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DYNAMIC STABILITY OF A SLIGHTLY CURVED PIPE RESTING ON A WINKLER ELASTIC FOUNDATION

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Abstract: This article investigates the stability of a slightly curved pipe conveying fluid, which is supported by a Winkler elastic foundation and fixed (cantilevered) at its left end. The Galerkin method is employed to solve this problem. The primary goal is to determine how the rigidity of the Winkler elastic foundation and the density of the fluid being conveyed affect the critical fluid velocity – the velocity at which the pipe becomes unstable. The numerical analysis presented in this work includes graphs that illustrate how the critical fluid velocity changes with the rigidity of the Winkler elastic foundation for various fluid densities. The results show that the stability of the system is influenced by both the elastic foundation's rigidity and the conveyed fluid's density.
Key words: Slightly curved pipes, Fluid, Critical velocity, Dynamic stability, Winkler elastic foundation, Galerkin method

INTRODUCTION

In engineering practice, a frequently used model for the soil foundation due to its simplicity is the Winkler model. In this model, the elastic foundation is modeled by a multitude of closely spaced linear springs.
Lotati and Kornecki [1] considered a clamped pipe resting on a Winkler elastic foundation, as well as a pipe simply supported at both ends. They proved that in both cases, the elastic foundation has a stabilizing effect. This effect manifests as an increase in the critical fluid velocity.
In 1970, Stein and Tobriner [2] also studied the vibrations of a pipe conveying fluid and resting on a Winkler elastic foundation.
P. Dzhondzhorov [3] investigated the stability of a pipeline clamped at one end and resting on an elastic foundation whose stiffness varies according to a linear law as a function of the axial coordinate. Linear functions for which the loading diagrams have equal areas were considered. It was established that for all such functions, the elastic foundation has a stabilizing effect on the pipeline. For a foundation where the stiffness variation is represented by a triangle with the maximum value of the Winkler elastic foundation coefficient at the free end of the pipe, this effect is the greatest. It was found that with an increase in the slope of the linear stiffness function of the soil foundation, its stabilizing effect also increases.
Other studies in the field of pipelines resting on an elastic foundation with varying stiffness were carried out by P. Dzhondzhorov, V. Vasilev, and V. Dzhupanov [4]. They considered a clamped pipe resting on an elastic foundation. The stiffness of the elastic foundation varies according to a quadratic parabola. The obtained critical velocities were compared with the critical velocities for the same pipe but without the presence of an elastic foundation. It was proven that the elastic foundation has a stabilizing effect on the system.
P. Dzhondzhorov [5] investigated the influence of a Winkler elastic foundation, applied to a part of the pipeline, on the dynamic stability of a clamped pipe conveying fluid. The author studied how the critical velocity of the conveyed fluid changes depending on the length and position of the section with the elastic foundation, as well as according to the stiffness of the foundation. The studies show that the elastic foundation can have a stabilizing or destabilizing effect on the system.
Sv. Lilkova-Markova [6] investigated the dynamic stability of a pipeline with flowing fluid. One end is free, and the other is clamped, simply supported, or free. A part of the pipeline rests on a Winkler elastic foundation. In all considered cases, the critical velocity of the flowing fluid was determined.

A recent study [7] examined the stability of fluid-conveying pipes supported by a two-parameter elastic foundation. Using the Euler-Bernoulli beam model under various boundary conditions, the authors applied the Differential Transform Method. Their findings show that the elastic foundation contributes to system stability, and increasing its parameters raises the critical flow velocity.
In a further recent investigation [8], the dynamic stability of a cantilevered pipe subjected to a lateral distributed load was analyzed. The Differential Quadrature Method was utilized in the analysis.
Ding and Ji [9] provide an extensive overview of recent advancements in vibration control techniques for fluid-conveying pipes.
The broad applications of nanoscale tubes in both scientific and industrial contexts have driven significant research efforts, as noted in [10] and [11]. To address the challenges and high costs associated with nanoscale experimentation, studies on fluid-structure interaction in carbon nanotubes often rely on continuum elastic models, including Euler and Timoshenko beam theories.

PROBLEM FORMULATION

The differential equation describing the transverse vibrations of an Euler-Bernoulli pipe with an initial geometric imperfection resting on the Winkler elastic foundation (Fig.1) is as follows [12]:

$$EI\frac{\partial^4 w}{\partial x^4}+\left(m_f+m_p\right)\frac{\partial^2 w}{\partial t^2}+2m_f V\frac{\partial^2 w}{\partial t\,\partial x}+m_f V^2\frac{\partial^2 w}{\partial x^2}+ \\ +kw=\frac{EA}{l}\int_0^l\left[\frac{dZ}{dx}\frac{\partial w}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right]dx\left(\frac{\partial^2 w}{\partial x^2}+\frac{d^2 Z}{dx^2}\right), \tag{1}$$

where EI is the bending rigidity and EA is the axial rigidity of the pipe, m_f is the mass of the fluid per unit length of the pipe, m_p is the mass of the pipe per unit length, k is the rigidity of the Winkler elastic foundation, x is the longitudinal coordinate along the pipe axis, t is the time and l is the length of the pipe. $w(x,t)$ is the transverse displacement of the pipe's cross-section. The remaining system characteristics are the velocity of the conveyed fluid V and the function of the initial imperfection of the axis of the pipe $Z(x)$.

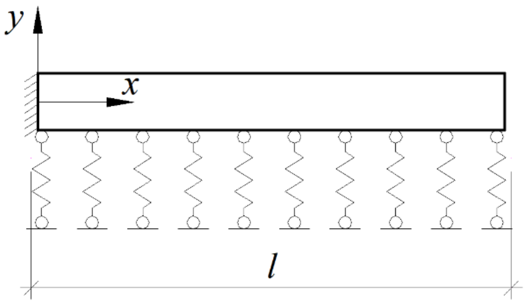

Fig. 1. *Static scheme of the pipe*

The assumption of a small rise to span ratio implies that a geometrically imperfect pipe is effectively a shallow arch [12].
In the present study for the function of initial imperfection $Z(x)$ is chosen a sinusoidal function satisfying the boundary conditions of the pipe depicted in Fig.1

$$Z(x) = b(1 - \cos x) \tag{2}$$

The boundary conditions for a cantilevered at its left end pipe (Fig.1) are as follows:

$$w(0,t) = 0 \text{ and } \frac{\partial w(0,t)}{\partial x} = 0 \tag{3}$$

To facilitate the analysis, the equation is non-dimensionalized by defining the following terms:

$$\eta = \frac{w}{r};\ \xi = \frac{x}{l};\ z = \frac{Z}{r};\ u = Vl\sqrt{\frac{m_f}{EI}};\ \beta = \frac{m_f}{m_f + m_p};\ \tau = \frac{t}{l^2}\sqrt{\frac{EI}{m_f + m_p}};\ \bar{k} = \frac{kl^4}{EI} \tag{4}$$

In (4) r represents the radius of gyration of the cross-section on the pipe.
The non- dimensional form of the differential equation (1) is as follows:

$$\frac{\partial^4 \eta}{\partial \xi^4} + u^2 \frac{\partial^2 \eta}{\partial \xi^2} + 2\sqrt{\beta} u \frac{\partial^2 \eta}{\partial \tau \partial \xi} + \frac{\partial^2 \eta}{\partial \tau^2} + \bar{k}\eta - \int_0^1 \left[\frac{dz}{d\xi}\frac{\partial \eta}{\partial \xi} + \frac{1}{2}\left(\frac{\partial \eta}{\partial \xi}\right)^2 \right] d\xi \left(\frac{\partial^2 \eta}{\partial \xi^2} + \frac{d^2 z}{d\xi^2} \right) = 0 \tag{5}$$

After the transformation to dimensionless variables, the boundary conditions (3) become:

$$\eta(0,\tau) = 0 \text{ and } \frac{\partial \eta(0,\tau)}{\partial \xi} = 0 \tag{6}$$

To solve this differential equation (1), the Galerkin method is applied.
The functions $\eta(\xi,\tau)$ is sought in the following form:

$$\eta(\xi,\tau) = \sum_{i=1}^{n} q_i(\tau) y_i(\xi) \tag{7}$$

In (7) $q_i(\tau)$ are generalized coordinates and $y_i(\xi) = \frac{Y_i(x)}{r}$ are basis functions, where $Y_i(x)$ are the eigenfunctions of the pipe. For the pipe in Fig.1 the eigenfunctions $Y_i(x)$ have the following form [13]

$$Y_i(x) = A_i \left[\sin\frac{\lambda_i x}{l} - \sinh\frac{\lambda_i x}{l} - \frac{\sin\lambda_i + \sinh\lambda_i}{\cos\lambda_i + \cosh\lambda_i}\left(\cos\frac{\lambda_i x}{l} - \cosh\frac{\lambda_i x}{l} \right) \right] \tag{8}$$

where $\cos\lambda_i \cosh\lambda_i + 1 = 0$ and $\lambda_i < \lambda_{i+1}$, $i = 1,2,...,n$. The constants A_i are chosen so that the eigenfunctions are orthogonal. The condition for orthogonality of the functions is represented by the following expression:

$$\int_0^l Y_i(x) Y_k(x) dx = \delta_{ik} \tag{9}$$

In (9) δ_{ik} is Kronecker delta.

Since expression (7) is not the exact solution of the differential equation (1), substituting (7) into (1) results in a residual function $R(\xi,\tau)$.

$$\sum_{i=1}^{n}\left\{\frac{\partial^4 y_i}{\partial \xi^4}q_i + y_i\ddot{q}_i + 2u\sqrt{\beta}\frac{\partial y_i}{\partial \xi}\dot{q}_i + u^2\frac{\partial^2 y_i}{\partial \xi^2}q_i + \right.$$
$$\left. +\bar{k}\,y_i q_i - \int_0^l\left[\frac{dz}{d\xi}\frac{\partial y_i}{\partial \xi}q_i + \frac{1}{2}\left(\frac{\partial y_i}{\partial \xi}q_i\right)^2\right]d\xi\left(\frac{\partial^2 y_i}{\partial \xi^2}q_i + \frac{d^2 z}{d\xi^2}\right)\right\} = R(\xi,\tau)\,, \tag{10}$$

The Galerkin method determines the unknown functions $q_i(\tau)$ by minimizing the residual function $R(\xi,\tau)$ over the region $\xi \in [0;1]$. This condition is expressed as:

$$\int_0^1 R(\xi,\tau)y_k(\xi)d\xi = 0; \quad k = 1,...,n \tag{11}$$

The condition (11) represents the requirement that the residual function $R(\xi,\tau)$ has to be orthogonal to all basis functions $y_i(\xi)$. Substituting (10) into (11) leads to a system of n differential equations with respect to the unknown functions $q_i(\tau)$. This system has the following expression:

$$\sum_{i=1}^{n}\int_0^1\left\{\frac{\partial^4 y_i}{\partial \xi^4}q_i + y_i\ddot{q}_i + 2u\sqrt{\beta}\frac{\partial y_i}{\partial \xi}\dot{q}_i + u^2\frac{\partial^2 y_i}{\partial \xi^2}q_i + \right.$$
$$\left. +\bar{k}\,y_i q_i - \int_0^l\left[\frac{dz}{d\xi}\frac{\partial y_i}{\partial \xi}q_i + \frac{1}{2}\left(\frac{\partial y_i}{\partial \xi}q_i\right)^2\right]d\xi\left(\frac{\partial^2 y_i}{\partial \xi^2}q_i + \frac{d^2 z}{d\xi^2}\right)\right\}y_k d\xi = 0\,, \tag{12}$$

These nonlinear differential equations can be solved using the seventh-eight order continuous Runge-Kutta method.
The non-linear dynamical behavior of the system is investigated based on the bifurcation diagrams.

RESULTS AND DISCUSSION

This paper investigates the influence of the Winkler elastic foundation's rigidity on the stability of the system by employing the Galerkin method, as detailed in the preceding section. Numerical simulations are conducted for the beam depicted in Fig. 1. The system's material and geometric properties include: the modulus of elasticity of the pipe material $E = 210\,GPa$, the density of the pipe’s material $\rho = 7800\,kg/m^3$, the outer and inner radii of the pipe's cross-section ($R_{out} = 0.014\,m$ and $R_{in} = 0.012\,m$, respectively), and the mass ratio $\beta = 0{,}3$.
The bifurcation diagram for the right end cross-section of the pipe is shown in Figure 2. This diagram was generated using $\bar{k} = 30$, with the following initial conditions for the cross-section: $\eta(1,0) = 10^{-6}$ and $\dot{\eta}(1,0) = 10^{-4}$. The Hopf bifurcation point signifies the loss of system stability, and the corresponding velocity $u_{cr} = 3{,}25$ is termed the critical velocity. Although a second bifurcation point appears at $u_{cr} = 4{,}34$, the first critical velocity holds the primary practical significance.

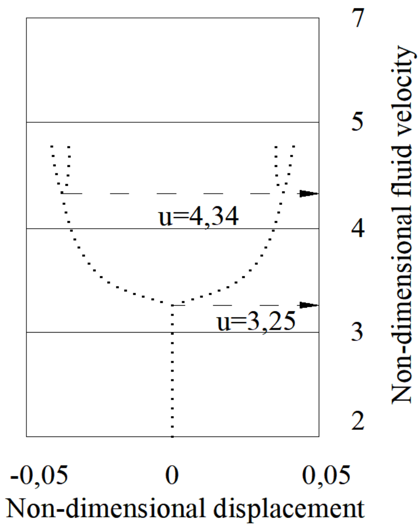

Fig. 2. *Bifurcation diagram of the right end cross-section for* $\beta = 0{,}3$ *and* $\bar{k} = 30$

Figure 3 illustrates the relationship between the non-dimensional critical velocity u_{cr} and the dimensionless rigidity of the Winkler elastic foundation $\bar{k}$ for two different mass ratios β.

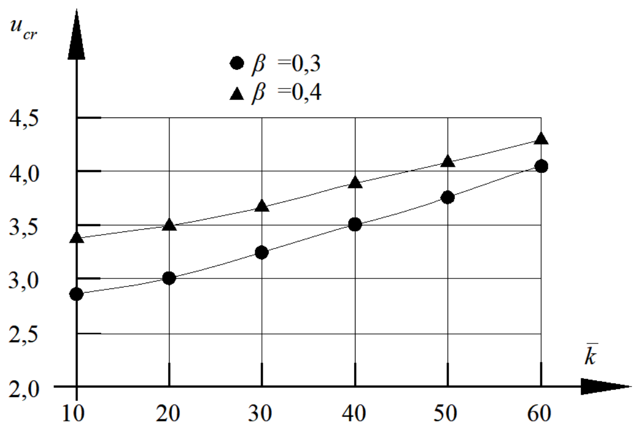

Fig. 3. *The relation between the non-dimensional critical velocity* u_{cr} *and the dimensionless rigidity of the Winkler elastic foundation*

CONCLUSIONS

In this paper, a slightly curved pipe conveying fluid and resting on a Winkler elastic foundation was investigated. The differential equation describing the lateral vibration of the pipe was solved using the Galerkin method, a widely used and powerful fundamental tool for the numerical solution of differential equations. The most crucial aspect of obtaining a solution through this method is the careful selection of the number of modes to include in the approximate solution, ensuring that the neglected modes do not significantly affect the solution's precision [12].

Based on the numerical investigations presented in this article, several major conclusions can be made.

- Figure 2 illustrates the findings of the numerical analysis, revealing that the pipe's stability could be compromised via a Hopf bifurcation, with subsequent bifurcations also occurring.

- The diagram in Figure 3 demonstrates that for the pipe under study, a stiffer Winkler elastic foundation enhances the system's stability, leading to higher critical fluid velocities.
- Numerical studies also suggest that a larger mass ratio β contributes to stabilizing the system.

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