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MRC COMBINER WITH THREE INPUTS IN THE PRESENCE OF FAST RICE FADING

ANA MATOVIĆ
Military Quality Control, Belgrade, ana.matovic4@gmail.com

MARIJA MATOVIĆ
Electrodistribution, Kraljevo, marija.matovic@es.rs

ELMEDIN BIBEROVIĆ
Telecom Serbia , N.Pazar, elmedin@telekom.rs

Abstract: *This paper analyzes a three-branch maximal-ratio-combining (MRC) receiver operating over independent and identically distributed fast, flat Rician fading channels. The model is stated directly in terms of the instantaneous branch signal-to-noise ratios (SNRs), which removes the ambiguity between Rician and Rayleigh fading in the original formulation. For ideal coherent MRC with perfect channel-state information and equal noise power in all branches, the output SNR is the sum of three independent Rician branch SNRs. This sum is a scaled noncentral chi-square random variable with six degrees of freedom and noncentrality parameter. Closed-form expressions are given for the output-SNR probability density function, cumulative distribution function, moment-generating and characteristic functions, and arbitrary raw moments. These results are then used to evaluate threshold-outage probability, ergodic spectral efficiency, coherent BPSK bit-error probability, noncoherent orthogonal BFSK error probability, and (M)-PSK symbol-error probability. Numerical results, supported by Monte Carlo validation, show the third-order diversity behavior of the three-branch receiver and quantify the reduction in deep-fade probability as the Rician (K)-factor increases at fixed average branch SNR.*

Keywords: *maximal-ratio combining; Rician fading; diversity; noncentral chi-square distribution; outage probability; bit-error probability; ergodic capacity.*

1. INTRODUCTION

Using the statistical characteristics of first-order systems—specifically, the system error probability, signal failure probability, average signal value, and channel capacity—we analyze the performance of a wireless telecommunication system. The failure probability is defined as the probability that the signal falls below a predefined failure threshold. This failure threshold is determined based on the required quality of service [12]. We also determine the channel capacity, i.e., the amount of information that can be transmitted from the transmitter to the receiver in a given time interval. In this study, we calculate the error probability for coherent systems, non-coherent systems, and systems using quadrature modulation [13].

Diversity techniques are commonly used to improve the performance of wireless signal transmission systems in the presence of various types of interference. In mobile telephony, spatial diversity is most often employed, using multiple antennas and combining received signals at the output of a combiner. In this study, we consider a wireless digital telecommunication system that employs maximal ratio combining (MRC) as a diversity technique to reduce the impact of fast Rician fading on system performance [4][5]. In an MRC receiver, the squared magnitude of the combined output signal equals the sum of the squared magnitudes of the signals at each input branch.

Specifically, we analyze an MRC combiner with three input branches under fast Rician fading. The Rician distribution models rapid fluctuations of the signal envelope in a line-of-sight environment [7-12]. A Rician random variable's square is the sum of the squares of two independent Gaussian random variables with equal variance, at least one having a non-zero mean.

In this case, the output of the three-branch MRC combiner is the sum of three squared Rayleigh-distributed random variables (equivalently, the sum of six squared Gaussian variables). As a result, the combined output signal is a random variable. We derive the probability density function (PDF) of the MRC output in equation (11). This PDF is used to compute the system's error probability (equation 24). Using the cumulative distribution function (equation 17) of the MRC output, we determine the probability of system failure (equation 23).

For completeness, we derive the moments of the MRC output signal: the first moment (mean), the second moment (average power, see equation 20), the third moment, and the general n-th moment. Finally, the channel capacity is computed as the expected value of Shannon's capacity expression (equation 22), representing

Corresponding author: A. Matović;
E-mail: ana.matovic4@gmail.com
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the average information transmitted per unit time.

2. MRC COMBINER WITH THREE INPUTS IN THE PRESENCE OF FAST RICE FADING

2.1 Probability density of MRC combiner with three inputs in the presence of fast Rice fading

The probability densities of x_1, x_2 and x_3 are:

$$p_{x_1}(x_1) = \frac{2x_1}{\Omega} e^{-\frac{x_1^2+A^2}{\Omega}} I_0\left(\frac{2Ax_1}{\Omega}\right),\ x_1 \geq 0 \quad (1)$$

$$p_{x_2}(x_2) = \frac{2x_2}{\Omega} e^{-\frac{x_2^2+A^2}{\Omega}} I_0\left(\frac{2Ax_2}{\Omega}\right),\ x_2 \geq 0 \quad (2)$$

$$p_{x_3}(x_3) = \frac{2x_3}{\Omega} e^{-\frac{x_3^2+A^2}{\Omega}} I_0\left(\frac{2Ax_3}{\Omega}\right),\ x_3 \geq 0 \quad (3)$$

The signal x is equal to:

$$x = x_1^2 + x_2^2 + x_3^2 \quad (4)$$

The signal of the variable z is:

$$z = \frac{x}{\sigma^2},\ x = z \cdot \sigma^2 \quad (5)$$

The probability density of z is:

$$p_z(z) = \frac{1}{2}\left(\frac{x}{\lambda}\right)^{\frac{1}{4}} e^{-\frac{\lambda+z}{2}} I_{1/2}\left((z\lambda)^{1/2}\right) \quad (6)$$

where is: $\lambda = \frac{3A^2}{\sigma^2},\ \sigma^2 = \frac{\Omega}{2},\ \lambda = \frac{A^2}{\Omega}$ (7)

The probability density of x is:

$$p_x(x) = \left|\frac{dz}{dy}\right| p_z\left(\frac{x}{\sigma^2}\right) \quad (8)$$

where is:

$$\frac{dz}{dy} = \frac{1}{\sigma^2} \quad (9)$$

By substituting (6) into (9) in (8) we get:

$$p_x(x) = \frac{1}{2\sigma^2\sqrt[4]{3A^2}}(x)^{\frac{1}{4}} e^{-\frac{3A^2+x}{3\sigma^2}} I_{1/2}\left(\frac{A\sqrt{3x}}{\sigma^2}\right) \quad (10)$$

The probability density of x can be written in the form:

$$p_x(x) = \frac{1}{2\sigma^2\sqrt[4]{3A^2}} x^{\frac{1}{4}} e^{-\frac{3A^2+x}{3\sigma^2}} I_{1/2}\left(\frac{A\sqrt{3}\,x^{1/2}}{\sigma^2}\right) =$$

$$\frac{1}{2\sigma^2\sqrt[4]{3A^2}} x^{\frac{1}{4}} e^{-\frac{3A^2+x}{2\sigma^2}} \sum_{i=0}^{\infty}\left(\frac{A\sqrt{3}}{2\sigma^2}\right)^{2i+1/2} \frac{1}{i!\Gamma(i+3/2)} x^{\frac{1}{4}} =$$

$$a\, x^{\frac{1}{4}} e^{-ax} \sum_{i=0}^{\infty} f_3(A,\sigma,i)\ x^{i+\frac{1}{4}} \quad (11)$$

where is:

$$a = \frac{1}{2\sigma^2\sqrt[4]{3A^2}} x^{\frac{1}{4}} e^{-\frac{3A^2}{2\sigma^2}}$$

$$\alpha = \frac{1}{2\sigma^2}$$

$$f_3(A,\sigma,i) = \left(\frac{A\sqrt{3}}{2\sigma^2}\right)^{2i+1/2} \frac{1}{i!\Gamma(i+3/2)} \quad (12)$$

The probability density of x can be written in the form:

$$p(a) = \frac{1}{2\sigma^2\sqrt[4]{3A^2}} x^{\frac{1}{4}} e^{-\frac{3A^2+x}{3\sigma^2}} I_{1/2}\left(\frac{A\sqrt{3}\,x^{1/2}}{\sigma^2}\right) =$$

$$\frac{1}{2\sigma^2\sqrt[4]{3A^2}} x^{\frac{1}{4}} e^{-\frac{3A^2+x}{2\sigma^2}} \sum_{i=0}^{\infty}\left(\frac{A\sqrt{3}}{2\sigma^2}\right)^{2i+1/2} \frac{1}{i!\Gamma(i+3/2)} x^{\frac{1}{4}} =$$

$$a\, x^{\frac{1}{4}} e^{-ax} \sum_{i=0}^{\infty} f_3(A,\sigma,i)\ x^{i+\frac{1}{4}} \quad (13)$$

where is:

$$a = \frac{1}{2\sigma^2\sqrt[4]{3A^2}} x^{\frac{1}{4}} e^{-\frac{3A^2}{2\sigma^2}}, \alpha = \frac{1}{2\sigma^2} \quad (14)$$

$$f_3(A,\sigma,i) = \left(\frac{A\sqrt{3}}{2\sigma^2}\right)^{2i+1/2} \frac{1}{i!\Gamma(i+3/2)} \quad (15)$$

The characteristic function of x is:

$$\mu_x(s) = \overline{e^{xs}} = \int_0^{\infty} e^{xs} f_x(x)\,dx =$$

$$\int_0^{\infty} e^{xs}\, a\, x^{\frac{1}{4}} e^{-ax} \sum_{i=0}^{\infty} f_3(A,\sigma,i)\ x^{i+\frac{1}{4}} dx =$$

$$a\sum_{i=0}^{\infty} f_3(A,\sigma,i) \int_0^{\infty} \sum_{r=0}^{\infty} \frac{s^k x^k}{k!}\, x^{\frac{1}{4}} e^{-ax}\, x^{i+\frac{1}{4}} dx =$$

$$a\sum_{i=0}^{\infty} f_3(A,\sigma,i) \sum_{r=0}^{\infty} \frac{s^k}{k!} \int_0^{\infty} x^{k+i+\frac{1}{2}} e^{-ax} dx =$$

$$a\sum_{i=0}^{\infty} f_3(A,\sigma,i) \sum_{r=0}^{\infty} \frac{s^k}{k!} \left(\frac{1}{a}\right)^{k+i+\frac{1}{2}} \Gamma(k+i+3/2) \quad (16)$$

2.2 Cumulative probability of MRC combiner with three inputs in the presence of fast Rice fading

The cumulative probability of a random variable x is:

$$F_x(x) = \int_0^x dt\, f_x(t) =$$

$$\frac{1}{2\sigma^2\sqrt[4]{3A^2}} e^{-\frac{3A^2}{2\sigma^2}} \sum_{i=0}^{\infty}\left(\frac{A\sqrt{3x}}{\sigma^2}\right)^{2i+1/2} \cdot$$

$$\frac{1}{i!\Gamma(i+3/2)} (2\sigma^2)^{i+3/2}\Gamma\left(i+3/2, \frac{x}{\sigma^2}\right) \quad (17)$$

2.3 Moments of MRC combiner with three inputs in the presence of fast Rice fading

The n-ti order moment of x is:

$$m_n = \overline{x^n} = \int_0^{\infty} x^n p_x(x)\,dx =$$

$$\int_0^{\infty} x^n\, a\, x^{\frac{1}{4}} e^{-ax} \sum_{i=0}^{\infty} f_3(A,\sigma,i)\ x^{i+\frac{1}{4}} dx =$$

$$a\sum_{i=0}^{\infty} f_3(A,\sigma,i) \int_0^{\infty} x^{n+i+\frac{1}{2}} e^{-ax} dx =$$

$$a\sum_{i=0}^{\infty} f_3(A,\sigma,i)\ \left(\frac{1}{a}\right)^{n+i+\frac{1}{2}} \Gamma(n+i+3/2) \quad (18)$$

For n=1, we get the first moment, that is, the mean value of *x*:

$$m_1 = \bar{x} = \int_0^\infty x\, p_x(x)\, dx =$$

$$a \sum_{i=0}^{\infty} f_3(A,\sigma,i) \left(\frac{1}{\alpha}\right)^{i+\frac{5}{2}} \Gamma\left(i+{}^5/_2\right) \quad (19)$$

For n=2, we get the second moment, or the mean square value of *x*:

$$m_2 = \overline{x^2} = \int_0^\infty x^2 p_x(x)\, dx =$$

$$a \sum_{i=0}^{\infty} f_3(A,\sigma,i) \left(\frac{1}{\alpha}\right)^{i+\frac{7}{2}} \Gamma\left(i+{}^7/_2\right) \quad (20)$$

For n=3, we get the third moment, or the average cubic value of *x*:

$$m_3 = \overline{x^3} = \int_0^\infty x^3 p_x(x)\, dx =$$

$$a \sum_{i=0}^{\infty} f_3(A,\sigma,i) \left(\frac{1}{\alpha}\right)^{i+\frac{7}{2}} \Gamma\left(i+{}^7/_2\right) -$$

$$\left(a \sum_{i=0}^{\infty} f_3(A,\sigma,i) \left(\frac{1}{\alpha}\right)^{i+\frac{5}{2}} \Gamma\left(i+{}^5/_2\right) \right)^2 \quad (21)$$

2.4 Capacity of the channel MRC combiner with three inputs in the presence of fast Rice fading

Channel capacity:

$$c = \int_0^\infty \left(ln(1+x^2)\right) p_x(x)\, dx = \int_0^\infty \left(ln(1+x^2)\right) a\, x^{\frac{1}{4}}\, e^{-\alpha x}$$

$$\sum_{i=0}^{\infty} f_3(A,\sigma,i)\, x^{i+\frac{1}{2}} dx =$$

$$\sum_{i=0}^{\infty} f_3(A,\sigma,i) \int_0^\infty x^{i+\frac{1}{2}}\, e^{-\alpha x} \left(ln(1+x^2)\right) dx \quad (22)$$

2.5 Probability of signal failure of MRC combiner with three inputs in the presence of fast Rice fading

The probability of signal failure is:

$$p_0 = \int_0^{x_0} p_x(x)\, dx = F_x(x_0) \quad (23)$$

2.6 Error probability of systems of MRC combiner with three inputs in the presence of fast Rice fading

Error probability for coherent systems:

$$p_{e_1} = \int_0^\infty a_1 (erfc(bx^2)) p_x(x)\, dx = \int_0^\infty a_1 (erfc(bx^2))$$

$$a\, x^{\frac{1}{4}}\, e^{-\alpha x} \sum_{i=0}^{\infty} f_3(A,\sigma,i)\, x^{i+\frac{1}{2}} dx =$$

$$a_1 a \sum_{i=0}^{\infty} f_3(A,\sigma,i) \int_0^\infty (erfc(bx^2))\, x^{i+\frac{i}{2}}\, e^{-\alpha x} dx \quad (24)$$

Error probability for incoherent systems:

$$p_{e_2} = \int_0^\infty a_1 e^{bx^2} p_x(x)\, dx = \int_0^\infty a_1 e^{bx^2}$$

$$a\, x^{\frac{1}{4}}\, e^{-\alpha x} \sum_{i=0}^{\infty} f_3(A,\sigma,i)\, x^{i+\frac{1}{4}} dx =$$

$$a_1 a \sum_{i=0}^{\infty} f_3(A,\sigma,i) \int_0^\infty x^{i+\frac{i}{2}}\, e^{-\alpha x}\, e^{bx^2} dx \quad (25)$$

Error probability for quadrature modulated signals is:

$$p_{e_2} = \int_0^\infty dx\, (a_1 erfc(b_1 x^2) + a_2 (erfc(b_2 x^2)) p_x(x) =$$

$$\int_0^\infty dx\, (a_1 erfc(b_1 x^2) + a_2 (erfc(b_2 x^2))$$

$$a\, x^{\frac{1}{4}}\, e^{-\alpha x} \sum_{i=0}^{\infty} f_3(A,\sigma,i)\, x^{i+\frac{1}{4}} \quad (26)$$

3. NUMERICAL RESULTS

Figure 1 shows the dependence of the probability density at the output of an MRC combiner with three inputs *p(x)* on the signal envelope *x*. For smaller values of the signal envelope *x*, the probability density *p(x)* increases, while for larger values of the signal envelope *x*, the probability density *p(x)* decreases. For a signal envelope *x* value from 1 to 7, the probability density *p(x)* increases from 0.001 to 0.09, while for a signal envelope *x* value from 7 to 20, the probability density *p(x)* decreases from 0.09 to 0.01.

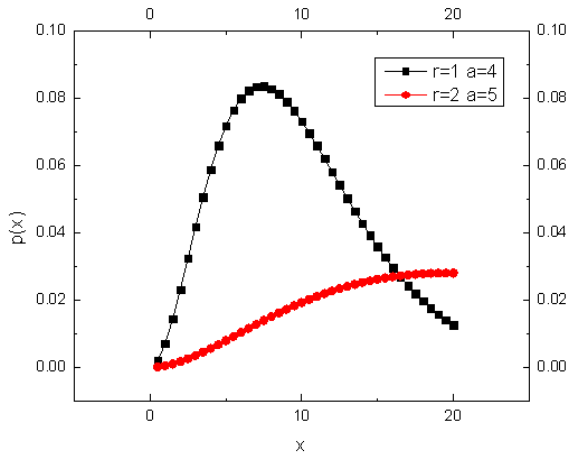


Figure 1. Dependence of the probability density at the output of the MRC combiner with three inputs on the parameter *x*

Figure 2 shows the dependence of the probability density at the output of an MRC combiner with three inputs on the parameter. For smaller values of the parameter, the probability density increases, while for larger values of the parameter, the probability density decreases. For parameter values from 0 to 5, the probability density rises from 0.048 to 0.078, while for parameter values from 5 to 10, the probability density decreases from 0.078 to 0.052.

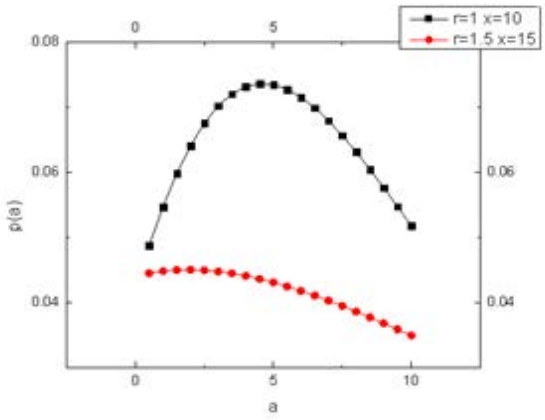


Figure 2. Dependence of the probability density at the output of an MRC combiner with three inputs on the parameter

Figure 3 shows the dependence of the mean square value of the signal at the output of the MRC combiner with three inputs, m_2, on the parameter a. For higher values of the parameter a, the mean square value of the signal m_2 increases. For a parameter a ranging from 0 to 10, the mean square value of the signal m_2 rises from 50 to 250.

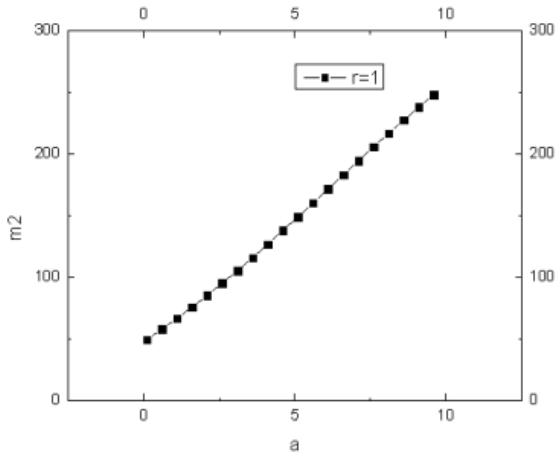


Figure 3. Dependence of the mean square value of the signal at the output of the MRC combiner with three inputs m_2 on the parameter a

4. CONCLUSION

This paper investigates an MRC combiner with three inputs in the presence of fast Rician fading. The probability density function and cumulative distribution function of the signal at the output of the three-input MRC combiner, as well as its first, second, and third moments, were derived under fast Rician fading conditions. Based on these results, the first-order statistical performance characteristics of the system were determined, including the system error probability, signal failure probability, average signal value, and channel capacity. Using the derived analytical expressions, the performance of the wireless communication system was analyzed, demonstrating the applicability of the MRC diversity technique for reducing the adverse effects of fast fading on system performance.

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