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PROBABILISTIC ANALYSIS OF PRESS FIT JOINT LOAD CARRYING CAPACITY

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Abstract

This paper presents a procedure for probabilistic analysis of joint load carrying capacity as a randomly variable quantity on the example of a cylindrical press fit joint of a bronze ring with teeth and a worm gear hub. By fitting the set of results obtained by a series of FEM simulations for characteristic values of input parameters (interference and material characteristics), the coefficients of a second-order multidimensional polynomial as a "response surface" for the output quantity - joint load carrying capacity were determined. By MC simulation, generating random parameters according to the known laws of statistical distributions and evaluating the previously obtained higher-order polynomial, a statistical set was obtained, the processing of which determines the distribution parameters of the randomly variable output quantity. The described procedure is applicable to a wide class of probabilistic design problems that are analyzed using FEM analysis.

***Keywords**: Probabilistic design, FEM simulation, Monte-Carlo simulation, statistical distribution*

1. INTRODUCTION

Design parameters such as load, mechanical or physical characteristics of materials, geometric characteristics (tolerances of length, shape or position) change in some range with previously unknown (small) deviations from nominal values and can be considered as randomly changing (stochastic) quantities. The deterministic concept implies that the design parameters have known nominal values with which calculations are performed. The ever-present uncertainty in the actual values of the parameters is taken into account by adopting an factor of safety. Compared to the previous one, the probabilistic concept is more advanced and more recent. It originated in the field of electronic equipment in aviation and gradually spread to other areas of technology. In this design approach, the desired reliability of the component (failure-free life) is adopted in advance and then, knowing the statistical laws of distribution of randomly variable construction parameters (factors) on which it depends, the necessary parameter values, e.g. dimensions that ensure the given, pre-adopted reliability for the adopted construction criterion, are determined in a complex calculation or numerical Monte-Carlo (MC) simulation. In our case, the strength or load carrying capacity of the press fit joint is an output quantity that depends on several design parameters that are randomly variable. Finite Element Method (FEM) has established itself as a reliable analysis method, but for more complex (non-linear) problems, the modeling procedure and its execution on a computer require a lot of time and computing resources. Even with the application of model symmetry and parametric programming of the FEM application (Parametric Design Languages or Python scripts), the computational scope of nonlinear simulations is still demanding and valid only for one set of input parameter values. Instead of such a computationally demanding approach (a series of dozens of FEM simulations), here, using a designed experiment for characteristic values of the input parameters, the minimum required number of FEM analyses is performed according to the plan of experiment, and for the obtained set of results (load capacity of the joint, F), the corresponding multidimensional polynomial of higher order (response surface) is determined by regression analysis. The general form of the second-order regression polynomial is given by (1):

$$F(X_1, X_2, \ldots X_k) = b_0 + \sum_{1\le i\le k} b_i X_i + \sum_{1\le i\le j\le k} b_{ij} X_i X_j + \ldots + \sum_{i_1, i_2, \ldots i_k} b_{i1,i2,\ldots ik} \times X_1^{i_1} X_2^{i_2} \ldots X_k^{i_k} \quad (1)$$

where Xi - input variables (factors) and b_0, b_i, b_{ij} are regression coefficients obtained as a solution to the corresponding system of linear equations. After that, by generating randomly variable input variables (X_i) according to the known laws of statistical distribution, in an external program, using the same polynomial (1), a larger number of output variable values are calculated (MC simulation) and the set of results is statistically processed in order to determine the parameters of the output variable distribution (F). The process is schematically, for the general case shown in Figure 1.

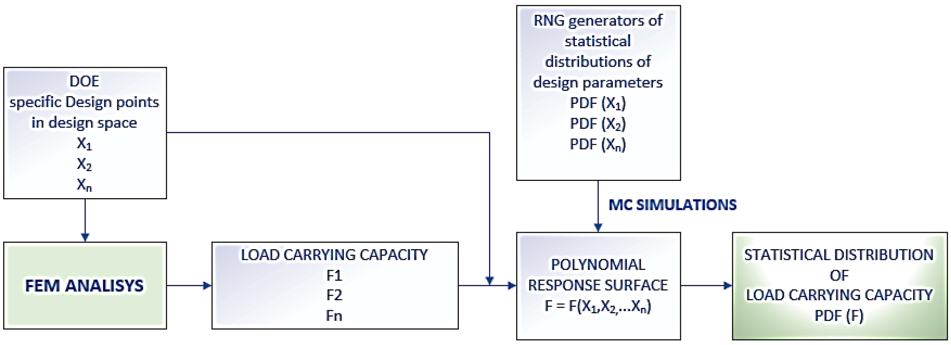


Figure 1. Process of probabilistic analysis of presss fit joints

The adequacy of this meta-model (the degree of agreement between the value of the load-bearing capacity (F) predicted by expression (1) and the value of the same for the case of performing FEM analysis with the same parameters) was confirmed after FEM simulations with randomly generated parameter values using Fisher's test (F distribution). Here, instead of a physical experiment, a computer (FEM) simulation is performed that is completely deterministic so the conditions for statistical tests also differ from the usual ones. Alternatively, instead of the regression polynomial (1) for the set of experimental points in the design space, a multi-dimensional polynomial obtained by expanding the output quantity 'F' in Taylor series in the vicinity of the central point in design space can be used using numerically calculated values of the higher-order derivatives of the output with respect to all input quantities. Although this 'variational technology' approach has some advantages, there are also significant difficulties associated with the numerical calculation of higher-order derivatives in FEM analysis.

2. FEM ANALYSIS OF PRESS FIT JOINT

A typical design of a press fit joint often used in mechanical engineering is shown in Figure 2. Saving on material leads to the use of joints with a bronze gear ring (CuSn12-C) and a cast iron hub (GG25, EN-GJL-250). In addition to the size of the interference, the dispersion of the achieved load capacity in pairs with different materials is also influenced by the elastic moduli E_1 and E_2, Poisson's constants μ_1 and μ_2, the operating temperature (thermal expansion), the dispersion of the of surface roughness, R_{z1} and R_{z2}, the coefficient of adhesion(friction), as well as deviations in the shape and position of both contact surfaces. The actual load-bearing capacity of joint is a random variable that depends on the previously mentioned input quantities (parameters, factors) Due to the complexity and the required minimum number of FEM analyses, the most influential input factors here are the interference (dimensions) and the mechanical properties of the material (E) of both parts in contact with three factors with three levels of values, so the experiment plan requires $N = 3^3 = 27$ FEM analyses.

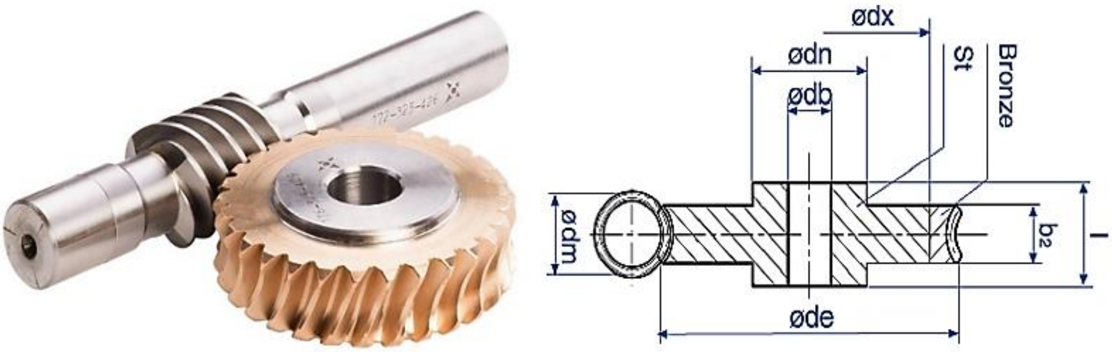
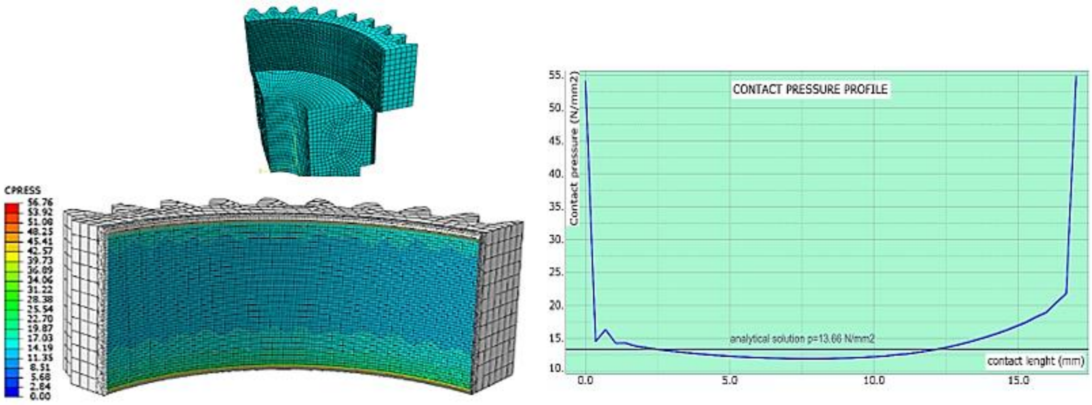


Figure 2. A typical design of a press fit joint, DIN 3975/3976 [1]

In Figure 3, for the nominal values of the parameters (central point of experiment) the distribution of the pressure over the cylindrical contact surface (a) and along the joint (b) is shown,. The axial symmetry and the even number of teeth (z=38) allow the analysis to be performed on 1/4 of the model. Due to the presence of the gear ring, the oscillation of the contact pressure along the contact surface was determined, as well as its significant increase at the exit edges, which is a consequence of the sudden change in stiffness, (b).



(a) (b)

Figure 3. Contact pressure (a) for the nominal values of input parameters, φ59 H7/s6, L=18 mm,

contact pressure along the joint (b)

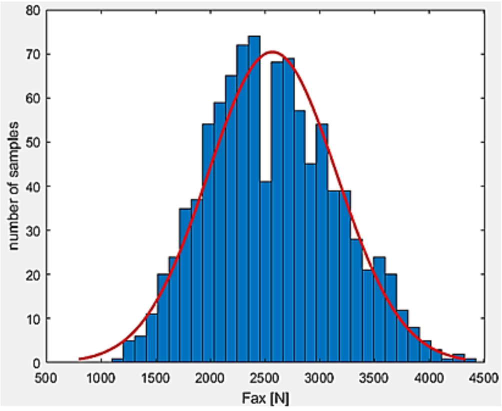
The approximate value of the load carrying capacity of a press-fit joint for cylindrical, axisymmetric geometry and for the elastic stress range can be determined from the theory of elasticity using the well-known Lame formulas (2) which form the basis of the standardized DIN 7190 procedure for the calculation of cylindrical press-fit joints [2]:

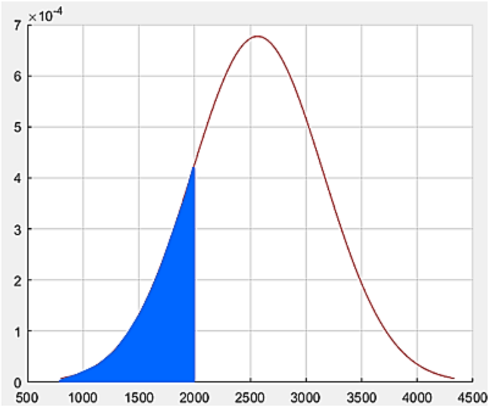
$$F_T = p \cdot A \cdot \mu = \frac{\left[\Delta - 0.8(R_{z1} + R_{z1}) - f(T, c_1, c_2)\right] \cdot \pi \cdot L \cdot \mu}{\frac{1}{E_I}\left(1 - \nu_I\right) + \frac{1}{E_o}\left(\frac{D_O^2 + D^2}{D_O^2 - D^2} + \nu_O\right)}; \qquad (2)$$

Formula (2), valid for coupled thick-walled cylinders of infinite length in the elastic region, gives approximate values for finite-length joints because they do not take into account a significant increase in pressure at the ends of the joint. In this case, it was found that Lame's analytical formula for contact pressure matches well with the results of the FEM analysis (Figure 3). Therefore, it is acceptable to use Lame's formula with 6 random parameters as the ‘response surface’ for the case of an axisymmetric press fit joint instead of the quadratic 3D polynomial (1) obtained by fitting a series of FEM simulations. It gives a nonlinear 6D 'response surface' and allows variations of six input (random) parameters to be taken into account in the MC simulation of the load carrying capacity distribution as an output quantity.

3. MONTE-CARLO SIMULATIONS

The following distributions were adopted for the selected parameters (factors):
• interference - normal distribution, with the geometric interference reduced by the shear size of the peaks of the asperities for which DIN 7190 recommends a value of $0.8*(R_{za}+R_{zi})$ where R_z is the height of the surface asperities on the contact surfaces.
• elastic moduli of joint materials (E_1 and E_2) - normal distribution with a coefficient of variation $\beta = 0.05$ for both materials.
In our own code, the mentioned distributions were implemented using a Pseudo-Random-Number-Generators and then, using a series of generated random values, expression (1) was evaluated as a 'response surface' for the axial force F as the output quantity. In this way, a set of results is obtained, the statistical processing of which determines the parameters of the distribution of the output quantity - the axial load carrying capacity of the press fit joint (μ_{Fax}=2.57 kN, σ_{Fax}=589.13 N, β_{Fax} = 23%). The histogram of the distribution of force F is shown in Figure 4 (a).





(a) (b)

Figure 4. Histogram of load carrying capacity F_{ax}, (a), probability of F_{ax} to be less then 2000 N (b)

4. CONCLUSION

Statistical processing of the data obtained according to the procedure in Figure 1. has determined an approximately normal distribution of the load carrying capacity F of the pair with parameters $F(F_m,\sigma_m)$. It allows us to determine the values of dispersion, the probability of the load capacity in an arbitrary part of the interval Fmin-F_{max}, the probability that the load capacity is outside the predicted interval (failure) and also the sensitivity of the output quantity to variations of each of the input quantities. This provides the possibility of a probabilistic estimate of the load carrying capacity value, which is certainly more useful than the unknown load capacity increased by an empirical factor of safety as is done in the deterministic concept. Procedure described in Figure 1. is general and often the only possible in the general case, when we do not have valid analytical solutions. It is applicable to a wide class of problems that are analyzed by FEM analysis.

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