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ANALYTICAL SOLUTION OF SPATIAL DISTRIBUTION OF AIR POLLUTION FROM POINT SOURCES

Stevo K. Jaćimovski[1], Jelena Lamovec[1], Dušan I. Ilić[2], Jovan P. Šetrajčić[3]
[1]University of Criminal Investigation and Police Studies, Zemun – Belgrade, Serbia,
e-mail: jacimovskis@gmail.com, jelena.lamovec@kpu.edu.rs;
[2]University of Novi Sad, Faculty of Technical Sciences, Novi Sad, Serbia,
e-mail: dusilic69@gmail.com
[3]Academy of Sciences and Arts of the Republic of Srpska, Banja Luka, Republic of Srpska – Bosnia and Herzegovina, e-mail: jovan.setrajcic@gmail.com

Abstract: This paper analyses the distribution of particulate air pollution originating from a specific location, modeled as a point source with periodic operation. The extent of the observed pollution is considered in one direction, accounting for diffusion and absorption processes in relation to current air flow conditions. The diffusion equation describing these processes is a partial differential equation and must ultimately be solved numerically for realistic parameters.

Key words: Periodic point source of pollution; particulate air pollution; diffusion-absorption processes; spatial distribution of pollution particles.

INTRODUCTION

The problem of environmental pollution is becoming acute due to rapid technical and technological development, especially in the air and particularly in large industrial centers.

Air pollution refers to the state of the atmosphere in which substances are present at concentrations above established normal values and are harmful to the environment and humans. This pollution results from natural phenomena and human activities and can therefore be divided into pollution from natural sources and pollution from anthropogenic sources [1].

Natural sources of air pollution have always existed in the biosphere and include: deflation—the transfer of soil and sand, usually in desert and forest-steppe zones; smoke from forest and steppe fires, which contains CO, soot, resin, tar, and other substances; volcanoes, as strong eruptions release large amounts of dust, gases, SO_2, CO_2, and more; mineral and thermal springs, which can emit CO_2, H_2S, methane, and similar gases; cosmic dust, especially if radioactive; large marine areas, which can release CO_2, CO, H_2S, chlorides, and other compounds; and natural disasters, such as storms and earthquakes, that are associated with significant emissions of air pollutants.

Anthropogenic sources of pollution can be classified as stationary or mobile sources. Mobile sources include motor vehicles with internal combustion engines. Indoor sources of air pollution include emissions from combustion and heating, as well as emissions from various materials or substances, such as the evaporation of organic compounds, various synthetic chemicals, and tobacco smoke.

Based on the number of sources and their spatial distribution, pollutants can be classified as individual, point (static or mobile), group (static or mobile), or linear. Depending on the type of emission, pollutant sources are divided into particle and gas emitters.

The greatest importance of understanding and predicting the spread of air pollution lies in its negative impact on human health. There is no direct link between the health of the population and the state of the environment, as various other factors also influence public health. However, the environmental factor, along with heredity and individual lifestyle characteristics, is important for the availability and effectiveness of health services. Air quality in urban areas is considered to have a greater impact on public health than other environmental factors, and outdoor air pollutants are among the most significant causes of health problems overall. Numerous epidemiological studies have clearly demonstrated that air pollution in the form of irrespirable particles is associated with increased morbidity and mortality from respiratory and cardiovascular diseases. The increase in certain diseases (cardiovascular diseases, hypertension, respiratory diseases, malignancies, infectious diseases, and parasitic infestations) may be due to modern lifestyles as well as air pollution. The APHEA project, "Air Pollution and Health: A European Approach," is one of the epidemiological studies monitoring the short-term effects of air pollution using various health parameters. Particular attention is given to daily variability in lung function, frequency of hospitalization, and mortality. In Paris, the risk of death from respiratory diseases increased by 17% with a 100 µg/m³ rise in particulate matter. Particulate matter, black smoke, and sulfur dioxide (SO_2) are associated with emergency admissions for respiratory diseases. A Spanish study found that oxides, particularly nitrogen dioxide (NO_2) and ozone, are linked to mortality from cardiovascular disease, especially in summer. The carcinogenic effect of many toxic compounds has also been proven.

To date, several hundred different pollutants have been identified (the most important and frequent are listed in Table 1), and the possible formation of new pollutants under the influence of sunlight and electrical discharge should be monitored. Air quality is determined by the concentration of pollutants in the air or by their deposition on surfaces over time. Pollutant concentration is the mass, volume, or quantity of substances contained in a given volume or mass of air.

Table 1. *Pollutant compounds and sources of pollution*

Pollutant compound	The main source of pollution
Sulfur dioxide SO_2	Combustion of coal, oil, heavy metallurgy and related
Hydrogen sulfide H_2S	Chemical processes, refineries
Carbon monoxide CO	Combustion
Nitrogen oxides NO_x	Combustion
$C_n H_{n+2}$	Evaporation of liquid fuels and exhaust gases
Soot	Burning
Particle suspensions	Technological processes, quarries, cement production
Vapors of organic compounds	Chemical processes, oil production, gasoline distribution

In addition to the concentration of pollutant compounds from pollutant sources, air quality in an area is also determined by meteorological elements and conditions: air pressure, wind direction and speed, eddy currents, humidity, fog, rainfall, air temperature, and temperature inversion. The highest concentration of pollutants spreads horizontally in the direction of the wind. During periods of "calm", when there is no air movement, all pollutants remain in the populated area. In the lower layers of the atmosphere, the air is warmer and moves towards the upper, colder layers, which allows normal dispersion. However, rapid cooling leads to a temperature inversion. The air at ground level is colder than the air in higher layers, so dispersion is not possible. Low air pressure, calm conditions, high humidity, haze, and temperature inversion reduce the dispersion of pollutants at height and distance, keeping them at ground level and concentrating them near the source of pollution. Smog can form along with compounds that are extremely toxic and dangerous to human health. The level of pollutants is determined by measurements [1,2].

Air pollution can be transported over long distances from its source. The distance depends on the rate of dispersion (diffusion) of the polluted air masses and the rate of deposition of the polluting compounds. For these reasons, it is necessary to predict the dispersion of air pollution under given conditions. This is achieved through modeling, which we have carried out and present the results in this paper.

To estimate the dispersion of air pollution, it is necessary to know the quality (physical and physicochemical properties) of the pollutants and the quantity (amount/contribution), as the effect of each pollutant depends on these factors.

AIR POLLUTION PROPAGATION MODEL

The distribution of particles in the atmosphere is of great interest to many areas of human activity. Growing awareness of the importance of these processes has enabled continuous measurement for monitoring and data collection. Empirical models of air pollution have been created based on these measurement results. Subsequently, considerable efforts have been made to develop methods for analyzing particle diffusion in the atmosphere. As a result, research findings are used to predict the consequences of emergencies, including various accidents involving the dispersion of hazardous compounds. Despite significant efforts, there are no generally recognized models for analyzing the distribution of air pollution. This is objectively due to the diversity and complexity of these processes. Consequently, there are many different types of models. The main characteristic by which the models are classified is whether they are empirical or theoretical [3].

The classical empirical model is the Pasquill-Gifford's model [4], while the Berlyander's model is an example of a theoretical model of turbulent diffusion [2,3]. In empirical models, the physics of atmospheric processes is either not considered or only very roughly approximated. Today, semi-empirical models provide the best results, though only to a limited extent. In these models, the empirical approach is supplemented by well-developed mathematical and numerical methods that allow the analysis of very complex situations and enable the synthesis of results from different experiments, such as meteorological and diffusion conditions, simultaneously.

However, the models also differ significantly depending on the mathematical apparatus used. Empirical models use explicit forms derived from numerous measurements and can be easily processed on microcomputers. Semi-empirical methods employ procedures for the numerical solution of partial differential equations that describe the observed spatial conditions. Theoretical models use various methods, such as dimensional analysis, analytical solutions of partial differential equations, and simulation methods such as Monte Carlo. It should be noted that, in the search for analytical solutions, mainly stationary processes are analyzed, whereas the numerical approach also allows for the analysis of non-stationary processes.

Current models can be categorized according to the spatial and temporal dimensions of their application: local, regional, and global. Local methods analyze air pollution in areas of up to ten kilometers over periods ranging from a few minutes to several hours. Regional models analyze the spread of air pollution within a radius of several tens to several hundred kilometers, with time intervals from several hours to several days. Global models address the spread of air pollution over areas of several hundred to several thousand kilometers, with the time dimension extending to several weeks [5].

Regarding the mathematical processing and treatment of the processes mentioned the models can be categorized into three types: Gaussian, Eulerian, and Lagrangeian models. The equations in these models are derived using different approximation approaches to solve the equations of turbulent diffusion. The simplest model for calculating the concentration of particles or pollutants on the ground is the statistical Gaussian model. This model assumes that particles emitted from continuous point sources form a plume in which the particle distribution follows a Gauss normal distribution (Fig. 1).

- In this model, wind speed is taken as a parameter from the nearest meteorological station, and it is assumed that this speed is maintained during the time required to transport the dirt particles over a distance of 20 – 30 km. The disadvantages of this model are:
- It assumes that the vertical and horizontal propagation of particles are independent;
- Wind speed components are not considered—a linear path of the particles is assumed;
- The shape of the dispersion, which describes the spread of the particle cloud, depends on the distance from the source;
- Initially, it is assumed that the concentration of particles at the source is infinitely high;
- This model cannot be used to calculate particle concentrations for low wind speeds (less than 1 m/s).

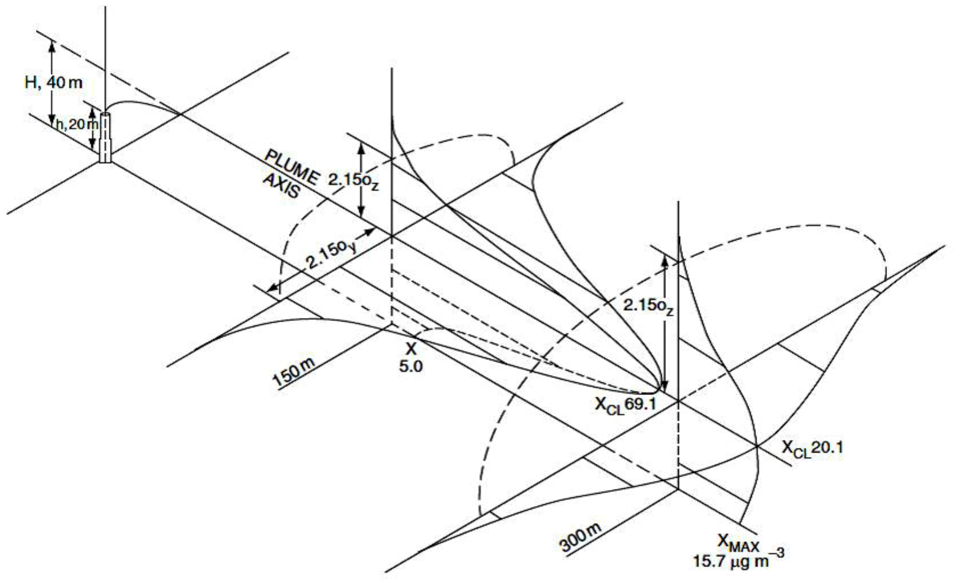

Fig. 1. Gaussian model of particle emission from a point source [2,6]

Euler's model is based on the solution of the semi-empirical equation for turbulent diffusion [5]:

$$\frac{\partial q}{\partial t} = -\vec{V}\cdot\nabla q + \frac{\partial}{\partial x}\left(k_x\frac{\partial q}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y\frac{\partial q}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z\frac{\partial q}{\partial z}\right), \qquad (1)$$

where $\vec{V}\cdot\nabla q$ defines the corresponding component of the equation, k_x, $k_y$and$k_z$are the turbulent diffusion coefficients, and q is the mean value of the impurity concentration.

Euler's model of impurity diffusion accounts for the fundamental characteristics of this process: the transport of particles along the direction of convection or flow. It considers turbulent diffusion, convection, spatio-temporal inhomogeneity of scattering parameters, interaction of particles with the Earth's surface, and other factors. This model enables calculation of particle concentration in an area 50 km from a point or other sources that continuously emit pollutants, under any meteorological conditions and over ground surfaces of any relief. It also provides data on pollutant concentrations in low wind speeds, even during stable weather. The model must yield results for the spatial distribution of contaminant concentrations that are consistent with empirical models. The turbulence coefficients depend on wind speed, atmospheric conditions, surface relief, the distance pollutants travel from the observation point, and the position of the pollution source:

$$k_z = U_z\frac{\kappa^2 h_{SBL}}{\ln(z_r/z_0)}\left(\frac{x}{x_1}\right)^b \quad ; \quad k_y = k_z P^{-1}, \qquad (2)$$

where U_z represents the wind speed measured at height z_r above the ground, z_0 is the characteristic of the surface relief, κ is the Karman constant, h_{SBL} is the height near the base level of the atmospheric layer, x is the distance travelled by the impurity particle from the source to the observation point, x_1 is the standard distance corresponding to 1000 m, b is the exponent of the power function, and P is the turbulence anisotropy coefficient.

In general, the task of mathematically predicting air pollution is defined as solving the corresponding turbulent diffusion equation with appropriate initial and boundary conditions. Equation (2) describes the spatial distribution of mean concentrations and their change over time, and can therefore be regarded as a prediction equation.

The analytical approach to solving the atmospheric diffusion equations is highly complex and can only be addressed through a series of approximations. Difficulties in obtaining analytical solutions arise even in simple cases. However, if it is assumed that wind speed varies logarithmically with height and the linear and/or energy dependence of the coefficients is removed, finding an analytical solution becomes easier. Studying atmospheric diffusion over a surface with complex relief requires accounting for the variation of many meteorological elements, each of which complicates and often renders an analytical solution practically impossible. Therefore, numerical approaches are the only viable method for integrating the turbulent diffusion equations. Regardless of the capacity and capabilities of computer resources, the parameters of the equation must be chosen carefully to successfully determine the distribution of air pollution [7].

SOLUTION OF A QUASI 1D PROBLEM

This section analyses the dispersion of air pollutants near a point source. The spatial distribution of the pollutant is examined when diffusion, absorption, and air flow processes are present. We focus on a quasi-linear, extended one-dimensional problem, as the aim is to estimate the spatio-temporal distribution of concentration from the air pollution source according to the specified pollution dynamics. Although the choice of a one-dimensional problem is an idealisation, it is justified by the inclusion of virtually all relevant physical mechanisms involved in the displacement of pollutant particles [8]. Furthermore, we have introduced the source function in an analytical form that accurately reflects the operation of an industrial plant. The turbulent diffusion equation for the case under consideration [9] is:

$$\frac{\partial q}{\partial t}+u\frac{\partial q}{\partial x}-b^2\frac{\partial^2 q}{\partial x^2}+cq=A\varphi(t)\delta(x), \tag{3}$$

where u is the wind speed in the direction of the x-axis, b is the impurity diffusion coefficient, $A\varphi(t)$ is a function of the amount of emitted impurity particles per unit time and$c$is the coefficient characterising the absorption of impurity particles (which can be determined after chemical analysis of the atmospheric processes leading to the transformation and absorption of impurities).

To make the process described by equation (3) more realistic, we assume that $\varphi(t)$ is a periodic function, defined by the following form [7]:

$$\varphi(t)=\begin{cases}0, & t\in(0,t_1), \quad t_1<0\,;\\ 1, & t\in(t_1,t_2), \quad t_2<2T\,;\\ 0, & t\in(t_2,2T), \quad t_2>t_1\;.\end{cases} \tag{4}$$

Therefore, the point source emits toxic compounds during the intervalt_2-t_1. In this case, the boundary and initial conditions are:$q(\pm\infty,t)=0$and$q(x,\pm\infty)=0$. It should also be noted that the postulated equations describe the meanconcentration values. The following parameter values are assumed in equation (3): the accepted value for the diffusion coefficient refers to CO_2. Using a numerical approach with the mathematics package *Mathematica*, the required solution was obtained and is shown graphically in the diagrams in Figure 2.

Fig. 2a shows the spatio-temporal distribution for a wind speed of 1 m/s, while Fig. 2b shows the same distribution for a wind speed of 4 m/s. Fig. 2c presents both cases together for easier comparison. The concentration decreases over time, more slowly at higher wind speeds. Additionally, the concentration decreases with distance from the source (in this example, 5 mg/m^3 for a distance of 3 km).

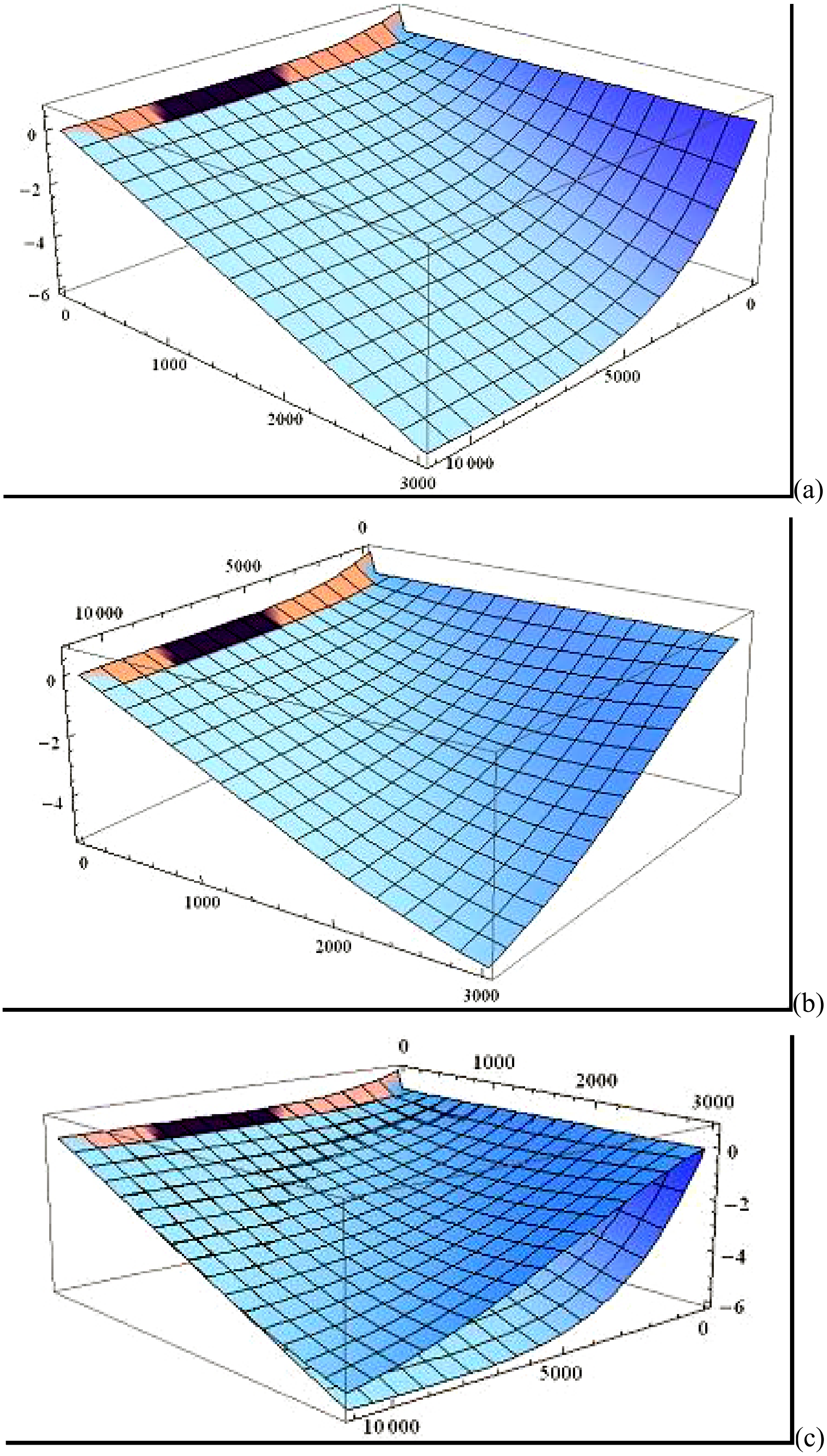

Fig. 2. *Spatial and temporal distribution of impurity concentrations for wind speeds of 1 m/s (a) and 4 m/s (b). Both cases are shown together in (c) for easier comparison*

CONCLUSION

The problem of air pollution is closely linked to the development of industry and transport. Environmental monitoring, particularly of the atmosphere, is carried out in all countries. The effectiveness of this process depends on two components: direct monitoring, which involves measuring air quality using specialized instruments integrated into a unified system that forms the basis of atmospheric monitoring; and indirect monitoring, which involves theoretical modeling of air pollution. This modeling enables the determination of toxic compound concentrations in areas not covered by direct measurements and allows the prediction of pollution distribution in the event of an incident. Thus, modeling is included as part of the overall air pollution monitoring system.

With improved mathematical methods for analyzing the transfer and dispersion of gaseous, liquid, and solid components of air pollution, theoretical modeling is becoming an increasingly effective tool for studying the state of the atmosphere. Determining the level of air pollution by applying an appropriate distribution model allows simulation of the dispersion of toxic compounds for different assumed emission intensities under topological, urban, and meteorological conditions.

With a theoretical approach and the development of a suitable mathematical-physical model, a realistic representation of the dispersion of pollutant particles into the surrounding atmosphere can be achieved, providing a basis for assessing hazard potential. Using a simplified one-dimensional model of a periodic (industrial) point pollutant, we have visualized the spatio-temporal distribution of pollutant concentration downwind (for two different velocities) from the source to a point 3 km away. We have shown that the pollutant concentration decreases over time from the moment the emission stops, and that this decrease is slower at higher wind speeds. Additionally, the pollutant concentration decreases almost linearly with distance from the source.

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