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MONTE CARLO BASED PRECISION ANALYSIS OF A LONG-RANGE ARTILLERY PROJECTILE

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Abstract: *The paper presents a preliminary assessment of the precision measures on the conceptual design of a long-range artillery projectile. The precision measures were calculated using the Monte Carlo method based on the projectile trajectory elements obtained using the flight model. The precision testing procedure is based on numerous simulations of projectile flight with six degrees of freedom, adjusted with selected perturbations of the initial firing conditions. Monte Carlo simulations consisted of 200 virtual firings with standard deviations of initial conditions. The initial conditions consisted of the standard deviation of the: muzzle velocity for different propellant charges, mass of the projectile, initial elevation and azimuth angles and estimated values of the angular velocities around transverse axes. The results of the precision expressed through probable deviations showed good agreement with referenced data of related artillery systems.*

Keywords: *Monte Carlo Method, Long-Range Artillery Projectile, Precision, Probable Deviation, Standard Deviation.*

1. INTRODUCTION

Contemporary artillery systems demand continuous evolution due to extend range, maintain and improve precision and accuracy and enhance operational effectiveness [1, 2]. The global trend is the standardization of mail caliber systems for tactical artillery support, particularly the 155 mm caliber, which has become widely adopted across many countries due to its favorable balance between range, payload, and system compatibility [1]. The artillery system 152 mm as artillery system with similar ballistics characteristics, also depends on technological and standardization design progress.

The research investigates the precision of the conceptual design of velocity enhanced long-range artillery projectile (V-LAP) 155 mm, as part of the ballistic performance.

The objective of this work is to provide a solid assessment of flight performance and precision to identify concurrent design directions for novel long-range artillery projectiles, based on reliable methods.

2. METHODOLOGY

The research consists of the flight model, Monte Carlo model for assessment of the deviations and determination of the precision measures, performed on the conceptual design of V-LAP 155 mm.

2.1. Conceptual Design of Projectile

The final phase of the projectile trajectory analysis is the determination of the precision and efficiency metrics of the projectile model using Monte Carlo simulations [3,4,5], Fig. 1. Due to determination of the trajectory parameters, it is necessary to employ the flight model.

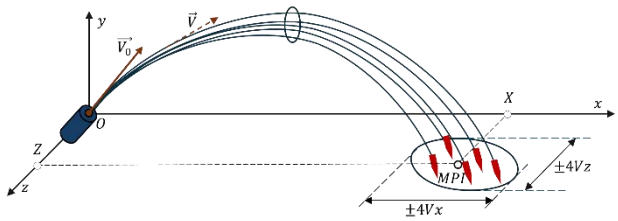


Figure 1. Exterior Ballistics Range Firing

This procedure simulates exterior ballistic range testing, during which the trajectory elements of an unguided projectile are measured [3]. Exterior ballistics firing in these tests varies depending on the type of equipment and the objective of the testing. Range firing is conducted

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using high-explosive projectiles, like the one proposed in this model; their impact can then be monitored and recorded from a safe distance [3,4,5].

The study was performed on the conceptual design of long-range projectile, developed to improve the maximal ranges for artillery system with top propellant charges, Fig. 2.

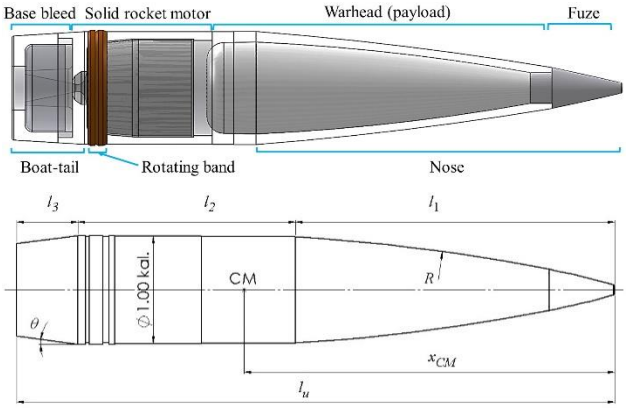


Figure 2. Conceptual design of long-range projectile

2.2. Flight Model

Flight model incorporates model of six degrees of freedom (6-DOF) [3,4,6] to determine projectile trajectory elements and the deviations of the elements.

Governing equations of the 6-DOF model:

$$\frac{d\overrightarrow{r_{CG}}}{dt} = \overrightarrow{V_r} \quad (1)$$

$$\frac{d(m\overrightarrow{V_r})}{dt} = \vec{R} + m\vec{g} + m\overrightarrow{a_c} \quad (2)$$

$$\frac{d(I\vec{\Omega})}{dt} = \overrightarrow{M_R} \quad (3)$$

$$\vec{\Omega} = \overrightarrow{\Omega_E} + \dot{\vec{\lambda}} + \dot{\vec{\varphi}} + \dot{\vec{\psi}} + \dot{\vec{\theta}} + \dot{\vec{\phi}} \quad (4)$$

Equations are applied on the model of long-range projectile, Fig. 1, with determined aerodynamic properties, [7, 8, 9,10].

Parameters in governing equations are:

$\overrightarrow{r_{CG}}$, vector of the center of gravity of projectile,

m, mass of projectile,

I, inertia tensor of projectile,

$\overrightarrow{V_r}$, velocity of the center of gravity of projectile,

$\vec{\Omega}$, angular velocity of projectile,

$\overrightarrow{\Omega_E}$, angular velocity of Earth coordinate frame,

λ, φ, geographic longitude and latitude,

ψ, θ, ϕ, angles of projectile,

$m\vec{g}$, gravity force,

$m\vec{a}_c$, Coriolis force,

$\vec{R}$, aerodynamic force on projectile and

$\overrightarrow{M_R}$, aerodynamic moment on projectile.

2.3. Precision measures

The standard deviation (σ), [11, 12] is a summary measure of the differences of each observation from the mean, i.e. each shot from the mean point of impact (MPI).

$$\sigma_x = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}} \quad (3)$$

$$\sigma_z = \sqrt{\frac{\sum_{i=1}^{n}(z_i - \bar{z})^2}{n-1}} \quad (4)$$

Based on the observer's position, it is possible to determine the coordinates of the projectile impact point (X, Z), and then, relative to the position of the firing asset and the direction of fire, calculate the probable deviation of the projectile in range and cross range (Vx, Vz) [3,4,5]. Fig. 1 shows an example of range firing [3].

Parameters Vx and Vz represent the limits of the interval containing 50% of the hits and are determined based on the standard deviation σ_x and σ_z as follows [3,4,5]:

$$V_x = 0.6745\,\sigma_x = 0.6745\sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}} \quad (5)$$

$$V_z = 0.6745\,\sigma_z = 0.6745\sqrt{\frac{\sum_{i=1}^{n}(z_i - \bar{z})^2}{n-1}} \quad (6)$$

Where are (x_i, z_i), coordinates of the i-th hit, ($\bar{x}$, $\bar{z}$), coordinates of the mean point of impact (MPI), and n is number of hits.

Circular error probable (CEP) [9-12] or circle of equal probability is measure of the precision that represents the radius of the circle, centered on the MPI, such that 50% of the impact points lie within it, Fig. 3. CEP is based on circular bivariate normal distribution (CBN), analog to normal (Gaussian) distribution.

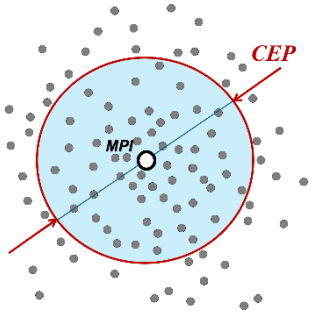


Figure 3. CEP

$$CEP = \begin{cases} \sigma_L(0{,}67 + 0{,}8w^2), & 0 < w < 0{,}5 \\ 0{,}59\sigma_L(1+w), & 0{,}5 \le w \le 1 \end{cases} \quad (7)$$

$$\sigma_L = max(\sigma_x, \sigma_y), \sigma_S = min(\sigma_x, \sigma_y), w = \frac{\sigma_S}{\sigma_L} \quad (8)$$

3. RESULTS

Monte Carlo simulations consist of applying disturbances to the initial firing conditions, as defined in Table 1. These include the muzzle velocity, azimuth and elevation angle of the barrel, the initial mass of the projectile, and the angular velocities q and r. Each Monte Carlo simulation comprised 200 firings, with pre-defined standard deviations for the analyzed disturbances. As a result, the disturbances varied randomly from one firing to another according to normal (Gaussian) distributions. The standard deviations considered were assumed based on available literature data for similar 155 mm projectiles [13,14].

Table 1. Initial conditions and standard deviations for the Monte Carlo simulation

Initial Condition		Unit	Mean value	Standard deviation
Muzzle velocity	V_{01}	[m/s]	830	2
	V_{02}		950	
Initial mass	m_0	[kg]	43.3	0.1
Elevation angle (barrel elevation)	θ_0	[mils]	996	1
Yaw angle (barrel azimuth)	ψ_0		0	1.25
Pitch angular velocity	q	[s^{-1}]	0	1
Yaw angular velocity	r		0	1

In real firing conditions, disturbances in initial velocity and projectile mass are generally considered systematic deviations. These are corrected using adjustments provided in firing tables. For example, the deviation given in the tables for initial velocity is ±10 m/s, while for the projectile mass, the deviation is related to the number of markings on the ammunition (the so-called weight zones). These systematic deviations also include the following initial conditions: wind speed (longitudinal and lateral), air temperature, atmospheric pressure, and propellant charge temperature. However, the measurement of these quantities during ballistic firing is also subject to random deviations due to the measurement uncertainties of the instruments used. Therefore, the Monte Carlo simulations in this analysis also include disturbances in initial velocity and projectile mass. Disturbances in the azimuth and elevation angle of the barrel are also considered random deviations, as they arise from mechanical factors (vibrations, recoil, etc.) or improper operator handling. Finally, disturbances in the angular velocities q and r are viewed as dynamic, random deviations that cannot be controlled and vary from one firing to another.

In this part of the study, the precision assessment was done to obtain a result of the probable deviation for hybrid model of long-range artillery projectile 155 mm. The initial set of disturbances was performed to muzzle velocity, mass, initial angles of the barrel and initial angular velocities.

4.1. Results of simulation for probable deviation

The Monte Carlo simulation results are shown in Figs. 4 through 13 for an elevation angle of 56° and muzzle velocities V_{01} (830 m/s) and V_{02} (950 m/s), corresponding to maximum ranges of 42 km and 57 km, respectively. Five firing scenarios were considered for each muzzle velocity. Muzzle velocity disturbances were accounted for in the first scenario (Figs. 4 and 6), while projectile mass disturbances were included in the second (Figs. 6 and 7).

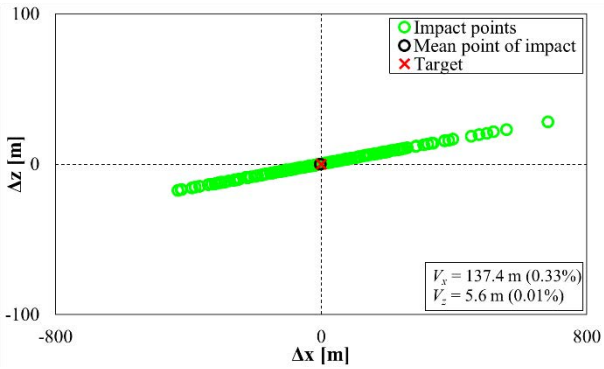


Figure 4. Probable deviation of hits due to deviation of muzzle velocity at 830 m/s

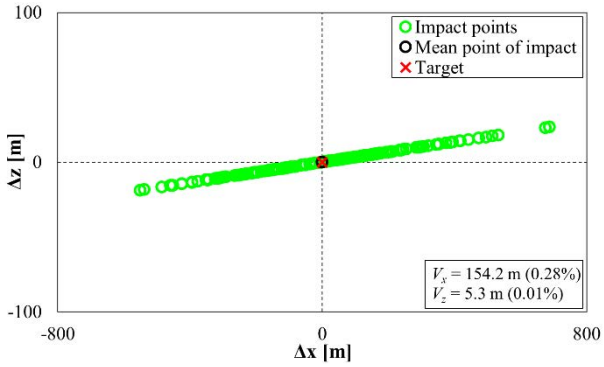


Figure 4. Probable deviation of hits due to deviation of muzzle velocity at 950 m/s

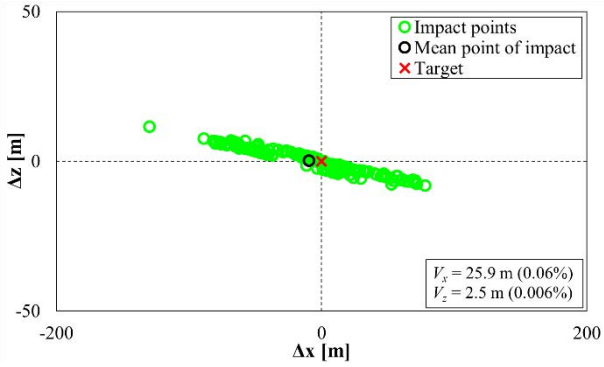


Figure 6. Probable deviation of hits due to deviation of projectile mass at 830 m/s

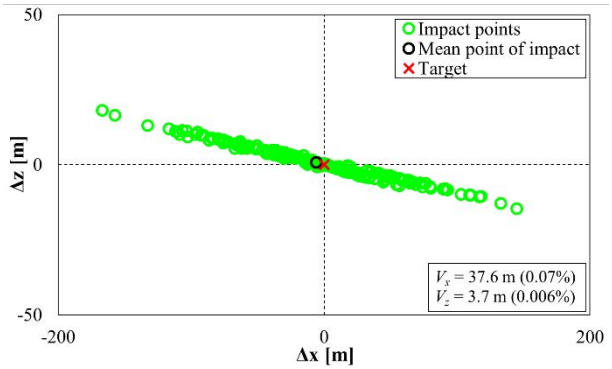


Figure 7. Probable deviation of hits due to deviation of projectile mass at 950 m/s

Azimuth and barrel elevation angle disturbances were examined together in the third scenario (Figs. 8 and 9).

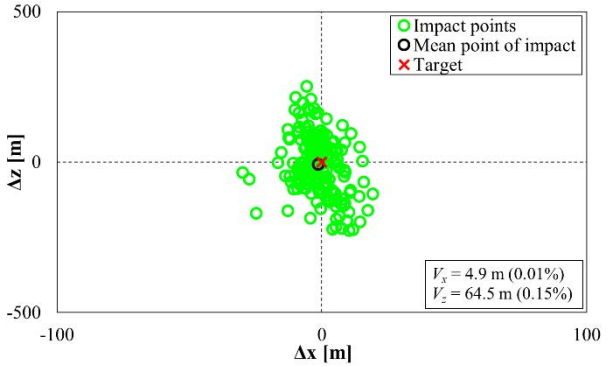


Figure 8. Probable deviation of hits due to deviation of elevation and yaw angles at 830 m/s

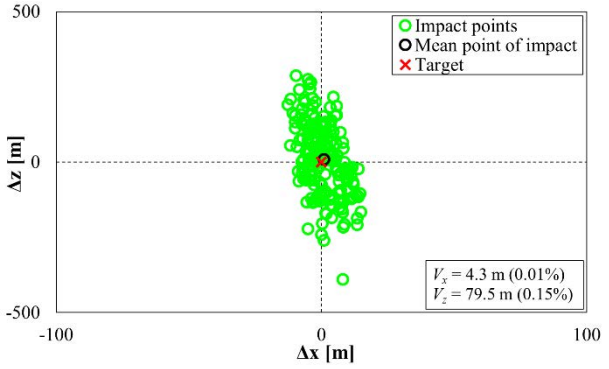


Figure 9. Probable deviation of hits due to deviation of elevation and yaw angles at 950 m/s

Similarly, disturbances in the angular velocities q and r were introduced in the fourth scenario (Figs. 10 and 11).

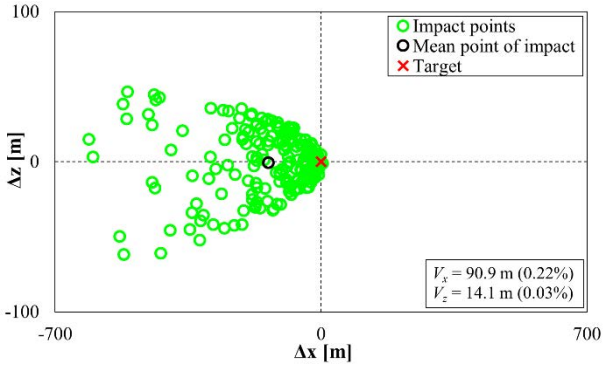


Figure 10. Probable deviation of hits due to deviation of pitch and yaw angular velocities at 830 m/s

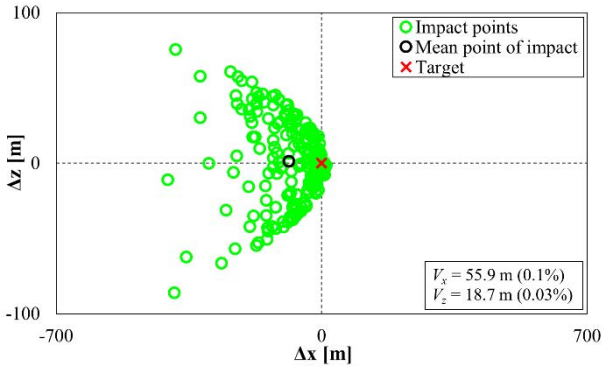


Figure 11. Probable deviation of hits due to deviation of pitch and yaw angular velocities at 950 m/s

Finally, the fifth scenario includes disturbances of all initial conditions considered in the first four scenarios (Figs. 12 and 13).

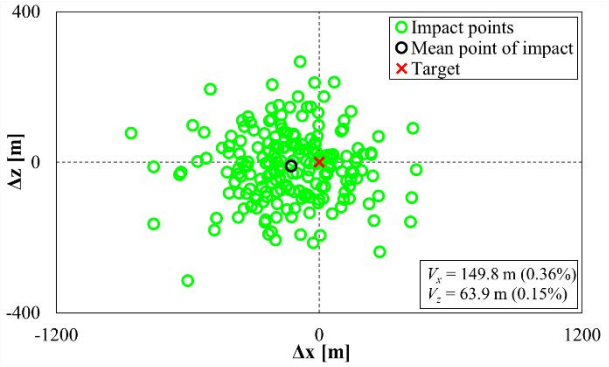


Figure 12. Probable deviation of hits due to deviation of all considered integral factors at 830 m/s

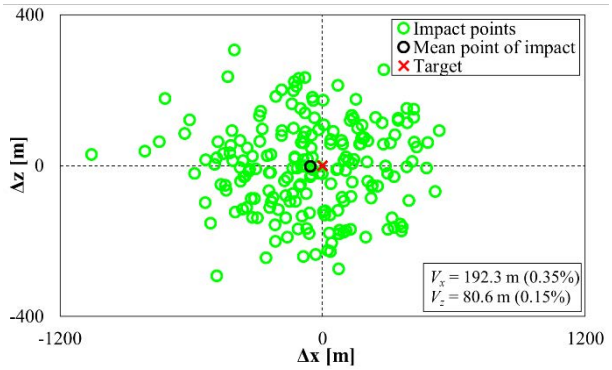


Figure 13. Probable deviation of hits due to deviation of all considered integral factors at 950 m/s

The target is located at point (0, 0) in each figure, and the projectile moves in the positive direction of the x-axis, from left to right. The probable deviation of the projectile across the range Vx and deflection Vz for each scenario is also illustrated in the figures. The percentages are calculated relative to the mean range.

The results in Figs. 4, 5, 6 and 7 show that disturbances of muzzle velocity and projectile mass have a significantly greater impact on range than on projectile deflection, with the mean points of impact nearly all centered on the target.

The results of the third scenario in Figs. 8 and 9 demonstrate that disturbances of the elevation angle cause minimal changes in range, whereas disturbances in the

barrel azimuth angle significantly affect projectile deflection. Although the considered standard deviations for both angles are nearly identical, their effects differ substantially. Furthermore, for this scenario, the mean point of impact coincides with the target point.

The hit plot for the fourth scenario shows that disturbances in the angular velocities q and r affect both the range and the projectile deflection (Figs. 10 and 11). All hits are located at distances less than or equal to the target range. The distribution of hits is nearly symmetrical with respect to the x-axis, with a boundary in the shape of an arc directed toward the target, which corresponds to the zone of highest hit density. The mean point of impact deviates from the target point, particularly for the initial velocity V_{01} (Fig. 10).

4.2. Results of simulation for CEP

The dimensions of the CEP metrics are presented in Table 3 under all disturbances.

Table 3. Results of CEP

Velocity [m/s]	CEP [m]	Vx [m]	Vz [m]
830	121.8	149.8	63.9
950	156.3	192.3	80.6

4.3. Analysis

The impact of all considered disturbances on the hit pattern is shown in Fig. 1. In the case of muzzle velocity V_{01}, the hit density is higher in the zone in front of the target, which is why the mean point of impact is at a greater distance from the target compared to the one obtained with velocity V_{02}, which is very close to the target. The hits for velocity V_{02} are more evenly distributed around the target than those associated with velocity V_{01}. This is a direct result of the stronger influence of disturbances in the angular velocities q and r in the case of velocity V_{01}. The probable deviation, as a measure of precision, amounts to 0.35% in range (Vx) and 0.15% in deflection (Vz) relative to the mean range for both velocity cases V_{01} and V_{02}, meaning that the projectile maintains the same precision even at a longer range. Based on the study presented in [9], which refers to 155 mm artillery system with 39 caliber barrel length and 52 caliber barrel length, the probable deviations Vx and Vz should not exceed 0.75% and 0.14%, respectively. Available data [15] on similar existing projectile models show that the probable error of the hybrid V-LAP projectile, which is currently in use, is 0.6% in range (Vx) and 0.13% in deflection (Vz) relative to the firing range. The probable error must remain within these limits to enable the successful application of potential 1D and 2D Course Correction Fuzes [4].

Available literature data show that the proposed projectile model has satisfactory precision, and even better in range. However, this comparative analysis is more indicative than fully objective, as the Monte Carlo simulations performed do not include all possible disturbances that can occur in real firing conditions. Furthermore, the standard deviations used were taken from similar studies available in the literature [13,14], which state that they are appropriate and representative for the analyzed model. However, it is not explained how these deviations were determined, nor are reference studies cited that would confirm this.

5. CONCLUSION

The study presents an investigation into ballistic performance of the long-rang artillery projectile 155 mm through precision assessment. The analysis showed that the considered disturbances of the initial conditions proportionally affect the trajectory parameters. The external ballistics model confirmed that the proposed projectile concept can achieve a substantial increase in range.

The results also demonstrated that range growth can be further improved through complementary approaches, including higher muzzle velocity, aerodynamic optimization, base-drag reduction, and rocket propulsion. The proposed approach provides a solid basis for future design optimization and experimental validation, particularly in relation to precision, flight performance, and range extension capability.

Acknowledgment

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