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TOOTH ROOT LOAD-CARRYING CAPACITY OF SPUR GEARS UNDER STANDARD SINGLE AND DOUBLE MESH CONDITIONS

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Abstract: *This paper presents and analyzes the tooth root load-carrying capacity of spur gears under standard single and double mesh conditions. A newly developed analytical model of the relative stress factor is used to approximately determine and compare the tooth root stresses. An Octave program is used to calculate the geometric and kinematic parameters of the pinion and wheel gear. A three-dimensional (3D) model of the pinion and wheel gear is created in CATIA, while two-dimensional (2D) model of their teeth is developed in AutoCAD Mechanical. Bending tests are performed on the gear teeth are performed at characteristic contact points. The tooth root stress values obtained using the finite element method (FEM) are compared with those calculated according to the ISO standard.*

Key words: *spur gear, tooth root stress, single and double mesh, finite element method (FEM).*

1. INTRODUCTION

The load-carrying capacity of gear teeth is limited, among other factors, by the bending strength of the teeth root [1–5]. The calculation of gear pairs has been based on many researches and analyses [6–15].

To accunt for the load distribution among simultaneously meshing tooth pairs, the load distribution factor K_α is defined in the literature [1, 2]. This factor represents the degree of participation of each simultaneously meshing tooth pair in transmitting the total load.

From the viewpoint of load distribution, two characteristic extreme cases may occur in simultaneously meshing tooth pairs. The first corresponds to an ideal uniform load distribution, whereas the second represents an extremely uneven load distribution. The actual load distribution lies between these two extreme cases (Fig. 1).

In the case of extremely uneven load distribution, the entire load is transmitted by a single tooth pair, even when two or more tooth pairs are simultaneously in mesh. In this case, the load distribution factor reaches its maximum value ($K_\alpha = 1$ for single mesh). In the case of ideal uniform load distribution, all simultaneously meshing tooth pairs equally participate in transmitting the total load of the gear pair ($K_\alpha = 1/2$ for double mesh, $K_\alpha = 1/3$ for triple mesh, etc.).

The transverse contact ratio ε_α represents the average number of simultaneously meshing tooth pairs transmitting the total load. In this paper, the case of $1 \le \varepsilon_\alpha \le 2$, corresponding to the range $0{,}5 \le K_\alpha \le 1$, is considered.

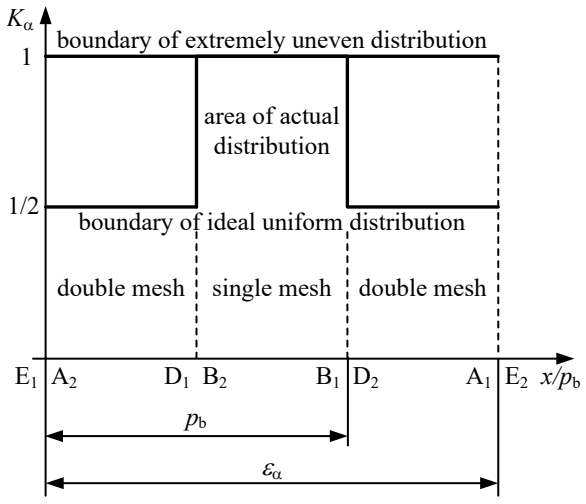


Figure 1. Boundary and actual load distributions in standard meshing gear tooth pairs

2. ANALYTICAL MODEL OF TOOTH ROOT STRESS

Based on the analyses presented in the literature [3], the general expression for the tooth root stress at any contact

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point during simultaneous meshing of the tooth profiles can be written as follows:

$$\sigma_{FX} = \sigma_0 \cdot K_{\alpha X} \cdot Y_{FX} \cdot Y_{SX} \qquad X \in (A,...,E) \qquad (1)$$

where σ_0 is the stress component that remains constant throughout the contact period from point A≡A_1≡A_2 to point E≡E_1≡E_2 (Fig. 1).

The variation of the tooth root stress during the contact period is analyzed by considering the ratio between the stress at an arbitrary contact point (X) and the stress at the outer point of the double mesh (point A in Fig. 1):

$$\frac{\sigma_{FX}}{\sigma_{FA}} = \frac{K_{\alpha X}}{K_{\alpha A}} \cdot \frac{Y_{FX} \cdot Y_{SX}}{Y_{FA} \cdot Y_{SA}} \qquad (2)$$

Equation (2) can also be written in the following form:

$$\frac{\sigma_{FX}}{\sigma_{FA}} = \frac{K_{\alpha X}}{K_{\alpha A}} \cdot Y_{\varepsilon X} \qquad (3)$$

where $Y_{\varepsilon X}$ is relative stress factor at the observed contact point.

2.1. Relative stress factor

The relative stress factor is defined as the ratio of the form factor and the stress correction factor at the observed contact point to the corresponding factors at the outer point of the double mesh:

$$Y_{\varepsilon X} = \frac{Y_{FX} \cdot Y_{SX}}{Y_{FA} \cdot Y_{SA}} \qquad (4)$$

According to Eq. (4), the relative stress factor reaches its maximum value at the tooth contact at the outer point of the double mesh, i.e. $Y_{\varepsilon A} = 1$.

The relative stress factor is determined separately for both the pinion and wheel gear tooth pairs:

$$Y_{\varepsilon Xi} = \frac{Y_{FXi} \cdot Y_{SXi}}{Y_{FAi} \cdot Y_{SAi}} \qquad (5)$$

where i=1 and 2 for the pinion tooth and wheel tooth.

Determination of the relative stress factor using Eq. (5) requires the values of the form factor and the stress correction factor at an arbitrary contact point, as well as the corresponding values at the outer point of the double mesh. Owing to the large number of influencing parameters, this procedure is suitable mainly for highly loaded or safety-critical gear pairs. For approximate calculations, the relative stress factor may be evaluated using simplified equations derived from appropriate approximations.

2.2. Approximate model of the relative stress factor

To simplify the determination of the relative stress factor, a linear variation during the contact period is assumed. In addition, it is assumed that the relative stress factor is identical for simultaneously meshing pinion and wheel gear teeth. Based on these assumptions, the relative stress factor during the contact period can be expressed as:

$$Y_{\varepsilon X} = 1 + \frac{Y_{\varepsilon E} - 1}{\varepsilon_\alpha} \cdot \frac{x}{p_b} \qquad (6)$$

where are $Y_{\varepsilon E}$ – the relative stress factor for the inner point of the double mesh (point E in Fig. 2) and x/p_b – position of the contact point relative to the outer point of the double mesh (point A in Fig. 2).

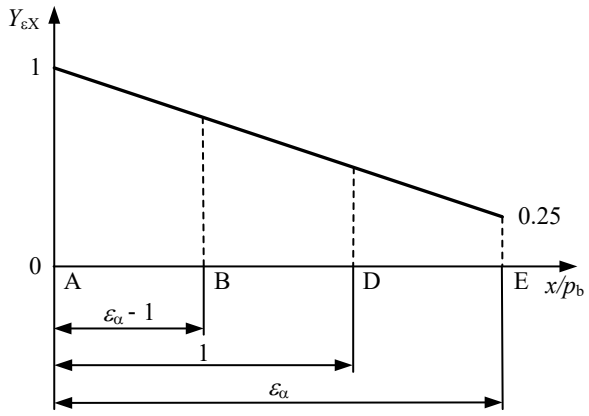


Figure 2. Approximation of the relative stress factor

Assuming that the minimum value of the relative stress factor occurs at the inner point of the double mesh $Y_{\varepsilon E} = 0.25$ (Fig. 2), the relative stress factor can be approximately determined for each characteristic point based on Eq. (6). Approximate equations for determining the relative stress factor are shown in Table 1.

Table 1. Approximate equations of the relative stress factor

point	$\frac{x}{p_b}$	$Y_{\varepsilon X}$	$Y_{\varepsilon\,apro}$	$Y_{\varepsilon\,apro}$ ($Y_{\varepsilon E} = 0.25$)
A	0	$Y_{\varepsilon A}$	$Y_{\varepsilon\,max}$	1
B	$\varepsilon_\alpha - 1$	$Y_{\varepsilon B}$	$1 + \frac{Y_{\varepsilon E} - 1}{\varepsilon_\alpha} \cdot (\varepsilon_\alpha - 1)$	$0.25 + \frac{0.75}{\varepsilon_\alpha}$
D	1	$Y_{\varepsilon D}$	$1 + \frac{Y_{\varepsilon E} - 1}{\varepsilon_\alpha}$	$\frac{\varepsilon_\alpha - 0.75}{\varepsilon_\alpha}$
E	ε_α	$Y_{\varepsilon E}$	$Y_{\varepsilon\,min}$	0.25

2.3. Analytical model of tooth root stress under ideal uniform load distribution

In the case of ideal uniform load distribution on simultaneously meshed teeth pairs, the values of the load distribution factor vary from $K_{\alpha X} = 1/2$ (for double mesh) to $K_{\alpha X} = 1$ (for single mesh). Substituting the limiting values of the load distribution factor into Eq. (1), analytical expressions for determining the tooth root stress at characteristic contact points at ideal uniform load distribution, according to the exact model, are obtained:

$$\sigma_{FX} = \frac{\sigma_0 \cdot Y_{FX} \cdot Y_{SX}}{2} \quad X \in (A \wedge E) \tag{7}$$

$$\sigma_{FX} = \sigma_0 \cdot Y_{FX} \cdot Y_{SX} \quad X \in (B \wedge D) \tag{8}$$

and by arranging Eq. (3), the tooth root stress at the characteristic contact points at ideal uniform load distribution can also be determined based on the following expressions, according to an approximate model:

$$\sigma_{FX} = Y_{\varepsilon X} \cdot \sigma_{FA} \quad X \in (A \wedge E) \tag{9}$$

$$\sigma_{FX} = 2 \cdot Y_{\varepsilon X} \cdot \sigma_{FA} \quad X \in (B \wedge D) \tag{10}$$

If it is assumed that the minimum value of the relative stress factor is at the inner point of the double mesh $Y_{\varepsilon E} = 0{,}25$, the tooth root stress at ideal uniform load distribution is, based on the expressions in Table 2 and Eqs. (9) and (10), given for:

- the outer point of the double mesh (point A)

$$\sigma_{FA} = 1 \cdot \sigma_{FA} \tag{11}$$

- the outer point of the single mesh (point B)

$$\sigma_{FB} = 2 \cdot \left(0.25 + \frac{0.75}{\varepsilon_\alpha}\right) \cdot \sigma_{FA} \tag{12}$$

- the inner point of the single mesh (point D)

$$\sigma_{FD} = 2 \cdot \left(\frac{\varepsilon_\alpha - 0.75}{\varepsilon_\alpha}\right) \cdot \sigma_{FA} \tag{13}$$

- the inner point of the double mesh (point E)

$$\sigma_{FE} = 0.25 \cdot \sigma_{FA} \tag{14}$$

When the minimum value of the relative stress factor at the inner point of the double mesh is $Y_{\varepsilon E} = 0.25$, for the tooth root stress with ideal uniform load distribution, based on Eqs. (11) to (14) according to the approximate model, it follows that $\sigma_{FB} > \sigma_{FA} > \sigma_{FD} > \sigma_{FE}$. The tooth root stress at ideal uniform load distribution on simultaneously meshed teeth pairs is greatest at the outer point of the single mesh.

2.4. Analytical model of tooth root stress under extremely uneven load distribution

In the case of extremely uneven load distribution, the values of the load distribution factor vary from $K_{\alpha X} = 0$ (when the tooth does not transmit the load at all) to $K_{\alpha X} = 1$ (when the tooth transmits the total load). By substituting only the limiting value of the load distribution factor $K_{\alpha X} = 1$ into Eq. (1), analytical expression for determining in the tooth root stress at characteristic contact points at extremely uneven load distribution when the total load is transmitted over one tooth pair, according to the exact model, is obtained:

$$\sigma_{FX} = \sigma_0 \cdot Y_{FX} \cdot Y_{SX} \quad X \in (A,..., E) \tag{15}$$

and by arranging Eq. (3), the tooth root stress at the characteristic contact points at extremely uneven load distribution when the total load is transmitted over one tooth pair can also be determined based on the following expression, according to an approximate model:

$$\sigma_{FX} = Y_{\varepsilon X} \cdot \sigma_{FA} \quad X \in (A,..., E) \tag{16}$$

If it is assumed that the minimum value of the relative stress factor is at the inner point of the double mesh $Y_{\varepsilon E} = 0.25$, in the tooth root stress when the entire load is transmitted through a single tooth pair is, based on the expressions in Table 2 and Eq. (16), given for:

- the outer point of the double mesh (point A)

$$\sigma_{FA} = 1 \cdot \sigma_{FA} \tag{17}$$

- the outer point of the single mesh (point B)

$$\sigma_{FB} = \left(0.25 + \frac{0.75}{\varepsilon_\alpha}\right) \cdot \sigma_{FA} \tag{18}$$

- the inner point of the single mesh (point D)

$$\sigma_{FD} = \left(\frac{\varepsilon_\alpha - 0.75}{\varepsilon_\alpha}\right) \cdot \sigma_{FA} \tag{19}$$

- the inner point of the double mesh (point E)

$$\sigma_{FE} = 0.25 \cdot \sigma_{FA} \tag{20}$$

When the minimum value of the relative stress factor at the inner point of the double mesh is $Y_{\varepsilon E} = 0.25$, for the tooth root stresses when the total load is transmitted over one tooth pair, based on Eqs. (17) to (20) according to the approximate model, it can be concluded that $\sigma_{FA} > \sigma_{FB} > \sigma_{FD} > \sigma_{FE}$. The tooth root stress at extremely uneven load distribution when the entire load is transmitted through a single tooth pair is greatest at the outer point of the double mesh.

3. 3D MODELS OF SPUR GEAR AND 2D MODELS OF GEAR TEETH

All geometric and kinematic parameters required for the 3D modelling of spur gear are determined using the Octave program, and the most relevant ones are presented in this paper. A spur gear pair with a pinion/wheel tooth number of z_1=29, z_2=118, standard module m_n=6,5mm, pressure angle α=20°, profile shift coefficients for the pinion and wheel gear x_1=0.2, x_2=0.2, center distance a=480.24mm, and transverse contact ratio ε_α=1.75 is considered. Based on these geometric and kinematic parameters, 3D models of the pinion and wheel gear are created in CATIA program is created (Figs. 3 and 4).

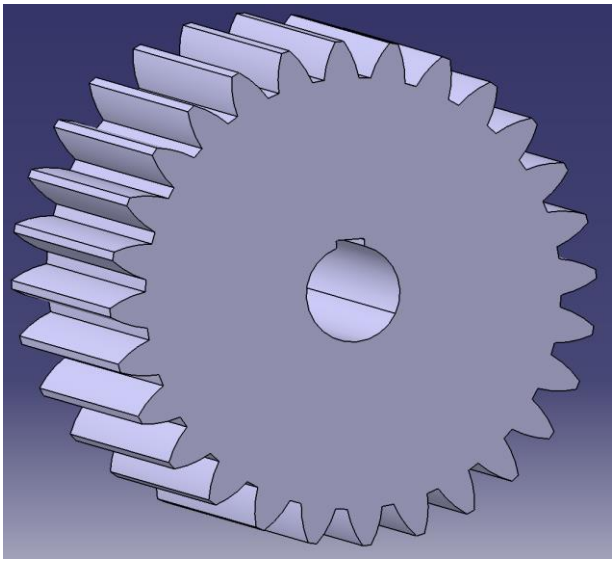


Figure 3. Model of the pinion gear

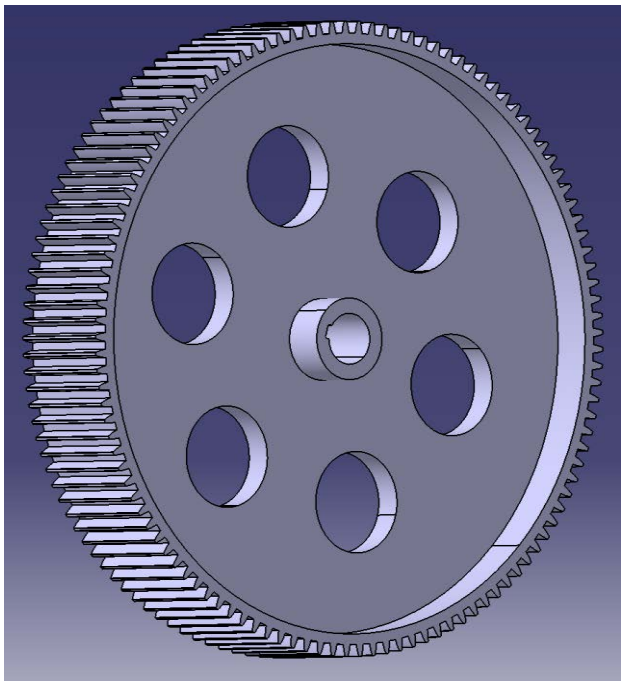


Figure 4. Model of the wheel gear

To determine the tooth root stress values, 2D tooth models of both gears are developed using FEM in AutoCAD Mechanical and are subjected to an identical load of 6518.73N (Figs. 5 to 8).

The tooth root stress is evaluated at two characteristic points, A and B, as these represent the most critically loaded locations.

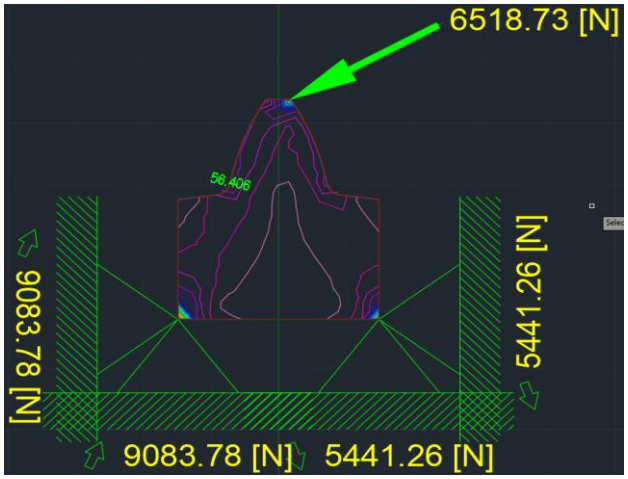


Figure 5. Pinion gear tooth model (load applied at point A)

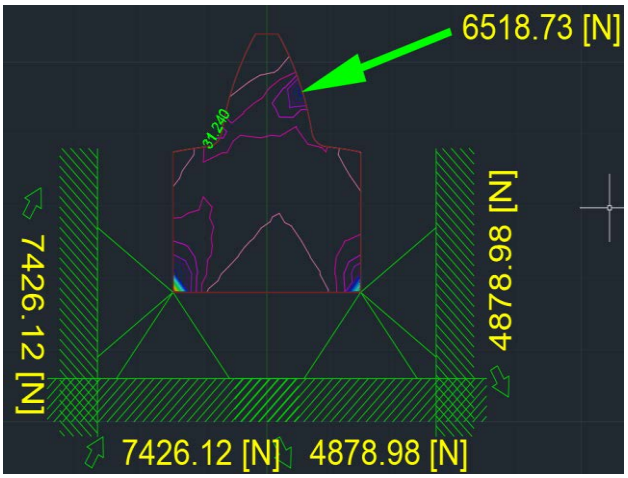


Figure 6. Pinion gear tooth model (load applied at point B)

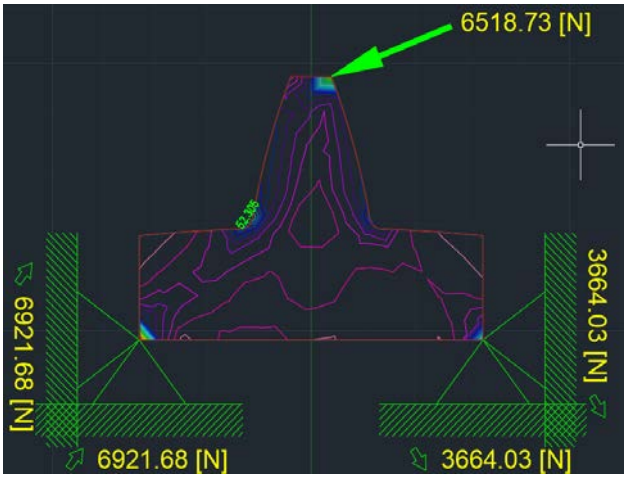


Figure 7. Wheel gear tooth model (load applied at point A)

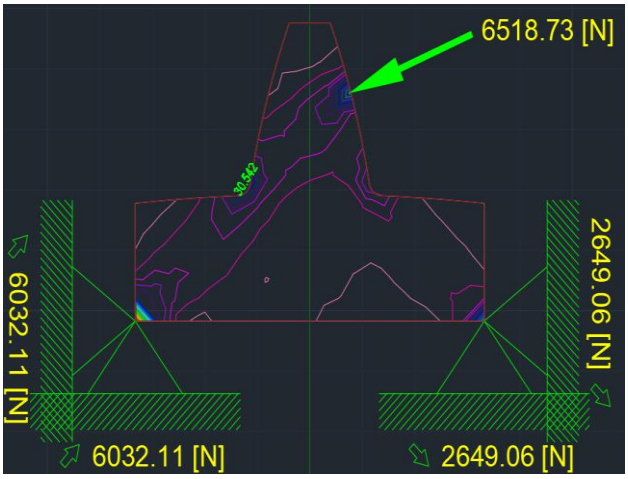


Figure 8. Wheel gear tooth model (load applied at point B)

Table 2 presents the tooth root stress values for both gears at points A and B, obtained using FEM.

Table 2.Tooth root stress values at points A and B

point	σ_F [N/mm^2]	pinion gear	wheel gear
A	σ_{FA}	56.41	52.31
B	σ_{FB}	31.24	30.54

4. ISO CALCULATION OF TOOTH ROOT STRESS AND COMPARATIVE ANALYSIS

For Method "B" specified in the ISO standard [16, 17], detailed guidelines for the calculation procedure are provided; therefore, only this method is considered in the present study. According to Method B, the tooth root stress is determined at point B, assuming an ideal uniform load distribution. The tooth root stress values at point B calculated according the ISO standard are presented in Table 3.

Table 3. Tooth root stress values at point B

point	σ_F [N/mm^2]	pinion gear	wheel gear
B	σ_{FB}	31.19	30.61

To validate the tooth root stress analysis performed according to Method B of the ISO standard, as well as the proposed analytical (approximate) model, a numerical model based on the FEM is developed in AutoCAD Mechanical.

A comparison of the tooth root stress values obtained using Method B of the ISO standard with those obtained from the FEM analysis shows excellent agreement. At point B, the deviation is less than 0.01%. This negligible difference results from the accurate generation of the numerical model and the appropriate selection of the analysis parameters, such as the material properties of the gears.

If the tooth root stress value at point A listed in Table 2 is divided by two to account for the double-mesh condition, i.e., if both the load distribution factor and the tooth geometry are taken into account, the stress at the outer point of the single mesh (point B) becomes greater than that at the outer point of the double mesh (point A). This behavior is also predicted by the developed analytical model for ideal uniform load distribution.

In the case of extremely uneven load distribution, where the entire load is transmitted through a single tooth pair, the tooth root stress at the outer point of the double mesh (point A) is greater than that at the outer point of the single mesh (point B). The same trend is observed in the FEM results (Figs. 5-8) and in the data presented in Table 2. This behavior is a consequence of neglecting the load distribution factor while considering only the influence of tooth geometry.

5. CONCLUSION

Based on the developed analytical and numerical models for tooth root stress, it can be concluded that the variation of the tooth root stress depends on the variation of the load distribution factor, the form factor, and the stress correction factor, all of which vary with the position of the contact point along the tooth profile. The developed analytical models can be used for the simplified determination of the relative stress factor and for the approximate evaluation of the tooth root stress.

A comparative analysis of the results obtained using Method B of the ISO standard and the numerical model showed excellent agreement between the two approaches. For engineering applications, it is recommended that both methods be used in combination: the ISO method for the rapid assessment and verification of less critical gear pairs, and 2D or 3D numerical models for the final analysis and optimization of highly loaded or safety-critical gear pairs.

This combined approach enables the design of gear pairs that are not only safe and reliable but also economical, ensuring efficient material utilization and an extended service life.

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