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AN EXAMPLE OF MISSILE-TO-TARGET INTERCEPTION

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Abstract: *Methods for calculating the interception point of the trajectory of a projectile and a moving target are the subject of many projects and are being solved in several directions. Initially, these were trigonometric methods, while nowadays probabilistic methods or methods of integrating differential equations are used. The paper proposes a classical method of integrating differential equations of projectile motion using the Runge-Kutta method, based on the initial azimuth and elevation angles of the projectile launch parameters, using the known initial projectile velocity. Future target positions are predicted based on the radar target tracking and calculated meeting time of the projectile and the target, using the interpolation method of the target's position coordinates and acceleration. The simulation results showed satisfactory accuracy for targets in the air at a distance of 2000 m with standard deviation of error position on 15 m.*

Keywords: *Target tracking, Kalman filter, Taylor extrapolation polynom, ballistic trajectory.*

1. INTRODUCTION

Intercepting an incoming artillery or mortar round is receiving interest as an area-protection mechanism. A problem has been that estimation, based on 6-degrees of freedom -(DoF) equations for both the incoming round and the interceptor round or missile, is to time consuming from a computational perspective. This calculation is used to find the maneuver control and the aimpoint or lead angle needed to intercept an incoming round. The method uses reduced dynamics flight models for the interceptor and the incoming round. Also, questions relating to the probability of a hit given a specific standard deviation and bias are addressed [1].

Guidance laws that aim to achieve desired impact angle, as well as zero miss distance, are pushed forward by a number of techniques [2]. Some authors derived guidance laws with impact angle constraint based on proportional navigation guidance (PNG). These PNG-based impact angle control guidance techniques are simple in form and cost little in the calculation, but some constraints on the impact angle cannot strictly be satisfied when the initial heading coincides with the line-of-sight (LOS). The sliding mode control (SMC) technique is another commonly used technique in the field of impact angle control guidance (IACG). Other authors use SMC to achieve desired impact angle. Guidance laws based on a conventional SMC can only ensure asymptotic convergence. A guidance law based on terminal sliding mode control was proposed in [3], which satisfied the finite-time convergence. So, it is also posible for combined SMC and virtual target strategy to control collision time and angle. Guidance laws based on SMC have strong robustness, which are even effective on maneuvering targets. However, the discrete terms in the guidance formula always cause inevitable oscillations in the guidance commands, which is not suitable for actual implementations. The SMC technique was introduced to design a guidance law that makes multiple missiles hit the target in a preset direction at the same time [4], and it utilized range-to-go and radial relative velocity rather than time-to-go to construct the sliding surface to avoid performance degradation. Some authors combined SMC, backstepping, and super-twist extend state observer to propose a robust guidance law [5] and verified its performance by a dynamics model.

This paper is aimed at proposing a novel algorithm, which calculates firing parameters (cannon barrel azimuth and elevation) only the data obatined from from radar sensor. When the target enters the projectile's zone of action, an algorithm is activated that, in the first step, extrapolates the future position of the target in the time interval required for the projectile to reach the target. Given that the projectile's speed is an order of magnitude greater than the target's speed of movement, the integration of the projectile's differential equations of motion calculates the azimuth and elevation angle of the gun barrel in order to meet the target, under

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the condition of a minimum distance between the target and the projectile.

The paper is organized as follows: after introductory considerations, theoretical models of the target and projectile are given. The third chapter describes the proposed algorithm for predicting the collision of the target and projectile, while the fourth chapter is dedicated to presenting the results of numerical simulations. Finally, concluding considerations are given.

2. PROBLEM STATEMENTS

2.1 Targets and measurements models

Consider a discrete-time stochastic hybrid system:

$$x_k = f(x_{k-1}) + q_{k-1} \tag{1}$$
$$y_k = h(x_k) + r_k \tag{2}$$

where $x_k \in R^n$ is the state vector in effect at sample time k with dynamic model function $f(.)$ and transition matrix F_k, $y_k \in R^m$ is the vector of observations in effect at time k, $q_{k-1} \approx N(0, Q_{k-1})$ is the Gaussian process noise, $r_k \approx N(0, R_k)$ is the Gaussian measurements noise.

2.2 Missile dynamic models

Consider a 3D motion scenario for a missile against a moving target. Inertial, moving and bound coordinate systems are used to model the movement of the target and projectile (Figure 1).

Positions are x, y and z and velocities are v_x,v_y and v_z and θ and φ their azimuth and elevation flight path angles, respectively. State vector is given by the $X = [x\ v_x\ y\ v_y\ z\ v_z]$. R stands for the distance between the missile and the target. The engagement kinematics for this problem can be given as follows [8]:

$$\begin{aligned} x &= x_0 + v_{x0}t \\ y &= y_0 + v_{y0}t \\ z &= z_0 + v_{z0}t - g\frac{t^2}{2} \end{aligned} \tag{3}$$

$$\begin{aligned} v_{x0} &= v_0 \cos\varphi \cos\theta \\ v_{y0} &= v_0 \cos\varphi \sin\theta \\ v_{z0} &= v_0 \sin\varphi \end{aligned} \tag{4}$$

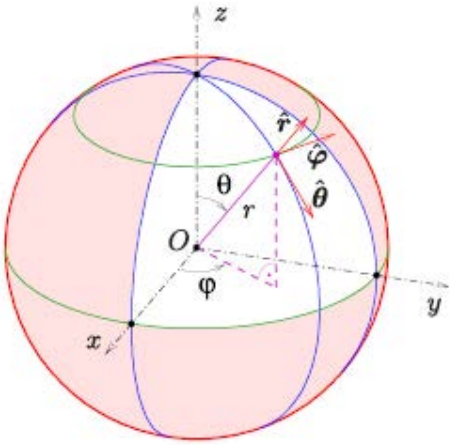


Figure 1. Kugel model

For ease of calculation, the problem can be reduced to 2D dimension (x,y). Due to the action of gravity, at time t the vertical component of the initial velocity decreases by the value of gravity, while the horizontal component remains unchanged. At that position, the velocity vector v makes an angle with the horizontal. The differential equations of motion in this case are [9]:

$$\begin{aligned} m\ddot{x} &= 0 \\ m\ddot{y} &= -mg \end{aligned} \tag{5}$$

or

$$\begin{aligned} \ddot{x} &= 0 \\ \ddot{y} &= -g \end{aligned} \tag{6}$$

The general solution of this system of equations is:

$$\begin{aligned} x &= C_1 t + C_2 \\ y &= -\frac{1}{2}gt^2 + C_3 t + C_4 \end{aligned} \tag{7}$$

The integration constants are determined by the initial conditions at the time:

$$\begin{aligned} x_0 &= 0, \\ y_0 &= 0, \\ (\dot{x})_0 &= v_0 \cos\alpha_0, \\ (\dot{y})_0 &= v_0 \sin\alpha_0, \end{aligned} \tag{8}$$

The final equations of motion of the projectile now are:

$$\begin{aligned} x &= v_0 t \cos\alpha_0 \\ y &= -\frac{1}{2}gt^2 + v_0 t \sin\alpha_0 \end{aligned} \tag{9}$$

By eliminating t from these parametric equations, the equations of the trajectory are obtained

$$y = -\frac{g}{2v_0^2 \cos^2\alpha_0}x^2 + tg\alpha_0 x \tag{10}$$

This is the parabola that lies symmetrically with respect to

the coordinates of the vertex of the trajectory, i.e. parabola with a vertical axis and a concave side downward. If it is assumed that $0<\alpha_0<\pi/2$ the value is positive and according to (5) the abscissa x increases with time i, i.e. the projectile is constantly moving to the right. To determine the range l, expresses the range of the projectile:

$$l=\frac{v_0^2}{g}\sin(2\alpha_0) \tag{11}$$

The total flight time t_A of the projectile from 0 to A is obtained by the equation:

$$t_A=\frac{l}{v_0\cos\alpha_0}=\frac{{v_0}^2\sin(2\alpha_0)}{gv_0\cos\alpha_0}=\frac{2v_0}{g}\sin\alpha_0 \tag{12}$$

The projections are twice the values for the abscissa of the vertex of the path and the duration of the flight to it, which must also be for reasons of symmetry. The height of the vertex of the path is obtained by the:

$$h=-\frac{g}{2}t_T^2+v_0\sin\alpha_0 t_T=\frac{v_0^2}{2g}\sin^2\alpha_0 \tag{13}$$

where t_T is time necessary to rich *h*.

3. DESCRIPTION OF PROPOSED ALGORITHM

3.1 Taylor polynomials

Definition.
Let *f* be a function that has all derivatives up to order *n* at *a*. Then we define the **Taylor polynomial** of *f* of degree *n* with center *a* as [10]:

$$T_n(x)=f(a)+f'(a)(x-a)+\frac{1}{2!}f''(a)(x-a)^2 +\ldots+\frac{1}{n} =\sum_{k=0}^{n}\frac{f^{(k)}(a)}{k!}(x-a)^2 \tag{14}$$

Theorem (Taylor's theorem).

Consider two distinct real numbers *a* and *x*. Let *I* be the closed interval with these two points as endpoints. Assume that a function *f* has continuous derivatives up to order *n* on *I* and $f^{(n+1)}$ on the interior Int(*I*). Then

$$Rn(x)=\int_a^x\frac{f^{(n+1)}(t)}{n!}(x-t)^n\,dt \tag{15}$$

Moreover, there exists a number *c* between *a* and *x* such that

$$Rn(x)=\frac{f^{(n+1)}(c)}{(n+1)!}(x-a)^{n+1} \tag{16}$$

The first result is called the **integral form** of the remainder, the second is called the **Lagrange form**. In fact, these precise results are also not exactly useful, since the integral may easily be too difficult to evaluate, while the second statement is existential, we know that such a *c* exists, but we have no idea what it is. However, the Lagrange form can be approximated from above and with a bit of luck it turns out that the upper estimate is small.

3.2 Projectile-target interception

Basic idea was devised to indicate the time needed to hit an incoming target, with a defined angle of hitting the target. The incoming round was modeled using the point interpolation equations and the interceptor round was modeled with a single dimension differential equation. Using these equations, it was possible to compute the lead angle needed for the gun interceptor round. This calculation is based on estimated time of arrival of the interceptor at predicted points of the target trajectory.

The expected time of the interceptor's arrival at each point can be calculated. At each point, the predicted time until arrival of the target and the interceptor are differenced. An aim point/lead angle can be selected for any positive difference by adjusting the firing time of the interceptor so that the time difference is zero. It is also possible to search the interval that goes from negative to positive for the zero delay aim point. The terminal geometry could also be considered in selecting an aim point. Post launch adjustments are made by selecting a different predicted target trajectory point for the interceptor to hit. After the interceptor is fired, it could only adjust its course by one degree at a time; thus, corrections could only take place if there was over one degree of error in the predicted impact point. The accuracy of the target track would have to meet certain criteria before an interceptor could be fired. Meeting this accuracy requirement would ensure that the target would be within the interceptor's maneuver space. Likewise the estimate of the interceptor's trajectory is of equal concern. As time increases, the accuracy of both the estimate of the target trajectory and the interceptor trajectory will increase. Many times, polynomial filters are discussed as candidates for trajectory estimation; these filters do not attempt to capture the dynamics of the situation and for longer prediction times are inadequate. Although filters based on *6-DoF* models are conceivable, they impose a computational load that cannot be met at this time [11].

3.3 Steps of proposed method

Let's assume the *3D* movement of a target in the air (e.g. a drone or quad copter), which moves in a straight line and with a slight maneuver (not larger than 3g). The target is tracked during the experiment by a Kalman filter (KF) using a CV (constant velocity) motion model. At the moment the target enters the weapon's engagement zone, an automatic 'encounter' calculation is activated. It is basically a three-dimensional problem of the kinematics

of the movement of the projectile and the target. It is necessary to solve the pure pursuit ballistic problem and its 'encounter' with the target that has entered the engagement zone of the weapon. For this calculation, the known laws of motion of a projectile fired from a known location, with a known initial velocity, are used. The algorithm should determine the angle at which the projectile is fired. The algorithm is performed in five main steps. These are the steps of the algorithm:

1. The target is tracked using a radar sensor in the C band, moving the state estimator (*Kalman filter*)

2. Calculation of the target distance from the weapon

3. Extrapolation by Taylor polynomial (most often of the third degree) of the future state of the target for the time it takes for the projectile to cover the calculated distance to the target. The order of the Taylor polynomial depends on the target trajectory until the moment of entering the zone of action. Based on the azimuth and elevation angles, the weapon is positioned towards the target

4. Integration of the differential equation of motion of the projectile by the *Runge-Kutta* method, using the ode 45 function. the parameter (i.e. condition) that needs to be met to complete the integration is the distance of the projectile to the target, which should be below 10m

5. Firing a sequentially with different (close) angles, so that the proximity fuses on the projectiles are activated

4. RESULTS OF SIMULATION

For experimental purpose we use the two type of scenario:

A) Simulated scenario with constant velocity of target (Fig.3)

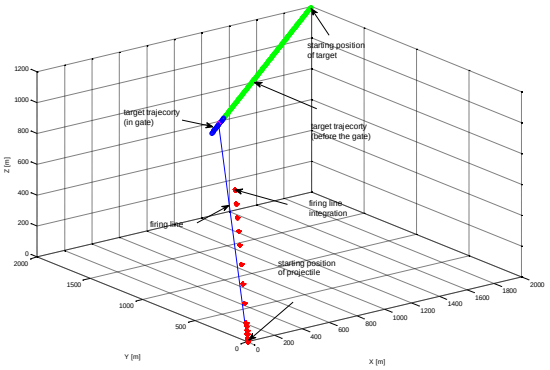


Figure 3. Simulated scenario with constant velocity

Using the known initial conditions, an expected projectile path can be generated. Range and velocity measurements can be used to improve the estimate of the projectile's path. The differential equation and the measurement process are combined to form a Kalman filter. The differential equation models the physics of the trajectory while the measurement updates the parameters of the equation based on least squares estimation.

$$F_{cv} = \begin{bmatrix} 1 & dt & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & dt & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & dt \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (17)$$

The simulation has 500 time intervals of duration dt=0.1[s] from the radar. The target state vector has $x=[x\ v_x\ y\ v_y\ z\ v_z\ ]$. The initial target parameters are: $x_t = 2000$, $v_{xt} = -80$, $y_t = 2000$, $v_{yt} = -60$, $z_t = 1200$, $v_{zt} = -5$. The cannon is launched at the coordinate origin. The target moves from the initial position, towards to the zone of action and when it enters the zone of action, the differential equation of motion of the projectile is integrated until the time of flight of the projectile until the encounter with the target is obtained.

B) Real data scenario (Fig.4).

These times are given in Table 1 for both scenarios. The distance of the cannon's effective range is 2000 m. The initial velocity of the projectile is 950 m/s. The target continues to move until the condition is met that it meets the projectile within the calculated time, in some delta environment. Transition matrix of constant acceleration (CA) target dynamic motion is:

$$F_{ca} = \begin{bmatrix} 1 & dt & dt^2/2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & dt & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & dt & dt^2/2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & dt & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & dt & dt^2/2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & dt \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (18)$$

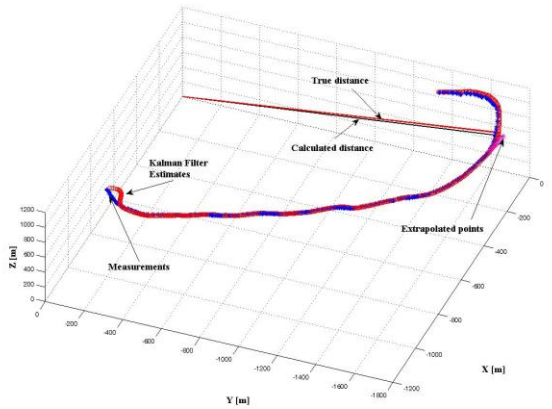


Figure 4. Simulation with real radar data

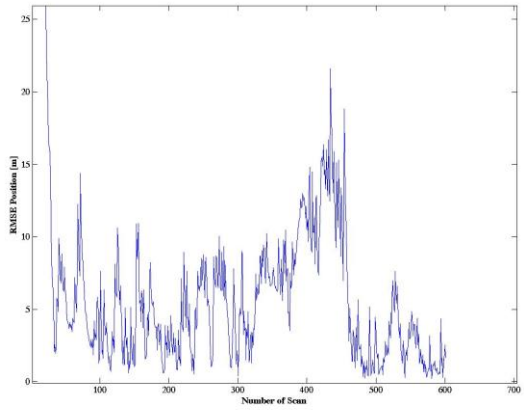


Figure 5. RMSE position diagram for real data scenario

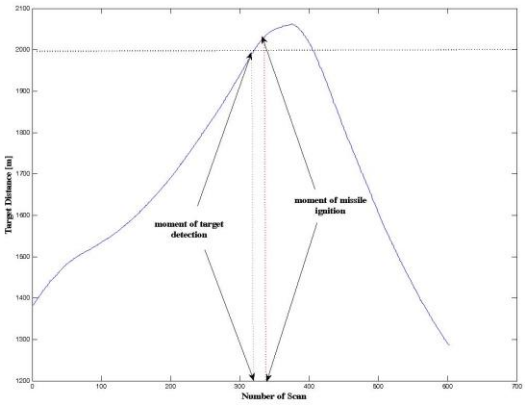


Figure 6. Distance of target diagram

The experimental results are given in the form of diagrams of the mean square error per position (real data – state estimate when tracking the target with the Kalman filter) given in Fig. 5. Figure 6. shows a diagram of the distance between the gun position (the initial position of the projectile) and a moving target. When the target enters the projectile's zone of action, an algorithm is activated that, in the first step, extrapolates the future position of the target in the time interval required for the projectile to reach the target. Given that the projectile's speed is an order of magnitude greater than the target's speed of movement, the integration of the projectile's differential equations of motion calculates the azimuth and elevation angle of the gun barrel in order to meet the target, under the condition of a minimum distance between the target and the projectile.

5. CONCLUSION

A complete algorithm for detecting, tracking and engaging low-flying targets using radar band range was proposed and tested in this paper. The algorithm showed good results in engaging moving targets in the air, using the Taylor polynomial (in the domain of extrapolation of future target states) and in the domain of predicting the missile and target collisions, using the Kalman filter. Preliminary research results are presented. The errors in approaching the bullet and targets are between 10 and 15 [m]. In future, a functional model for the complete detection, tracking and engaging low-flying targets in the air is envisaged.

References

[1] N. A. Shneydor, Missile Guidance and Pursuit: Kinematics, Dynamics and Control. Elsevier, Jan. 1998.

[2] Z. Radosavljevic, D. Ivkovic and B. Kovacevic: The Prediction of Missile-Target Interception Using C-band Radar, 13th International Conference on Electrical, Electronics and Computer Engineering (IcETRAN), Silver lake, Republic of Serbia, 8-11 June 2026.

[3] I. S. Jeon, J. I. Lee, and M. J. Tahk, “Impact-time-control guidance law for anti-ship missiles,” IEEE Transactions on Control Systems Technology, vol. 14, pp. 260–266, Mar. 2006.

[4] M. J. Tahk, C. K. Ryoo, and H. Cho, “Recursive time-to-go estimation for homing guidance missiles,” IEEE Transactions on Aerospace and Electronic Systems, vol. 38, pp. 13–24, Jan. 2002.

[5] H. S. Shin, H. Cho, and A. Tsourdos, “Time-to-go Estimation using Guidance Command History,” IFAC Proceedings Volumes, vol. 44, pp. 5531–5536, Jan. 2011.

[6] N. Dhananjay and D. Ghose, “Accurate Time-to-Go Estimation for Proportional Navigation Guidance,” Journal of Guidance, Control, and Dynamics, vol. 37, pp. 1378–1383, July 2014.

[7] S. R. Kumar and D. Ghose, “Sliding mode control based guidance law with impact time constraints,” in 2013 American Control Conference, (Washington, DC), pp. 5760–5765, IEEE, June 2013.

[8] R. Tekin, K. S. Erer, and F. Holzapfel, “Polynomial Shaping of the Look Angle for Impact-Time Control,” Journal of Guidance, Control, and Dynamics, vol. 40, pp. 2668–2673, Oct. 2017.

[9] R. Tsalik and T. Shima, “Circular Impact-Time Guidance,” Journal of Guidance, Control, and Dynamics, vol. 42, pp. 1836–1847, Aug. 2019.

[10] J. I. Lee, I. S. Jeon, and M. J. Tahk, “Guidance law to control impact time and angle,” IEEE Transactions on Aerospace and Electronic Systems, vol. 43, pp. 301–310, Jan. 2007.

[11] T. H. Kim, C. H. Lee, I. S. Jeon, and M. J. Tahk, “Augmented Polynomial Guidance With Impact Time and Angle Constraints,” IEEE Transactions on Aerospace and Electronic Systems, vol. 49, pp. 2806–2817, Oct. 2013.