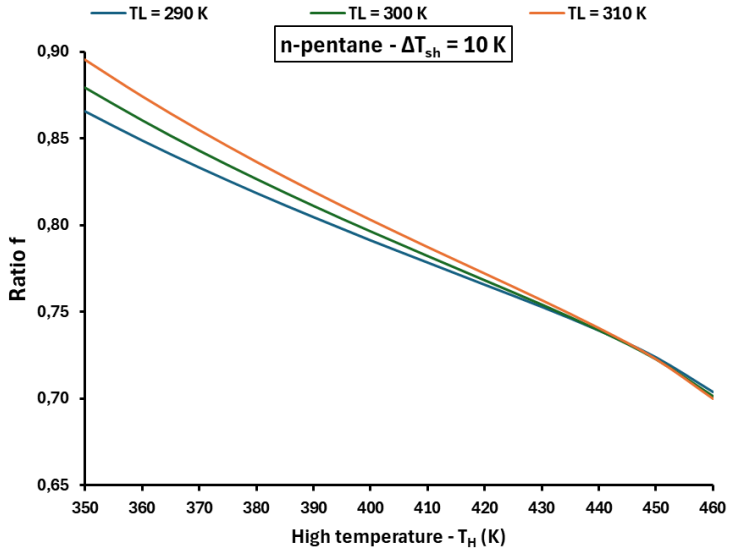

Figure 5. Ratio (f) for n-pentane with low cycle temperatures at 290-300-310 K, 10 K superheating degree and variable high cycle temperatures

Moreover, the development of some linear regression equations regarding the factor (f) is something very interesting. The following approximation equations were developed for three working fluids.

A) n-pentane (0.50% mean deviation and R^2=99.09%)

$$f = 1.2401 - 0.001554 \cdot T_H + 0.000566 \cdot T_L + 0.001012 \cdot \Delta T_{sh} \quad (28)$$

Applicability for: $350\ K \le T_H \le 460\ K$, $290\ K \le T_L \le 310\ K$, $0\ K \le \Delta T_{sh} \le 20\ K$

B) Toluene (0.40% mean deviation and R^2=98.60%)

$$f = 1.0821 - 0.001024 \cdot T_H + 0.000605 \cdot T_L + 0.000496 \cdot \Delta T_{sh} \quad (29)$$

Applicability for: $350\ K \le T_H \le 460\ K$, $290\ K \le T_L \le 310\ K$, $0\ K \le \Delta T_{sh} \le 20\ K$

C) R600 (0.67% mean deviation and R^2=98.46%)

$$f = 1.4362 - 0.002312 \cdot T_H + 0.00077 \cdot T_L + 0.00121 \cdot \Delta T_{sh} \quad (30)$$

Applicability for: $350\ K \le T_H \le 420\ K$, $290\ K \le T_L \le 310\ K$, $0\ K \le \Delta T_{sh} \le 20\ K$

Figures 6, 7 and 8 show the results regarding the accuracy of the model for the (f) compared to the real analytical model values for n-pentane, toluene and R600 respectively. It is clear that the suggested formulas have high accuracy, and they can easily be applied to thermodynamic calculations.

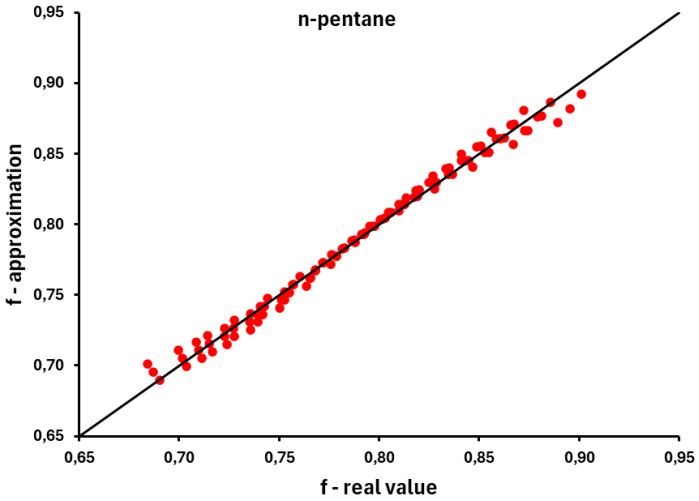

Figure 6. Comparison of the approximation model for the (f) ratio for n-pentane as working medium

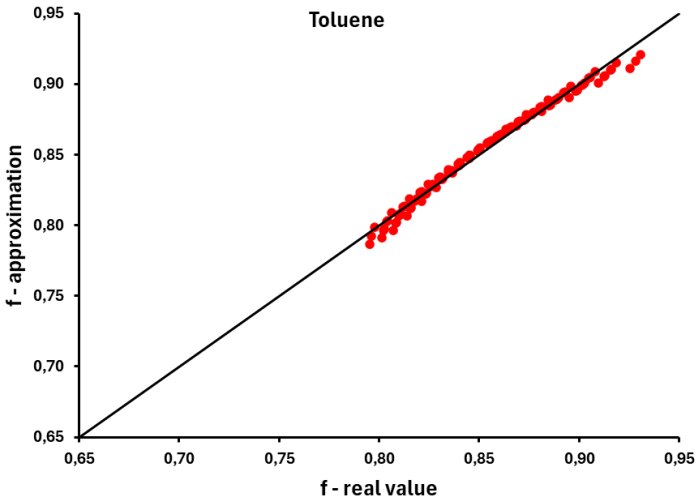

Figure 7. Comparison of the approximation model for the (f) ratio for Toluene as working medium

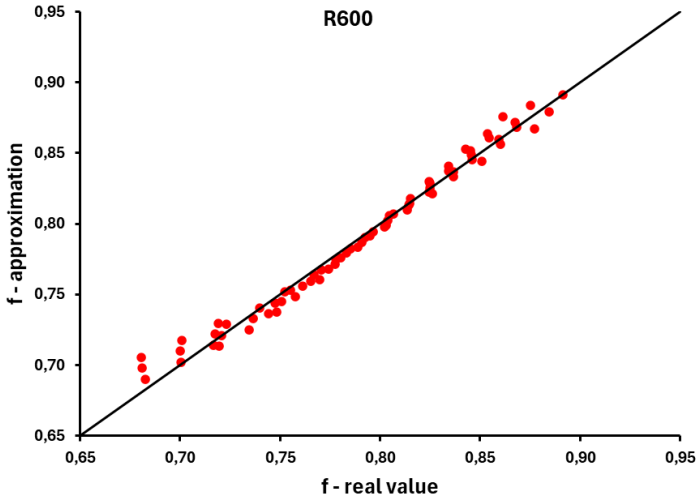

Figure 8. Comparison of the approximation model for the (f) ratio for R600 as working medium

4. Conclusions

The present work suggests an analytical model regarding the thermal efficiency calculation of a basic ORC. This model needs the low cycle and the high cycle temperatures, the superheating degree, the specific heat capacities of the fluid in liquid and vapor phases, as well as the latent heat in the high-temperature level. This model is able to determine the efficiency of the cycle with a 5% accuracy compared to the traditional thermodynamic models. Moreover, this model can be used for quick simulations regarding ORCs, for example, in dynamic simulations and in optimization procedures.

Furthermore, in the present work, extra studies were conducted by exploiting the developed model. Specifically, the (f) factor, which is the ratio of the ORC to the respective Carnot cycle was calculated for different operating conditions with n-pentane. Also, linear approximation equations were developed for the (f) for n-pentane, toluene and R600. The results of this investigation can be used for the analysis of systems that include ORC aiming to simplify the calculations and to deacrease the computational time. Also, in the future, the present model can be extended to other ORC architectures and to other working fluids.

Nomenclature

$c_{p,l}$ Specific heat capacity of the liquid, kJ/kgK

$c_{p,v}$ Specific heat capacity of the vapor, kJ/kgK

f Factor for ORC efficiency ratio

h Specific enthalpy, kJ/kg

q Specific heat, kJ/kg

q_{in} Specific heat input in the heat recovery system, kJ/kg

q_{out} Specific heat rejection in the condenser, kJ/kg

T Temperature, K

T_H High cycle temperature, K

T_L Low cycle temperature, K

T_m Mean temperature, K

s Specific entropy, kJ/kgK

w Specific work, kJ/kg

w_{net} Net specific work production, kJ/kg

w_{pump} Specific word demand by the pump, kJ/kg

w_{exp} Specific work production by the expander, kJ/kg

Greek Symbols

Δs Specific entropy variation, kJ/kgK

ΔT_{sh} Superheating degree in the expander inlet, K

ΔT_{cool} Superheating degree in the expander outlet, K

η_{carnot} Carnot cycle efficiency

$\eta_{model,ORC}$ ORC efficiency with the suggested model

$\eta_{real,ORC}$ ORC efficiency with the traditional thermodynamic analysis

Abbreviation

ORC Organic Rankine Cycle

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