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Development of a Mathematical Model of a Drum Steam Boiler by Using the Automatic Control System – Second Step

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Abstract: Steam boiler or steam generator is widespread equipment for energy conversion in the industry for obtaining steam and later electricity. Processes in energy states are complex and require the application of automatic control theory, which basically relies on mathematical models of objects and processes given in the state space. This approach requires knowledge of both the static and dynamic properties of the processes taking place in the thermoelectric plant. The general approach to mathematical modeling consists in setting bilas equations as well as an appropriate number of supplementary equations which, together with the previous ones, give a closed system of equations to be solved. In this paper, the second step in the modeling of a steam boiler is presented, and for the specific case, a simulation of the mathematical model was performed on the basis of experimentally collected data.

Keywords: steam boiler, mathematical model, combustion dynamics, automatic control

1. Introduction

The demand for energy, and consequently electricity, is rapidly increasing both globally and within our country, leading to a greater reliance on thermal power plants for overall electricity generation. Advances in modern thermal power plants are evident in enhanced working conditions, optimized steam block operating parameters, improved signaling and safety measures, the emergence of more effective technical designs, and the increasing implementation of computer-controlled machinery in production processes. Significant attention is focused on enhancing the dynamic performance of both individual units and the overall thermal power plant. This improvement translates into more stringent requirements for operating and managing the steam unit, particularly the steam boiler [1]. Consequently, the control units of the thermal power plant are expected to meet specific criteria, enabling them to monitor substantial changes in operating conditions and maintain relevant parameters within narrow tolerances. Furthermore, their economy and reliability are called into question, which comes to the fore when the plant is put into operation, the sudden disconnection of certain units from the operation, a sudden change in load and the event of an emergency mode. Classic control systems in thermal power plants are not designed for such strict requirements. They are designed to act independently of each other, as there are a large number of influences of the relevant physical quantities that need to be regulated. The design of control units for thermal power plants includes knowledge of the processes' static and dynamic properties. Additionally, concerns about their economic efficiency and reliability arise, particularly during the commissioning of the plant, abrupt disconnections of specific units, sudden load changes, or emergency situations. Traditional control systems in thermal power plants do not meet these stringent demands [2]. They typically function independently, responding to various factors that must be controlled. Designing control units for thermal power plants requires a deep understanding of both the static and dynamic characteristics of the processes involved [3].

A steam boiler is a facility designed to generate steam at pressures exceeding atmospheric levels [4]. From an energy perspective, the steam boiler functions as an open, flowing system, facilitating numerous energy exchanges. In thermodynamic terms, it operates as a heat exchanger. However, when considering the boiler from the standpoint of automatic control, it is regarded as a complex entity due to the various processes occurring within the exchanger [5].

These processes are intricate owing to their occurrence in spaces with varying dimensions, and they involve both single-phase and two-phase flows. From a control and regulation perspective, this complexity arises from the interconnections among multiple physical quantities that need to be maintained simultaneously. As a result, it necessitates the implementation of advanced management concepts, including local, cross, and feedback strategies.

Numerous mathematical models have been developed in the literature to characterize the dynamics of steam boilers. Some of these models focus on monitoring the stability of steam boilers under various initial operating conditions, while also controlling key parameters such as drum level, boiler pressure, and steam pressure [6][7]. Additionally, some models are non-linear, providing insights into the dynamic processes occurring within power plants [8][9]. Furthermore, there are models that track the dynamics of processes in plants [10][11] as well as those specifically designed for steam generators utilized in heat recovery within combined cycle power plants [12][13].

In this paper, a mathematical model was developed that follows the dynamics of drum steam boilers when the combustion process takes place in a layer. The model is formed for each part, which includes the boiler plant, drum, downcomer pipe and grate. Also, balance equations were formed for each element individually. During the formation of the model, additional simplifications were introduced to obtain an adequate mathematical model, which was the goal of the work.

2. Formation of a structural diagram

When forming the structural diagram of the boiler plant, it started by defining the tracts of the primary working media and the position of the basic heating surfaces along their path. However, in addition to the previous one, output (control) quantities are also defined. By defining them, the requirements that are imposed during the operation of the steam boiler are stated, and the basic parameters are the superheated steam θ_p and the pressure of the superheated steam p_p. Figure 2.1 shows a diagram of the boiler plant.

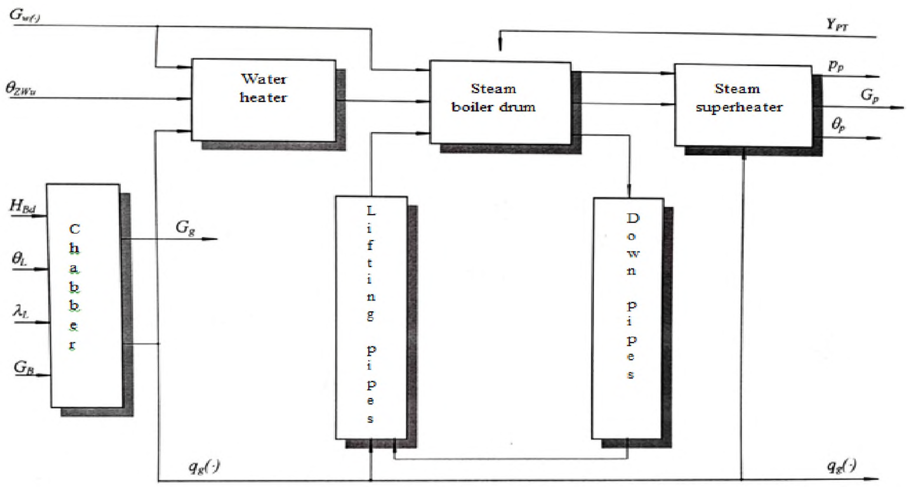

Figure 1. Global structural diagram of a steam boiler (Adopted from [3])

Disturbances that significantly affect the operation of the boiler plant are the lower heating value of the fuel H_B, the temperature of the supplied water θ_w, the temperature of the primary air θ_L i of the cooling water θ_{rw}, the change in the position of the regulating valve of the steam turbine Y_{PT} and excess air λ_L. During the formation of the model, these quantities were taken into account because of the management concept. The second group of disturbances, such as changes in fuel temperature θ_B, changes in specific heats of air c_{pl}, fuel c_B, and cooling water c_{rw}, are classified in the second order of importance when forming a mathematical model.

3. Mathematical model of the boiler

Equations of the mathematical model with the introduction of relative deviations according to the author [14] were formed for the nominal operating mode, while neglecting non-linear terms.

During the formation of the mathematical model, a number of simplifications were introduced in order to solve the problem. Those simplifications are as follows:

1. The dynamics of the water heater is neglected for two reasons: firstly, due to the flow of incompressible fluid, and secondly, due to the small amount of absorbed heat from the gaseous products of combustion. This stems from the fact that modeling the water heater would significantly but unnecessarily increase

the performance of the entire model. With this simplification, it was practically adopted that the entrance to the boiler plant is at the point of entry where the feed water enters the drum.

2. The process of complete combustion was considered, which implies that boiling water enters the drum and dry saturated steam leaves it.
3. The primary and secondary steam superheaters are treated as one heating surface. This is due to the unnecessary increase in the order of the system of equations that describe the process.
4. The dynamics of the surface steam cooler is neglected.
5. The speed of movement of the chain grate is adopted for the control effect although there is a regulator of the height of the coal layer. This was done with the aim of avoiding the dynamics of the grid as a classic conveyor because then there is a pure time delay in the control, which in the mathematical sense makes the problem much more difficult and introduces differential equations with a delay.
6. All geometric characteristics of the boiler plant are determined from the drawing and thermal calculation of the boiler.

After defining the simplification of the mathematical model and knowing that the considered boiler consists of several parts, below are the equations for each part separately.

Drum

$$\frac{d(\overline{\Delta\rho_{pczp}})}{dt}+\frac{d(\overline{\Delta V_{dzp}})}{dt}=\frac{x_N G_{pcN}}{\rho_{pczpN}V_{dzpN}}(\overline{\Delta x}+\overline{\Delta G_{pc}})+\frac{G_{wpN}}{\rho_{pczpN}V_{dzpN}}\overline{\Delta G_{wp}}-\frac{G_{ppuN}\cdot\overline{\Delta G_{pp}}}{\rho_{pczpN}V_{dzpN}} \tag{1}$$

$$\frac{d(\Delta V_{dw})}{dt}+\frac{d(\overline{\Delta\rho_w})}{dt}=\frac{G_{eN}}{V_{dwN}\rho_{wN}}\overline{\Delta G_e}+\frac{(1-x_N)G_{pcN}}{V_{dwN}\rho_{wN}}\overline{\Delta G_{pc}}-\frac{x_N G_{pcN}}{V_{dwN}\rho_{wN}}\overline{\Delta x}-$$
$$-\frac{G_{scN}}{V_{dwN}\rho_{wN}}\overline{\Delta G_{sc}}-\frac{G_{wpN}}{V_{dwN}\rho_{wN}}\overline{\Delta G_{wp}} \tag{2}$$

$$\frac{d(\Delta V_{dw})}{dt}+\frac{d(\overline{\Delta\rho_w})}{dt}+\frac{d(\overline{i_w})}{dt}=\frac{i_{eN}G_{eN}}{V_{dwN}\rho_{wN}i_{wN}}(\overline{\Delta G_e}+\overline{\Delta i_e})+\frac{G_{pcN}i_{pcN}}{V_{dwN}\rho_{wN}i_{wN}}\overline{\Delta i_{pc}}+$$
$$\frac{G_{pcN}i_{pcN}}{V_{dwN}\rho_{wN}i_{wN}}\overline{\Delta G_{pc}}-\frac{x_N G_{pcN}i_{pcN}}{V_{dwN}\rho_{wN}i_{wN}}(\overline{\Delta i_{pc}}+\overline{\Delta G_{pc}}+\overline{\Delta x})-$$
$$\frac{G_{scN}i_{scN}}{V_{dwN}\rho_{wN}i_{wN}}(\overline{\Delta G_{sc}}+\overline{\Delta i_{sc}})-\frac{G_{wpN}i_{pczpN}}{V_{dwN}\rho_{wN}i_{wN}}(\overline{\Delta G_{wp}}+\overline{\Delta i_{pczp}}) \tag{3}$$

Downpipe

$$\frac{d(\Delta G_{sc})}{dt}=\frac{p_{dN}A_{sc}}{i_{sc}G_{sc}N}\overline{\Delta p_d}-\frac{p_{scN}A_{sc}}{i_{sc}G_{scN}}\overline{\Delta p_{sc}}-\frac{4\rho_{sc}G_{scN}}{d_{sc}A_{sc}\rho_{wN}}\overline{\Delta G_{sc}}-\frac{G_{scN}}{A_{sc}i_{sc}\rho_{wN}}\overline{\Delta G_{sc}}+g\rho_{wN}\frac{A_{sc}}{G_{scN}}\overline{\Delta\rho_w}$$

$$\frac{d(\overline{\Delta\rho_{pc}})}{dt}=\frac{G_{scN}}{\rho_{pcN}V_{pvc}}\overline{\Delta G_{sc}}-\frac{G_{pcN}}{\rho_{pcN}V_{pc}}\overline{\Delta G_{pc}} \tag{4}$$

$$\frac{d(\overline{\Delta\rho_{pc}})}{dt}+\frac{d(\overline{\Delta i_{pc}})}{dt}=\frac{G_{scN}i_{scN}}{V_{pc}\rho_{pcN}i_{pcN}}(\overline{\Delta G_{sc}}+\overline{\Delta i_{sc}})-\frac{G_{pcN}i_{pcN}}{V_{pc}\rho_{pcN}i_{pcN}}(\overline{\Delta G_{pc}}+\overline{\Delta i_{pc}})+\frac{Q_{pcN}}{V_{pc}\rho_{pcN}i_{pcN}}\overline{\Delta Q_{pc}} \tag{5}$$

$$\frac{d(\overline{\theta_{zpc}})}{dt}=\frac{Q_{gpcN}}{m_{zpc}c_{zpc}\theta_{zpcN}}\overline{\Delta Q_{gpc}}-\frac{Q_{pcN}}{m_{zpc}c_{zpc}\theta_{zpcN}}\overline{\Delta Q_{pc}} \tag{6}$$

$$\frac{d(\overline{\Delta\rho_{pp}})}{dt}=\frac{G_{ppuN}}{V_{pp}\rho_{ppN}}\overline{\Delta G_{ppu}}-\frac{G_{Tvp}}{V_{pp}\rho_{ppN}}\overline{\Delta G_{Tvp}} \tag{7}$$

$$\frac{d(\overline{\Delta\rho_{pp}})}{dt}+\frac{d(\overline{\Delta\theta_{pp}})}{dt}=\frac{G_{ppuN}i_{ppuN}}{V_{pp}\rho_{ppN}i_{ppN}}(\overline{\Delta G_{ppu}}+\overline{\Delta i_{ppu}})-\frac{G_{TvpN}i_{TvpN}}{V_{pp}\rho_{ppN}i_{ppN}}(\overline{\Delta G_{Tvp}}+\overline{\Delta\theta_{pp}})+$$
$$+\frac{Q_{ppN}}{V_{pp}\rho_{ppN}i_{ppN}}\overline{\Delta Q_{pp}}+\frac{d(\overline{\Delta\theta_{zpp}})}{dt}=\frac{Q_{gppN}}{m_{zpp}c_{zpp}\theta_{zppN}}\overline{\Delta Q_{gpp}}-\frac{Q_{ppN}}{m_{zpp}c_{zpp}\theta_{zppN}}\overline{\Delta Q_{gpp}} \tag{8}$$

Stokehole with grate

$$0.6T_{RŠ}\frac{d(\overline{\Delta\lambda})}{dt}-0.4T_{RŠ}\frac{d(\overline{\Delta G_{Bsg}})}{dt}=\overline{\Delta G_{Bsg}}-K_{RŠ2}\overline{\Delta W_{RŠ}} \tag{9}$$

Previously formed equations (1 - 9) represent a linearized mathematical model of a drum boiler with combustion in a layer.

Selection of state quantities, disturbance control and managed quantities

Knowing the processes that take place in a steam boiler, and based on the obtained system of equations and in order to perform a simulation of the operation of the boiler as well as the subsequent analysis of the obtained results, the following selection of sizes is adopted:

Sizes of the balance	Disturbing sizes	Management sizes	Managed sizes
$\overline{\Delta p_d}(t) = x_1(t)$	$\overline{\Delta x}(t) = z_1(t)$	$\overline{\Delta G_e}(t) = u_1(t)$	$x_{i1}(t) = \overline{\Delta p_d}(t)$
$\overline{\Delta h_{dw}}(t) = x_2(t)$	$\overline{\Delta w_{pc}}(t) = z_2(t)$	$\overline{\Delta W_{RŠ}}(t) = u_2(t)$	$x_{i2}(t) = \overline{\Delta h_{dw}}(t)$
$\overline{\Delta \theta_w}(t) = x_3(t)$	$\overline{\Delta \theta_{pczp}}(t) = z_3(t)$		$x_{i3}(t) = \overline{\Delta \theta_{pp}}(t)$
$\overline{\Delta G_{sc}}(t) = x_4(t)$	$\overline{\Delta w_{ppu}}(t) = z_4(t)$		
$\overline{\Delta \rho_{pc}}(t) = x_5(t)$	$\overline{\Delta i_e}(t) = z_5(t)$		
$\overline{\Delta i_{pc}}(t) = x_6(t)$	$\overline{\Delta i_{sc}}(t) = z_6(t)$		
$\overline{\Delta \theta_{zpc}}(t) = x_7(t)$	$\overline{\Delta p_{sc}}(t) = z_7(t)$		
$\overline{\Delta \rho_{pp}}(t) = x_8(t)$	$\overline{\Delta \theta_{pc}}(t) = z_8(t)$		
$\overline{\Delta \theta_{pp}}(t) = x_9(t)$	$\overline{\Delta w_{ppi}}(t) = z_9(t)$		
$\overline{\Delta \theta_{zpp}}(t) = x_{10}(t)$			
$\overline{\Delta G_{sg}}(t) = x_{11}(t)$			
$\overline{\Delta Q_g}(t) = x_{12}(t)$			
$\overline{\Delta \lambda}(t) = x_{13}(t)$			

The final forms of the linearized mathematical model of the considered boiler were obtained by the selection of state, disturbance, control and controlled quantities using the equations shown in the paper [15]. The differential vector equation of state reads:

$$E\underline{\dot{x}}\,(t) = A^*\underline{x}(t) + B^*\underline{u}(t) + F^*\underline{z}(t) \tag{10}$$

and the output equation:

$$\underline{x_i}(t) = C\underline{x}(t) \tag{12}$$

Since the elements of the matrix E are such that $\det E \neq 0$, for the adopted steam drum steam boiler, equation (10) reduces to the standard form of the form:

$$\underline{\dot{x}}(t) = E^{-1}A^*\underline{x}(t) + E^{-1}B^*\underline{u}\,(t) + E^{-1}F^*\underline{z}(t) \tag{13}$$

Ie:

$$\underline{\dot{x}}(t) = A\underline{x}(t) + B\underline{u}(t) + z(t) \tag{4}$$

Where the $A = E^{-1}A^*; B = E^{-1}B^*; F = E^{-1}F^*$, and are the matrices of the system. The elements of the matrix are determined on the basis of the operational and structural characteristics of the considered boiler.

4. Results and discussion

Based on the vector equations of state (4) and the output equation (2), a simulation of the operation of the steam boiler was performed. When conducting the simulation, only state variables in transition modes created under the influence of control variables were considered.

The dynamic behavior of the steam boiler was considered with simultaneous changes of the jump type of control variables, and with all zero initial conditions.

On the ordinates of the diagram, the variables of the state in the label were given as $x(j),\ j = 1, 2, \ldots, 13$ which further gives the state sizes x_j. The time in seconds is given on the abscissa, and the simulation time interval is 120 seconds

Bearing in mind the choice of controlled (output) quantities, the changes in state quantities $x_1, x_2, and\ x_9$ represent at the same time the quantities of changes in output quantities, i.e. changes in pressure in the drum of the steam boiler, water level and temperature at the outlet of the steam superheater, so the discussion of the obtained simulation is directed on these three sizes.

The simulation results are shown on the change diagrams which are marked as Figure 2 to Figure 14

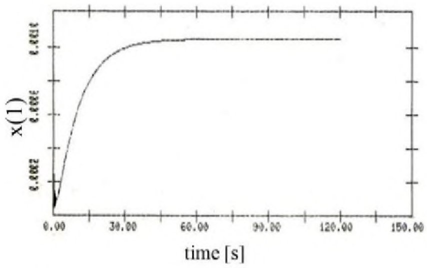

***Figure 2**. Change of pressure in the drum steam boiler*

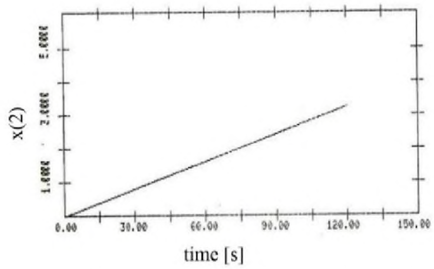

***Figure 3**. Change in water level at the output from steam superheater*

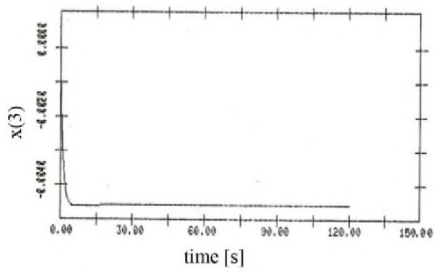

***Figure 4**. Change in water temperature at the output of the steam superheater*

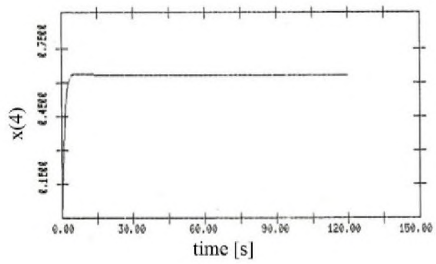

***Figure 5**. Change in water flow downpipes*

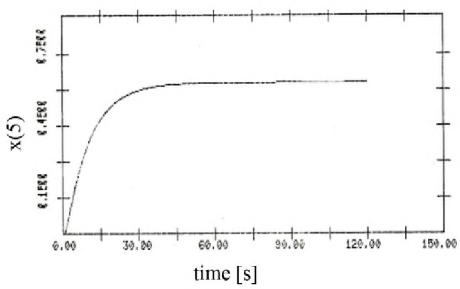

***Figure 6**. Change in the density of the lift pipe mixture*

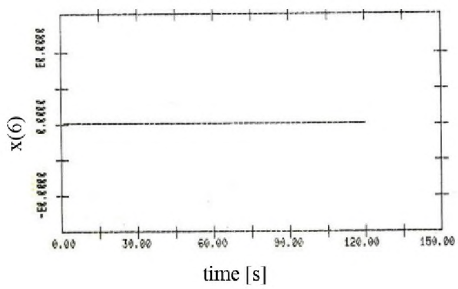

***Figure 7**. Enthalpy of the lift pipe mixture*

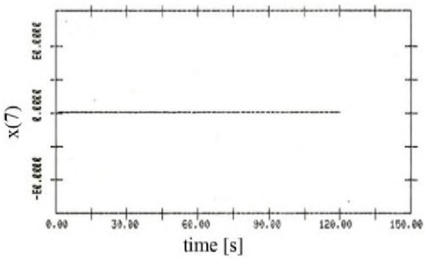

Figure 8. Change in the temperature of the lift pipe wall

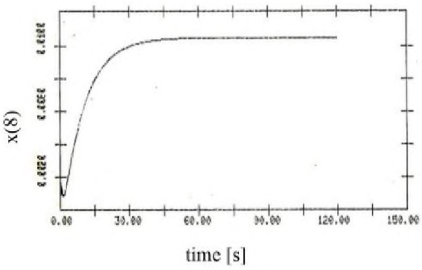

Figure 9. Change in steam density at the output of the steam superheater

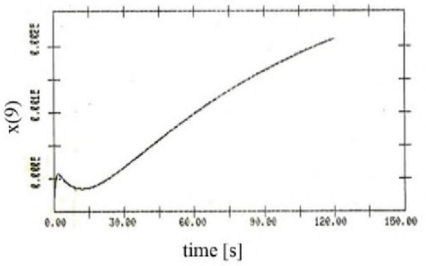

Figure 10. Temperature change at the output of the steam superheater

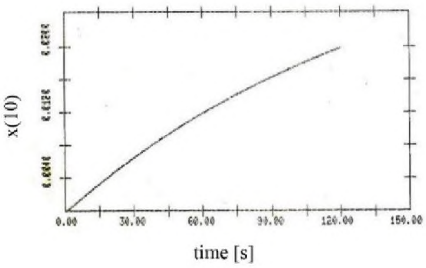

Figure 11. Change in the temperature of the steam superheater tube walls

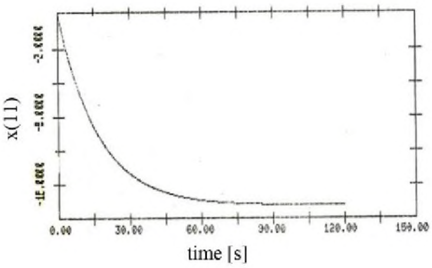

Figure 12. Change in fuel mass flow through the combustion chamber

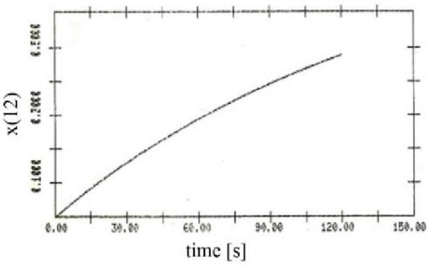

Figure 13. Change in the total amount of heat supplied to the combustion chamber

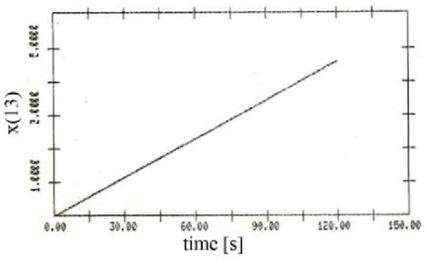

***Figure 14.** Change of excess air in the* combustion chamber

Diagrams 2 and 3 show that with simultaneous changes in water flow and fuel flow (increase), the pressure in the drum will grow up to some new stationary value, while the water level will grow indefinitely, showing the integral nature of the transmission. This result arises because the simulation was performed on a steam boiler as an object of automatic control without the presence of a control system that certainly reacts in such situations.

Diagram 10 shows that with the same changes in the control parameters, the temperature of the steam at the outlet of the steam superheater rises and it stabilizes at the new value.

It should be pointed out that the used simulation time was not enough to cover the entire duration of the transition process, although the tendency towards a new stationary state can easily be concluded. It can be seen that the transient process in the steam superheater takes a very long time, that is, it has a large settling time value. This result stems from the location of the steam superheater, that is, the last exchanger is in the water-steam tract and has a large volume, as well as the fact that it contains a compressible fluid - superheated steam.

The obtained results, shown in the other diagrams, correspond to the results that can be found in the literature. They are qualitatively satisfactory, and a quantitative assessment could only be given after comparison with the experimental determined transition curves on the boiler itself.

5. Conclusion

In this paper, a mathematical model of a drum steam boiler with combustion in a layer is presented. During the implementation of the simulation, the mathematical model was first concretized in concrete numerical values, which required the knowledge of a whole series of working and constructive sub-systems of a particular boiler with a chain grate. A large number of operational and structural parameters of this model were adopted due to the impossibility of their exact determination. The jump responses of all state variables as well as output variables of the steam boiler (pressure in the drum, water level and temperature of superheated steam at the superheater inlet) gave results that correspond to real conditions. In some cases, due to the insufficient duration of the transition process and the large values of the settling time of some physical quantities, the conclusions were not drawn to the end.

Nomenclature

List of symbols		Bottom marks
A - flow surface	[m^2]	B – fuel
c – specific heat	[kJ/kgK]	d – drum
d – pipe diameter	[m]	e – economizer
G – mass flow	[kg/s]	g – gaseous combustion products
H_B - lower heating value	[kJ/kg]	gB – gaseous products – fuel
h – water level	[m]	ge – from gas to economizer
i – enthalpy	[kJ/kg]	gpc – from gases to lifting pipes
		i – at the exit
K – heat transfer coefficient	[kW/m^{2}K	L – air
k – constant		lož – fireplace
l – length	[m]	N – nominal mode
m – mass	[kg]	pc – lifting pipes
p – pressure	[kN/m^2]	pp – primary steam superheater
Q – heat flow	[kW]	sc – down pipes
T - time constant	[s]	sg – combustible part
t – time	[s]	TVP – high pressure turbine
		u – at the entrance
V – volume	[m^3]	w – water
w – flow rate	[m/s]	wp – from water to steam
ρ - density	[kg/m^3]	z – pipe wall
ξ – spatial coordinate	[m]	zc – the wall of the economizer pipe
θ – temperature	[K]	zpc – the wall of the riser pipes
b – width	[m]	zpp – primary superheater tube wall
λ – excess air		zr – radiation
		rš – grid

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