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Averaged Axisymmetric Flow Surfaces in Hydraulic Turbomachines

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Abstract: Hydraulic turbomachines represent a group of turbomachines that operate with incompressible fluid, mostly water. They have a huge application in different hydraulic systems, such as water supply, irrigation, in cooling systems, hydraulic transport, as part of technological procedures in the industry etc. The optimal design of the hydraulic turbomachine is crucial for its optimal operation, i.e. work with maximum efficiency. In the process of turbomachine designing one of the basic assumptions is that fluid flow passing the impeller is axisymmetric, and on that bases the elementary stages of the impeller are determined. This paper presents the procedure for determining the averaged axisymmetric flow surfaces based on the results obtained using numerical simulations of flow in the impeller. Numerical simulations are used to verify and correct results of flow velocity in impeller domain, enabling the correction of the impeller geometry in accordance with the obtained results of the numerical analysis. According to the obtained results, the correction of specific works of elementary stages can be calculated in order to compare with the assumption made in the previous iteration.

Keywords: hydraulic turbomachines, averaged flow surface, numerical simulations.

1. Introduction

Ever since the design of the first turbomachines, the design process has relied heavily on previous experience. Due to the complex geometry of flow domain and the impossibility of explicitly calculating the flow parameters in it, this task is complex and requires some assumptions to be made. The assumptions may be more or less correct, which must be confirmed in the turbomachine testing phase. The entire process of designing a turbomachine is a long and demanding job, and each correction requires the creation of a new prototype whose working parameters must be checked on the test stand. Therefore, any correction that can be made before experimental testing of the prototype greatly reduces design time and cost.

The main designing element is certainly impeller, which is the main intermediary in the transformation of energy, and it is designed with special attention.

In the designing process the basic assumptions are that fluid flow is stationary and that the fluid flow is axisymmetric, and the theory is the theory refers to an impeller with an infinite number of infinitely thin blades [1-3]. Also, fluid flow in the impeller is continual, while flow energy increases in the non-turbine impellers and decreases in turbine impellers [4, 5]. Impeller calculation usually involves defining a certain number of elementary stages (min 5) which usually have equal specific work. There are also studies showing that in some cases it is shown that it is better to have different values of the specific work of elementary stages [6].

Different methodologies for optimization of turbomachinery impellers have been for turbomachines designing purpose [7].

The procedure of averaging the flow velocities according to the circular coordinate enables us to reduce the flow in the impeller to the flow in a fictitious impeller with an infinite number of thin blades that achieve the same flow deflection [8-10].

Numerical simulation of flow in hydraulic turbomachines are valuable tool which allows us to obtain all flow parameters in the flow domain. In this way, relatively quickly, we obtained all the values of the flow parameters of impeller and determine the averaged flow surfaces. Obtaining flow parameters in the flow domain and operating parameters of the turbomachine using CFD software and methodology that has shown good results in practice [11-14].

With the help of CFD techniques, the flow parameters obtained in the impeller of the hydraulic turbomachine can be used to recalculate the averaged flow surfaces [9, 10]. Meridian streamlines are calculated using the integral continuity equation for previously averaged flow parameters [8]. The presented methodology is used to obtain the fictive impeller with infinite number of infinitely thin blades, what allows to compare meridian streamlines with assumed ones.

2. Averaging of flow parameters

Here we will consider the curvilinear orthogonal coordinate system which suits better the fluid flow inside the impeller. The coordinate surfaces are: q_3=const. - meridian planes (planes passing through the axis of the rotating impeller), q_2=const. - axisymmetric surfaces approximate to averaged flow surfaces and q_1=const. - axisymmetric surfaces perpendicular to q_3=const. and q_2=const, shown in Fig.1 [8]. It is the right (positive) coordinate system and the direction of circumferential coordinate $q_3(\vec{e}_3^{\,o})$ did not need to follow the direction of the impeller rotation.

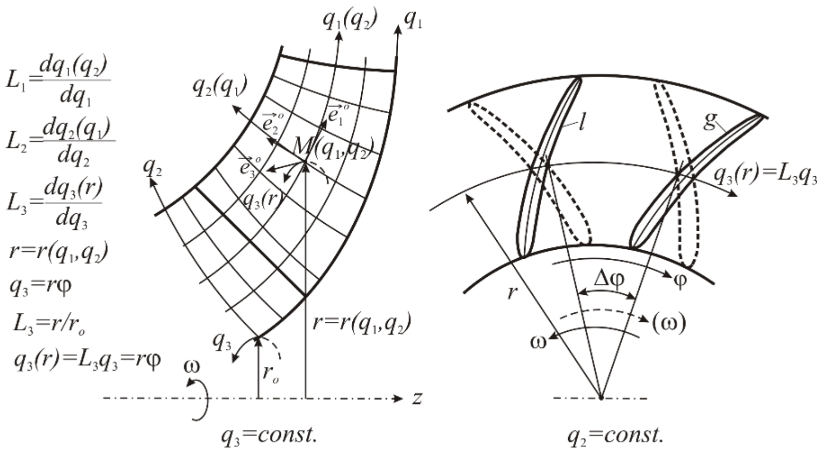


Figure 1. Curvilinear orthogonal coordinate system applied for centrifugal hydraulic turbomachine

Figure 1 shows the cross-section of a centrifugal pump, where dashed line defines pump blades for impeller rotation in the clockwise direction. Lame's coefficients are function of curvilinear coordinates, $L_1(q_1,q_2)$, $L_2(q_1,q_2)$ and $L_3=r/r_o$, where $r=r(q_1,q_2)$ is a radial distance from the origin of the cylindrical coordinate system (r, φ, z) [8]. For turbomachinery it is commonly used the cylindrical coordinate system (r, φ, z): $q_1=z$, $q_2=r$ i $q_3=r_o\varphi$ ($L_1=1$, $L_2=1$, $L_3=r/r_o$), where r_o is the radial distance in inlet cross-section of the turbomachine.

Averaging of flow parameters according to circular coordinate is a method that implies the use of the following mathematical expression for averaging a scalar function $f(q_1,q_2,q_3)$:

$$\tilde{f}\left(q_1,q_2\right)=\frac{1}{\Delta q_3}\int_{q_{3a}(q_1,q_2)}^{q_{3b}(q_1,q_2)} f\left(q_1,q_2,q_3\right)\cdot \mathrm{d}q_3 , \tag{1}$$

where $\Delta q_3=\Delta q_3(q_1,q_2)=q_{3b}(q_1,q_2)-q_{3a}(q_1,q_2)$ and a and b are two next blade surfaces in the blade passage.

Scalar flow parameters, such as pressure $p(q_1,q_2,q_3)$ and flow velocity components $w_j(q_1,q_2,q_3)$ and $c_j(q_1,q_2,q_3)$ ($j=1,2,3$) can be averaged according to circumferential coordinate $q_3=r_o\varphi$, using equation (1).

In Fig.1 circumferential coordinate line $q_3=L_3q_3=r\varphi$ passes through the point M(q_1,q_2) in meridian cross-section of pump impeller and equation (1) can be written:

$$\tilde{f}\left(q_1,q_2\right)=\frac{1}{\Delta q_3}\int_{q_{3a}(r)}^{q_{3b}(r)} f\left[q_1,q_2,q_3(r)\right]\cdot \mathrm{d}q_3(r)=\frac{1}{\Delta\varphi(r)}\int_{\varphi_a(r)}^{\varphi_b(r)} f\left[q_1,q_2,\varphi(r)\right]\cdot \mathrm{d}\varphi(r) , \tag{2}$$

where: $r=r\left(q_1,q_2\right)$, $\Delta q_3(r)=r\varphi(r)$, $\Delta\varphi(r)=\varphi_b(r)-\varphi_a(r)$.

Velocities in the impeller satisfy the equation:

$$\vec{c}=\vec{w}+\vec{u}=\vec{w}+\left[\vec{\omega},\vec{r}\right] , \tag{3}$$

where $\vec{c}$ is absolute, $\vec{w}$ is relative velocity and $\vec{u}$ is circumferential velocity.

Formula (2) leads to the following equation:

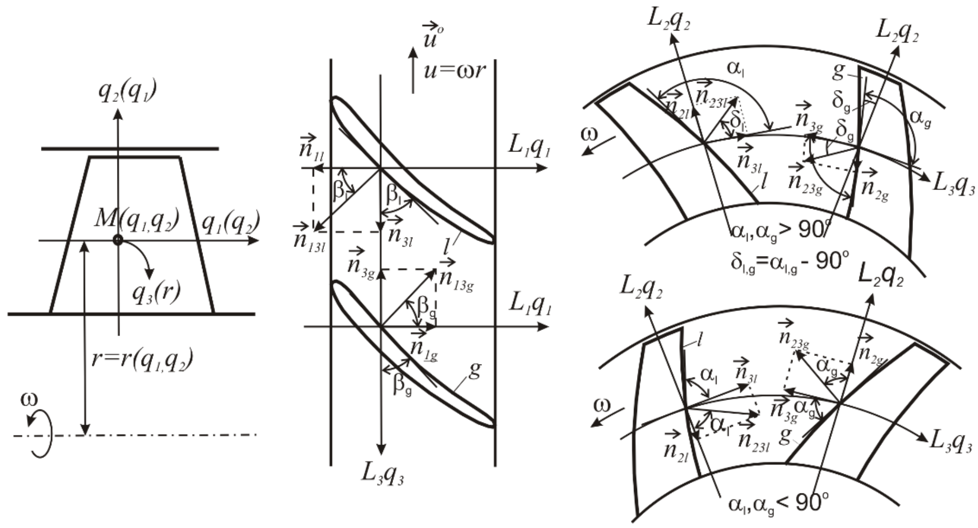
$$\operatorname{rot}\vec{c} = \operatorname{rot}\vec{w} + 2\vec{\omega}, \quad i.\,e. \quad \operatorname{rot}\vec{w} = \operatorname{rot}\vec{c} - 2\vec{\omega}\,. \tag{4}$$

For $\vec{e}_3^{\,o} = -\vec{u}^{\,o}$, there is: $u = -\omega \cdot r < 0$, $c_3 = -c_u < 0$, $w_3 = c_3 + \omega \cdot r = -c_u + \omega \cdot r > 0$.

For $\vec{e}_3^{\,o} = \vec{u}^{\,o}$, there is: $u = -\omega \cdot r > 0$, $c_3 = c_u > 0$, $w_3 = c_3 - \omega \cdot r = c_u - \omega \cdot r < 0$.

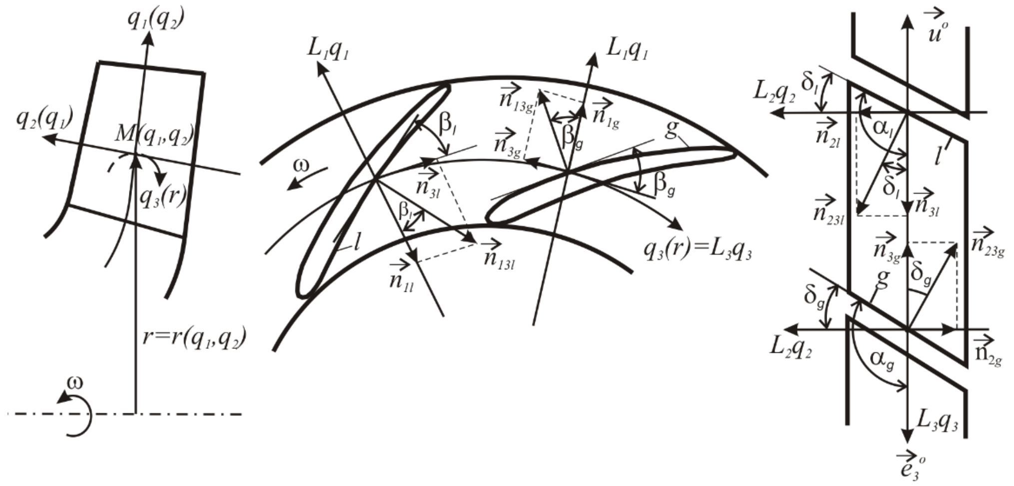
Index *a* becomes *l* and *b* becomes *g* for $\vec{e} = -\vec{u}$, while *a*=*g* and *b*=*l* for $\vec{e} = -\vec{u}$.

Averaging of scalar values $f(q_1, q_2, q_3)$ is obtained according to its function distribution over a circular arc and scalar values $\bar{p}(q_1, q_2),\ \bar{w}_j(q_1, q_2),\ \bar{c}_j(q_1, q_2),\ j = 1,2,3$ can be determined using numerically obtained distribution of these values over circular arcs in the blade passage. Since numerical simulations give us numerous data for those values, equation (2) can be calculated using the trapezoid rule with good accuracy.



(a) q_3=const. **(b)** q_2=const. **(c)** q_1=const.

Figure 2. Impeller of axial flow turbopump, where $\vec{e}_3^{\,o} = -\vec{u}^{\,o}$ (a=l, b=g)



a) q_3=const. ***b)*** q_2=const. ***c)*** q_1=const.

Figure 3. Impeller of centrifugal turbopump, where $\vec{e}_3^{\,o} = -\vec{u}^{\,o}$ (a=l, b=g)

Using equation (2) there are following expressions derived:

$$\left(\frac{\partial f}{\partial q_3}\right) = \frac{1}{\Delta q_3}\Delta f \quad \text{where} \quad \Delta f = f_b(q_1, q_2) - f_a(q_1, q_2) \tag{5}$$

and

$$\left(\widetilde{\frac{\partial f}{\partial q_{1,2}}}\right)=\frac{1}{\Delta q_3}\frac{\partial\left(\Delta q_3\tilde{f}\right)}{\partial q_{1,2}}-\frac{1}{\Delta q_3}\Delta\left(f\frac{\partial q_3}{\partial q_{1,2}}\right), \tag{6}$$

where $\Delta\left(f\frac{\partial q_3}{\partial q_{1,2}}\right)=\left(f\frac{\partial q_3}{\partial q_{1,2}}\right)_{\mathrm{b}}-\left(f\frac{\partial q_3}{\partial q_{1,2}}\right)_{\mathrm{a}}$

For the case $\vec{e}_3^{\,\mathrm{o}}=-\vec{u}^{\,o}$, when $a=l$and $b=g$ the following expressions can be made:

$$\left(\frac{dq_3}{dq_1}\right)_{\mathrm{a,b}}=\frac{L_1}{L_3}\mathrm{ctg}\beta_{\mathrm{a,b}}=-\frac{L_1}{L_3}\left(\frac{n_1}{n_3}\right)_{\mathrm{a,b}}\quad\text{and}\quad\left(\frac{dq_3}{dq_2}\right)_{\mathrm{a,b}}=\frac{L_2}{L_3}\mathrm{ctg}\alpha_{\mathrm{a,b}}=-\frac{L_2}{L_3}\left(\frac{n_2}{n_3}\right)_{\mathrm{a,b}}, \tag{7}$$

Where: $L_1=L_1(q_1,q_2)$, $L_2=L_2(q_1,q_2)$ and $L_3=r(q_1,q_2)/r_{\mathrm{o}}$ are Lamé's coefficients.

According to equation (7) equation (6) becomes:

$$\left(\widetilde{\frac{\partial f}{\partial q_{1,2}}}\right)=\frac{1}{\Delta q_3}\frac{\partial\left(\Delta q_3\tilde{f}\right)}{\partial q_{1,2}}+\frac{1}{\Delta q_3}\frac{L_{1,2}}{L_3}\Delta\left(f\frac{n_{1,2}}{n_3}\right), \tag{8}$$

where for point M(q_1,q_2), $\Delta\left(f\frac{n_{1,2}}{n_3}\right)=\left(f\frac{n_{1,2}}{n_3}\right)_b-\left(f\frac{n_{1,2}}{n_3}\right)_a$.

Furthermore, formulas for $\mathrm{grad}f\left(q_1,q_2,q_3\right)$, $\mathrm{div}\vec{v}\left(q_1,q_2,q_3\right)$ and $\mathrm{rot}\vec{v}\left(q_1,q_2,q_3\right)$ are obtained:

$$\left.\begin{aligned}
\widetilde{grad}f&=\frac{1}{\Delta q_3}grad\left(\Delta q_3\tilde{f}\right)+\frac{1}{L_3\Delta q_3}\Delta\left(f\frac{\vec{n}}{n_3}\right)=\mathrm{grad}\tilde{f}+\frac{1}{L_3\Delta q_3}\Delta\left(f'\frac{\vec{n}}{n_3}\right)\\
\widetilde{div}\vec{v}&=\frac{1}{\Delta q_3}div\left(\Delta q_3\tilde{\vec{v}}\right)+\frac{1}{L_3\Delta q_3}\Delta\left(\frac{\vec{n}}{n_3},\vec{v}\right)=\mathrm{div}\tilde{\vec{v}}+\frac{1}{L_3\Delta q_3}\Delta\left(\frac{\vec{n}}{n_3},\vec{v}'\right)\\
\widetilde{rot}\vec{v}&=\frac{1}{\Delta q_3}rot\left(\Delta q_3\tilde{\vec{v}}\right)+\frac{1}{L_3\Delta q_3}\Delta\left[\frac{\vec{n}}{n_3},\vec{v}\right]=\mathrm{rot}\tilde{\vec{v}}+\frac{1}{L_3\Delta q_3}\Delta\left[\frac{\vec{n}}{n_3},\vec{v}'\right]
\end{aligned}\right\} \tag{9}$$

Considering turbulent flow in turbomachinery, scalar and vector values can be defined as:

$$f'=f-\tilde{f}\quad\text{and}\quad\vec{v}'=\vec{v}-\tilde{\vec{v}}, \tag{10}$$

and equations (9) become:

$$\overline{\mathrm{grad}f'}=\widetilde{grad}f'=\frac{1}{L_3\Delta q_3}\Delta\left(f',\frac{\vec{n}}{n_3}\right),\ \widetilde{div}\vec{v}'=\frac{1}{L_3\Delta q_3}\Delta\left(\frac{\vec{n}}{n_3},\vec{v}'\right),\ \widetilde{rot}\vec{v}'=\frac{1}{L_3\Delta q_3}\Delta\left[\frac{\vec{n}}{n_3},\vec{v}'\right]. \tag{11}$$

In the flow domain without blades, equations (11) are equal to zero.

3. Averaging of flow equations

As previously described, the flow in turbomachines is assumed to be axisymmetric and stationary. Since the flow is extremely turbulent, kinematic characteristics of time averaged turbulent flow are in a good agreement with the calculation obtained for inviscid fluids.

Continuity equation for incompressible fluids is:

$$\mathrm{div}\vec{w}=0. \tag{12}$$

Using equation (9), continuity equation becomes:

$$div\left(\Delta q_3\cdot\tilde{\vec{w}}\right)+\frac{1}{L_3}\Delta\left(\frac{\vec{n}}{n_3},\vec{w}\right)=0. \tag{13}$$

Since blade surfaces are perpendicular to the flow surfaces, the first term is equal to zero, then equation (13) becomes:

$$\frac{\partial}{\partial q_1}\left(L_2 L_3 \Delta q_3 \tilde{w}_1\right)+\frac{\partial}{\partial q_2}\left(L_3 L_1 \Delta q_3 \tilde{w}_2\right)=0. \tag{14}$$

Previous equation can be transformed, by multiplying to $2\pi/\tau$, where $\tau=2\pi/z_1$:

$$\frac{\partial}{\partial q_1}\left(2\pi r_o k L_2 L_3 \tilde{w}_1\right)=-\frac{\partial}{\partial q_2}\left(2\pi r_o k L_3 L_1 \tilde{w}_2\right), \tag{15}$$

and

$$\frac{\partial \tilde{\psi}_m}{\partial q_1}=-2\pi r_o k L_3 L_1 \tilde{w}_2 \quad \text{and} \quad \frac{\partial \tilde{\psi}_m}{\partial q_2}=2\pi r_o k L_2 L_3 \tilde{w}_1, \tag{16}$$

where k – blockage factor, which is the coefficient of flow reduction due to the thickness of the blades.

The flow equation for inviscid fluid, if the gravitational influence is neglected, can be define as:

$$\left[\vec{w},\mathrm{rot}\vec{c}\right]=\mathrm{grad}E \tag{17}$$

where: $\mathrm{rot}\vec{c}=\mathrm{rot}\vec{w}-2\vec{\omega}$ and E is the Bernoulli's integral for relative fluid flow,

$$E=\frac{p}{\rho}+\frac{w^2}{2}-\frac{(r\omega)^2}{2},\quad E=E\left(q_1,q_2,q_3\right). \tag{18}$$

Equation (18) for turbulent flow becomes:

$$\left[\tilde{\vec{w}},\widetilde{rot\vec{c}}\right]+\left[\overline{\vec{w}',(rot\vec{c})'}\right]=\widetilde{gradE}. \tag{19}$$

Using equation (9) the previous formula can be written:

$$\left[\tilde{\vec{w}},rot\tilde{\vec{c}}\right]+\vec{F}^{(1)}+\vec{F}^{(2)}+\vec{F}^{(3)}=grad\tilde{E}, \tag{20}$$

where;

$$\vec{F}^{(1)}=\left[\tilde{\vec{w}},\frac{1}{L_3\Delta q_3}\Delta\left[\frac{\vec{n}}{n_3},\vec{w}'\right]\right],\ \vec{F}^{(2)}=-\frac{1}{L_3\Delta q_3}\Delta\left(E',\frac{\vec{n}}{n_3}\right),\ \vec{F}^{(3)}=\left[\overline{\vec{w}',rot\vec{w}'}\right] \text{ and } \tilde{E}=\frac{\tilde{p}}{\rho}+\frac{\tilde{w}^2}{2}-\frac{\omega^2 r^2}{2}.$$

4. Using numerical simulation results

CFD software, such as ANSYS CFX, can be used to obtain numerical results of fluid flow passing through the hydraulic turbomachine, and as a result all flow parameters (pressure, velocity, etc) in the whole geometric domain can be calculated. This this allows us to use the obtained numerical data for averaging these values by circular coordinate.

For symmetrical impeller, it is enough to observe only one blade passage, whether it is a space between two blades, or we consider one blade and half of blade passages on both sides of the blade (Figure 4).

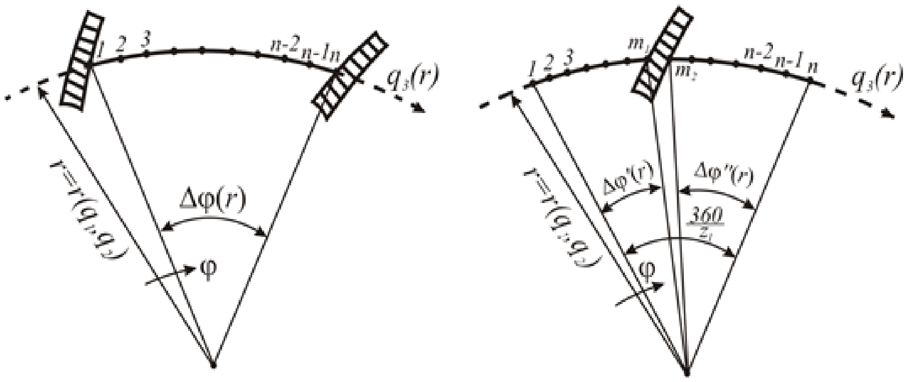


Figure 4. Averaging over circular coordinate q3 (between and around the blade)

As shown in Figure 4, points 1, 2, 3, 4, ..., n are discrete densely distributed points on the circular arc in the blade passage, which passes through the point M(q_1,q_2) in the meridian plane, which corresponds to radius vector $r = r(q_1, q_2)$. Denoting by $f_1, f_2, f_3, ..., f_a$ the scalar quantities f numerically obtained on the circular arc, using the trapezoidal integration rule, equation (2) becomes:

$$\tilde{f}\left(q_1, q_2\right) = \frac{1}{\Delta\varphi}\left(\frac{1}{2}\sum_{j=2}^{n}\left(f_j + f_{j-1}\right)\left(\varphi_j - \varphi_{j-1}\right)\right) \tag{21}$$

where $\Delta\varphi\left(r\right) = \varphi_n - \varphi_1$ and $r = r(q_1, q_2)$, $\varphi_j = \varphi_j(r)$, $f_j = f_j(r)$, $j = 1,2,3,...,n$.

When Ansys CFX is used, due to the symmetry of the impeller geometry, the flow domain can be only one blade with half of blade passage on both sides of the blade. In that case, the angle of circular coordinate is 360º/z_l (z_l – number of blades), and the equation for averaging flow parameters is:

$$\tilde{f}\left(q_1, q_2\right) = \frac{1}{\Delta\varphi'(r)}\int_{\varphi_1(r)}^{\varphi_{m_1}(r)} f\left(q_1, q_2, \varphi(r)\right) d\varphi(r) + \frac{1}{\Delta\varphi''(r)}\int_{\varphi_{m_2}(r)}^{\varphi_n(r)} f\left(q_1, q_2, \varphi(r)\right) d\varphi(r) \tag{22}$$

where $\Delta\varphi'(r) = \varphi_{m_1}(r) - \varphi_1(r)$ and $\Delta\varphi''(r) = \varphi_n(r) - \varphi_{m_2}(r)$ (Fig.2).

Using trapezoidal integration rule, previous equation becomes:

$$\tilde{f}\left(q_1, q_2\right) = \frac{1}{\Delta\varphi'(r)}\left(\frac{1}{2}\sum_{j=2}^{m_1}\left(f_j + f_{j-1}\right)\left(\varphi_j - \varphi_{j-1}\right)\right) + \frac{1}{\Delta\varphi''(r)}\left(\frac{1}{2}\sum_{j=m_2+1}^{n}\left(f_j + f_{j-1}\right)\left(\varphi_j - \varphi_{j-1}\right)\right) \tag{23}$$

where $r = r(q_1, q_2)$, $\varphi_j = \varphi_j(r)$, $f_j = f_j(r)$, $j = 1,2,3,...,m_1,m_2,...,n$.

4.1. Obtaining meridian streamlines of averaged flow and calculation of the flow rate

For obtaining meridian streamlines of averaged flow using integral continuity equation, it is necessary to calculate averaged flow velocities $\tilde{w}_1 = \tilde{w}_1\left(q_1, q_2\right)$ and $\tilde{w}_2 = \tilde{w}_2\left(q_1, q_2\right)$ in point M(q_1,q_2) (Figure 5).

For known components od averaged meridian velocities $\tilde{w}_1 = \tilde{w}_1\left(q_1, q_2\right)$ and $\tilde{w}_2 = \tilde{w}_2\left(q_1, q_2\right)$, and known function of the coefficient of flow reduction due to the thickness of the blades, so called blockage factor $k\left(q_1, q_2\right)$, the meridian streamlines $\tilde{\psi}_m\left(q_1, q_2\right) = const.$ can be determined using equation (16). Meridian streamlines represent traces of the cross-section in the meridian plane.

In Figure 5 (a, b) the cylindrical coordinate system is shown, where q_1=z, q_2=r, q_3=$r_o\varphi$, $\tilde{w}_1 = \tilde{w}_z$, $\tilde{w}_2 = \tilde{w}_r$ .Assuming form meridian flow on the inner part of the impeller disk $\tilde{\psi}_m\left(q_1, q_2\right) = 0$, the meridian flow on the outer part of the disk is $\tilde{\psi}_m\left(q_1, q_2\right) = Q$. Figure 5.c shows the curvilinear orthogonal coordinate system, where $q_2 = const.$ corresponds to an axisymmetric flow surface, while coordinate $q_1(q_2)$ corresponds to meridian streamline of the averaged fluid flow.

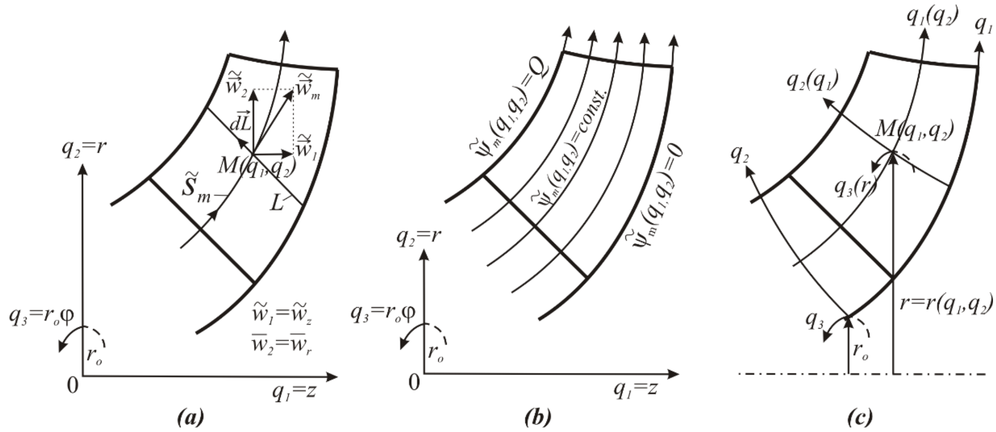


Figure 5. Meridian streamline in centrifugal turbopump

Equation (16) gives an expression for the volume flow rate through the axisymmetric flow surface of the meridian trace dL:

$$dQ = 2\pi k r_o \left(L_2 L_3 \tilde{w}_1 dq_2 - L_3 L_1 \tilde{w}_2 dq_1 \right), \tag{24}$$

where $k = \Delta\varphi / \tau = \Delta q_3 / \tau r_o = k\left(q_1, q_2\right)$, i.e. $\Delta q_3 = k\tau r_o = 2\pi k r_o$.

Velocity components $\tilde{w}_1$ and $\tilde{w}_2$ in cylindric coordinate system and integral formulas for flow rate distribution on the flow surfaces L_1, L_2, L_3,...,L_{10} (Figure 6), lead to the following equation:

$$dQ = 2\pi k r \left(\tilde{w}_z dr - \tilde{w}_r dz \right), \tag{25}$$

where $k = \Delta\varphi / \tau = k\left(z, r\right)$.

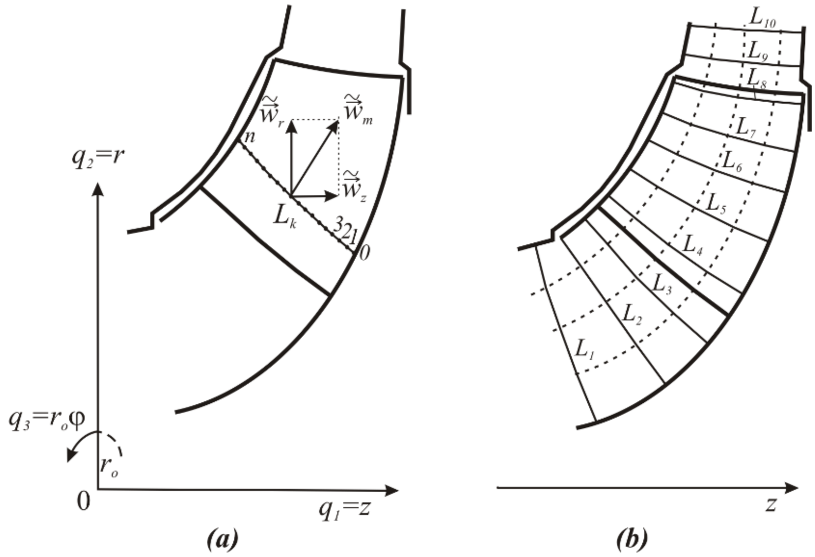


Figure 6. Meridian traces of flow cross-section L_k

The previous equation can be written:

$$Q_j^{(L_k)} = 2\pi \int_{r(L_{k.o})}^{r(L_{k.j})} k r \tilde{w}_z \cdot dr + \left(-2\pi \int_{z(L_{k.o})}^{z(L_{k.j})} k r \tilde{w}_r \cdot dz \right), \quad j = 1,2,...,n, \tag{26}$$

for the chosen points $L_{k.j}$ (j=0,1,2,...,n) in meridian surface, defined by coordinates $z(L_{k.j})=z_{k.j}$ and $r(L_{k.j})=r_{k.j}$, the following functions are defined:

$$\begin{aligned} f_j &= \left(k r \tilde{w}_z\right)_{L_{k.j}} = f\left(z_j, r_j\right) = k\left(z_j, r_j\right) \cdot r\left(z_j, r_j\right) \cdot \tilde{w}_z\left(z_j, r_j\right), \quad j = 1,2,...,n \\ F_j &= \left(k r \tilde{w}_r\right)_{L_{k.j}} = F\left(z_j, r_j\right) = k\left(z_j, r_j\right) \cdot r\left(z_j, r_j\right) \cdot \tilde{w}_r\left(z_j, r_j\right), \quad j = 1,2,...,n \end{aligned} \tag{27}$$

Using trapezoid rule volume flow rate can be calculated:

$$Q_j^{(L_k)} = \pi \sum_{i=1}^{j} \left(f_i + f_{i-1}\right)\left(r_i - r_{i-1}\right) + \pi\left(-\sum_{i=1}^{j} \left(F_i + F_{i-1}\right)\left(z_i - z_{i-1}\right) \right), \quad j = 1,2,...,n.. \tag{28}$$

4.2. Calculation of torque, power and specific work of the turbomachine impeller

The meridian trace of the flow cross-section at the impeller inlet (1-1) is L_1, and the meridian trace of the flow cross-section at the impeller outlet (2-2) is shown with line L_2 (Figure 7), for pump and turbine impeller.

The torque of the turbopump impeller can be calculated using equation:

$$M_k^{(P)} = \int_{A_2} \rho r \omega \tilde{c}_u \left(\vec{\tilde{c}}_m, d\vec{A} \right) - \int_{A_1} \rho r \omega \tilde{c}_u \left(\vec{\tilde{c}}_m, d\vec{A} \right), \tag{29}$$

where A_1 and A_2 are control cross-sections of axisymmetric flow surface on the inlet (A_1) and outlet (A_2).

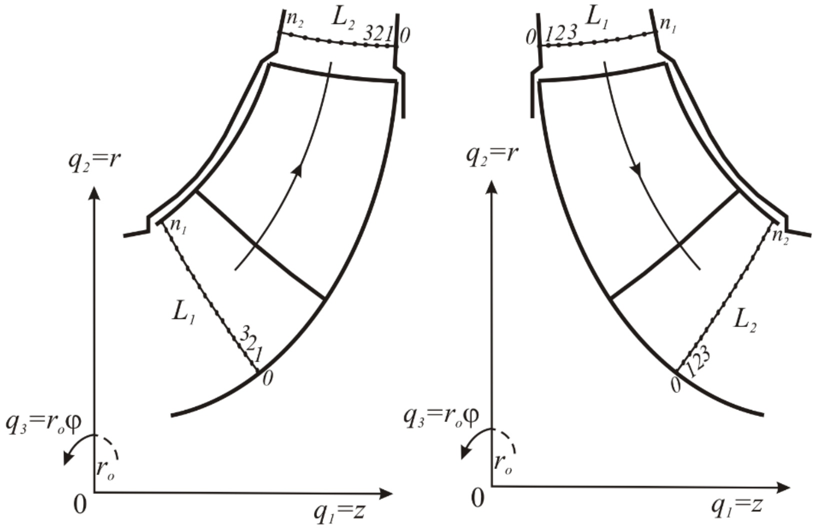


Figure 7. Meridian traces of flow surfaces L_k (turbopump left and turbine right)

Previous equation can be also written:

$$M_k^{(P)} = M_k(2) - M_k(1)\,, \tag{30}$$

where,

$$\left.\begin{aligned} M_k(2) &= 2\pi\rho\int_{L_2} r^2\tilde{c}_u\tilde{c}_z dr - 2\pi\rho\int_{L_2} r^2\tilde{c}_u\tilde{c}_r dz,\ \left(L_2 = L_2(z,r)\right) \\ M_k(1) &= 2\pi\rho\int_{L_1} r^2\tilde{c}_u\tilde{c}_z dr - 2\pi\rho\int_{L_1} r^2\tilde{c}_u\tilde{c}_r dz,\ \left(L_1 = L_1(z,r)\right) \end{aligned}\right\} \tag{31}$$

Calculation points on the meridian traces of control sections L_1 and L_2 can be denoted by:

$$L_{1.j} = L_{1.j}\left(z_{1.j}, r_{1.j}\right),\ \text{j=0,1,2,...,n}_1 \ \text{ and } \ L_{2.j} = L_{2.j}\left(z_{2.j}, r_{2.j}\right),\ \text{j=0,1,2,...,n}_2\,. \tag{32}$$

Defining the following functions:

$$z_j^{(1)} = z_{1.j},\ r_j^{(1)} = r_{1.j}\left(\text{ j=0,1,2,...,n}_1\right) \ \text{ and } \ z_j^{(2)} = z_{2.j},\ r_j^{(2)} = r_{2.j}\left(\text{ j=0,1,2,...,n}_2\right),$$

$$g_j^{(1)} = r^2\tilde{c}_u\tilde{c}_z = g_j^{(1)}\left(z_j^{(1)}, r_j^{(1)}\right),\quad G_j^{(1)} = r^2\tilde{c}_u\tilde{c}_r = G_j^{(1)}\left(z_j^{(1)}, r_j^{(1)}\right)\quad \left(\text{ j=0,1,2,...,n}_1\right),$$

$$g_j^{(2)} = r^2\tilde{c}_u\tilde{c}_z = g_j^{(2)}\left(z_j^{(2)}, r_j^{(2)}\right),\quad G_j^{(2)} = r^2\tilde{c}_u\tilde{c}_r = G_j^{(2)}\left(z_j^{(2)}, r_j^{(2)}\right)\quad \left(\text{ j=0,1,2,...,n}_2\right),$$

And the trapezoidal integration rule, equation (30) becomes:

$$\left.\begin{aligned} M_k(2) &= \pi\rho\sum_{j=1}^{n_2}\left(g_j^{(2)} + g_{j-1}^{(2)}\right)\left(r_j^{(2)} - r_{j-1}^{(2)}\right) - \pi\rho\sum_{j=1}^{n_2}\left(G_j^{(2)} + G_{j-1}^{(2)}\right)\left(z_j^{(2)} - z_{j-1}^{(2)}\right), \\ M_k(1) &= \pi\rho\sum_{j=1}^{n_1}\left(g_j^{(1)} + g_{j-1}^{(1)}\right)\left(r_j^{(1)} - r_{j-1}^{(1)}\right) - \pi\rho\sum_{j=1}^{n_1}\left(G_j^{(1)} + G_{j-1}^{(1)}\right)\left(z_j^{(1)} - z_{j-1}^{(1)}\right) \end{aligned}\right\} \tag{33}$$

The torque of the turbine impeller can be calculated using equation:

$$M_k^{(T)} = M_k(1) - M_k(2)\,. \tag{34}$$

Power of the pump ($P_k^{(P)}$) and power of the turbine ($P_k^{(T)}$) can be determined:

$$P_k^{(P)} = \omega_P M_k^{(P)} \ \text{ and } \ P_k^{(T)} = \omega_T M_k^{(T)}\,. \tag{35}$$

Specific work of the pump impeller and turbine impeller are:

$$Y_k^{(P)} = \frac{P_k^{(P)}}{\dot{m}} = \frac{\omega_P M_k^{(P)}}{\rho Q} \quad \text{and} \quad Y_k^{(T)} = \frac{P_k^{(T)}}{\dot{m}} = \frac{\omega_T M_k^{(T)}}{\rho Q}\,. \tag{36}$$

Conclusion

In this paper the procedure of averaging axisymmetric flow surfaces in hydraulic turbomachines is presented. Averaging the operating parameters of hydraulic turbomachines obtained by numerical simulations are used for determining the averaged flow surfaces and determining the meridian streamlines, flow rate, specific work, torque and power of the turbomachine impeller.

The procedure shown in the paper is simple and uses the numerical method of the trapezoidal integration rule.

This procedure of averaging flow surface, using numerical simulations and mathematical calculation lead to obtain useful information about the hydraulic turbomachine, allowing the designers to compare those results with assumptions made in the first stage of designing the hydraulic turbomachine impeller.

According to the distribution of the calculated operating parameters (for example specific works of elementary stages) the correction of the geometry of the impeller can be performed before making and testing the prototype of the turbomachine. The presented procedure greatly reduces designing time and costs.

Nomenclature

Latin symbols

$\vec{c}$ – absolute velocity [m/s].

$\vec{w}$ – relative velocity [m/s].

$\vec{u}$ – circumferential velocity [m/s].

c_u – circumferential component of absolute velocity [m/s].

c_m – meridian component of absolute velocity [m/s]

p – pressure [Pa]

k – blockage factor

Q – volume flow rate [m^3/s]

$\dot{m}$ – mass flow rate [kg/s]

z_l – number of blades

M_k – torque of the impeller [Nm]

P_k – power of the impeller [W]

Y_k – specific work of impeller [J/kg]

L_i – Lamé’s coefficients, i=1,2,3

g – pressure side of the blade

l – suction side of the blade

Greek symbols

ψ – stream function.

τ – angular blade pitch.

ω – angular velocity [rad/s]

Subscripts

a, b – surfaces of the two next blades

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